PETROV TYPE D ISOLATED NULL SURFACES

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Introduction

The theory of isolated null surfaces is a part of the local approach to black holes (BH) [1-3]. Intrinsic geometry of those surfaces, that is the induced metric and the induced covariant derivative, has physically relevant features of the geometry of stationary black hole horizons. The theory is applicable to cosmological horizons and to the null boundaries of the conformally compactified asymptotically flat spacetimes. Moreover, it may be applied to the black hole holograph construction of spacetimes about Killing horizons. There are analogies between the properties of isolated null surfaces and the properties of BHs. Isolated null surfaces admit their mechanics, an analog of the BH "thermodynamics" [3], the rigidity theorem, and the uniqueness theorems. The key difference between the BHs and the isolated null surfaces lies in the degrees of freedom. While the families of stationary BH solutions are finite dimensional, the intrinsic geometry of isolated null surfaces has local degrees of freedom. That makes their theory far more general. We develop an observation that at each non-extremal isolated null surface the spacetime Weyl tensor can be determined by the intrinsic geometry via the Einstein equations and some stationarity assumption. We focus on the case of the Petrov type D, derive and investigate the type D equation on data defined on a 2-slice of an isolated null surface: a Riemannian metric g_{AB} and a co-vector ω_A modulo gradient.

Isolated null surfaces

Consider an n-dimensional spacetime (M, g) of the signature $-+...+$. Suppose H is a non-expanding, shear-free null surface of the co-dimension 1 embedded in M. Then, the degenerate metric tensor g_{ab} induced in H satisfies

$$\mathcal{L}_\ell g_{ab} \;=\; 0 \;=\ell^a g_{ab} \tag{1}$$

for every null vector field ℓ defined on and tangent to H. Moreover, the spacetime covariant derivative ∇_α (metric and torsion free) preserves the tangent bundle of H and via the restriction endows it with a torsion free covariant derivative ∇_a. The derivative satisfies the pseudo-metricity condition

$$\nabla_c g_{ab} \;=\; 0, \tag{2}$$

however it is not sufficient to determine ∇_a. The pair (g_{ab}, ∇_a) sets the intrinsic geometry of H. Moreover, for every null vector field ℓ tangent to H, there is a 1-form $\omega^{(\ell)}$ such that

$$\nabla_a \ell \;=\; \omega^{(\ell)}_a \ell. \tag{3}$$

In particular

$$\ell^a \nabla_a \ell^b \;=\; \kappa^{(\ell)} \ell^b, \qquad \kappa^{(\ell)} \;=\; \omega^{(\ell)}_a \ell^a, \tag{4}$$

hence $\kappa^{(\ell)}$ is a self acceleration of ℓ. Surface H admitting a null vector ℓ such that

$$[\mathcal{L}_\ell, \nabla_a] \;=\; 0 \tag{5}$$

is called isolated. We focus on isolated null surfaces (H, ℓ) diffeomorphic to $\mathbb{R}\times S$, where S is a compact manifold, and such that every slice of H corresponding to S is spacelike (such surfaces are often referred to as isolated horizons). In that case the degenerate metric tensor g_{ab} and the rotation 2-form invariant Ω_{ab} are determined by a (Riemannian) metric tensor g_{AB} and an exact 2-form Ω_{AB} defined on the quotient manifold S via a pullback of the map $\Pi : H \mapsto S$

$$\Pi : g_{ab} \;=\; \Pi^*{}_{ab}{}^{AB} g_{AB}, \qquad \Omega_{ab} \;=\; \Pi^*{}_{ab}{}^{AB}\Omega_{AB}. \tag{6}$$

The Petrov type D equation

The Petrov type D condition can be expressed by the invariant

$$\Psi \;=\; -\frac{1}{2}(\,K \;+\; i\Omega\,), \tag{7}$$

where K is the Gaussian curvature and pseudo-scalar Ω characterizes the rotation 2-form Ω_{AB} in terms of the area 2-from η_{AB}:

$$\Omega_{AB} = \Omega\eta_{AB}. \tag{8}$$

The metric tensor is represented by a complex null 2-co-frame $(\bar{m}_A dx^A, m_A dx^A)$,

$$g_{AB} \;=\; m_A\bar{m}_B \;+\; \bar{m}_A m_B.$$

Suppose H is a 3-dimensional non-extremal isolated null surface stationary to the second order in a 4-dimensional spacetime such that the vacuum Einstein equations with cosmological constant Λ are satisfied. Then, the necessary and sufficient condition for the spacetime Weyl tensor to be of the Petrov type D at each point of the null geodesic $x \in S$ is that the invariant Ψ (7) satisfies the following two conditions [4]:

$$\Psi(x) \;\neq\; -\frac{\Lambda}{6}, \tag{9}$$

and

$$\bar{m}^A\bar{m}^B\nabla_B\nabla_A\left(\Psi(x)+\frac{\Lambda}{6}\right)^{-\frac{1}{3}} = 0. \tag{10}$$

Local no-hair theorem

On a topological 2-sphere S, every axisymmetric solution g_{AB} and Ω_{AB} to the Petrov type D equation (10) with a cosmological constant Λ is uniquely determined by a pair of numbers: the area A, and the angular momentum J [4, 5]. The range of the pairs (A, J) corresponding to the solutions depends on the cosmological constant as follows:

- for $\Lambda > 0$, $J \in (-\infty, \infty)$ for $A \in \left(0, \frac{12\pi}{\Lambda}\right)$ and $|\,J\,| \in \left[0, \frac{A}{8\pi}\sqrt{\frac{\Lambda A}{12\pi}-1}\right)$ for $A \in \left(\frac{12\pi}{\Lambda}, \infty\right)$;
- for $\Lambda \leq 0$, $J \in (-\infty, \infty)$ and $A \in (0, \infty)$.

Every solution defines a type D isolated horizon whose intrinsic geometry (g_{ab}, ∇_a) coincides with the intrinsic geometry of a non-extremal Killing horizon contained in one of the following (locally defined) spacetimes:

(i) the generalized Kerr - (anti) de Sitter spacetime,

(ii) the generalized Schwarzschild - (anti) de Sitter spacetime,

(iii) the near horizon limit spacetime near an extremal horizon contained either in the generalized Kerr - (anti) de Sitter spacetime or in the generalized Schwarzschild - (anti) de Sitter spacetime.

Solutions to the Petrov type D equation for genus ≥ 1 sections of IHs

A pair (g, ω) is a solution to the Petrov type D equation with a cosmological constant Λ on a compact, orientable 2-surface of genus ≥ 1 if and only if g has constant Gauss curvature (Ricci scalar)

$$K \;=\; \text{const} \neq \frac{\Lambda}{3}$$

and ω is closed

$$d\omega \;=\; 0.$$

The corresponding isolated horizons are non-rotating, meaning that their angular momentum $J = 0$. It follows that every rotating Petrov type D isolated horizon stationary to the second order and contained in a 4-dimensional spacetime that satisfies the vacuum Einstein equations with possibly non-zero cosmological constant, has spacelike section of the topology of a 2-sphere. In other words, for spacelike sections of genus > 0 there are no rotating Petrov type D isolated horizons stationary to the second order and contained in a 4-dimensional spacetime satisfying the vacuum Einstein equations with possibly non-zero cosmological constant [6].

The Petrov type D equation and the near horizon geometry equation

Suppose a differential 1-form ω_A and a Riemannian metric tensor

$$g_{AB} \;= m_A\bar{m}_B \;+\; \bar{m}_A m_B,$$

both defined on a 2-dimensional manifold S satisfy the following (near horizon geometry) equation:

$$\nabla_{(A}\omega_{B)} \;+\; \omega_A\omega_B \;-\; \frac{1}{2}R_{AB} \;+\; \frac{1}{2}\Lambda g_{AB} \;=\; 0, \tag{11}$$

with a constant Λ. Then, the invariant Ψ defined by (7) with

$$\Omega_{AB} \;:=\; \partial_A\omega_B - \partial_B\omega_A$$

satisfies the equation

$$\bar{m}^A\bar{m}^B\nabla_A\nabla_B\left(\Psi(x)+\frac{\Lambda}{6}\right)^{-\frac{1}{3}} = 0. \tag{12}$$

This result may be applied just as an integrability condition for the near horizon geometry equation to investigate the space of solutions [4].

References and acknowledgments

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