Testing loop quantum cosmology

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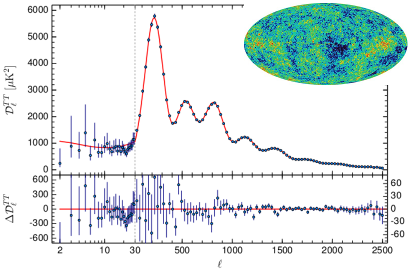
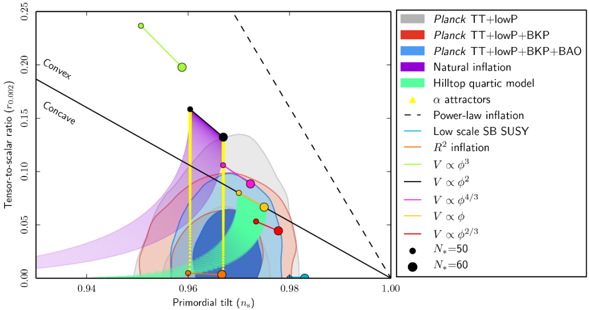
Loops '17

The Cosmic Microwave Background

High precision observations of the cosmic microwave background (CMB) have taught us a lot about the early universe.





[Planck2015+BICEP2/Keck]

In fact, it has been possible to rule out a number of cosmological models, including some of the simplest models for inflation, as well as alternatives to inflation.

Could these observations be used to constrain theories of quantum gravity?

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A major difficulty for any theory of quantum gravity is to obtain predictions that can be confronted to experiment or observations.

One of the best opportunities in this direction lies in the very early universe, where quantum gravity effects are expected to be strong and may have left some imprints on the CMB. What form could these imprints have?

I will consider this question coming from the perspective of loop quantum cosmology (LQC).

Loop Quantum Cosmology

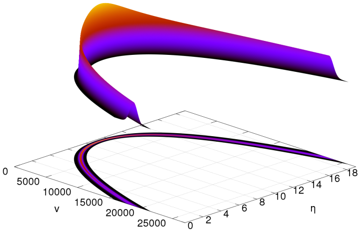
In homogeneous loop quantum cosmology, one first imposes the symmetries of the cosmological space-time of interest, then performs a loop quantization of the reduced system.

For homogeneous and isotropic space-times, the gravitational phase space becomes two-dimensional. The Hamiltonian constraint also simplifies

$$\mathcal{C}_H = -\frac{E^a_i E^b_j}{16\pi G\gamma^2\sqrt{q}}\,\epsilon_{ij}{}^k F_{ab}{}^k + \mathcal{H}_{\rm matter}, \qquad \text{for } k = 0.$$

Then, $\mathcal{C}_H$ is expressed in terms of areas and holonomies—in particular, the field strength is expressed as the holonomy of the Ashtekar-Barbero connection around a loop of minimal area—which is then promoted to an operator [Bojowald; Ashtekar, Bojowald, Lewandowski; Ashtekar, Pawłowski, Singh; . . .].

Singularity Resolution



[Pawłowski, Pierini, WE]

The quantum dynamics can be solved numerically (and in some cases analytically). An important result is that the big-bang and big-crunch singularities are generically resolved and replaced by a bounce [Ashtekar, Pawłowski, Singh; Ashtekar, Corichi, Singh, ...].

For sharply-peaked states, an 'effective Friedmann equation' provides an excellent approximation to the dynamics of the expectation value of the scale factor, [Ashtekar, Pawłowski, Singh; Taveras; Rovelli, WE, ...]

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho\left(1 - \frac{\rho}{\rho_c}\right).$$

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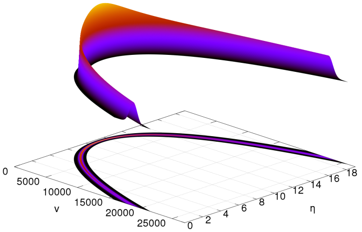

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Quantum gravity effects are important at the bounce. Could they leave an imprint on the CMB? Can LQC be tested via observations?

Relation to Loop Quantum Gravity

LQC is not derived from LQG, but rather motivated by it. For this reason, it is important to better understand the relation between the two theories:

- Relating the kinematical Hilbert spaces of LQG & LQC [Engle, Fleischhack, Hanusch, Thiemann; Beetle, Engle, Hogan, Mendonca; ...],
- Spin foam cosmology [Bianchi, Krajewski, Rovelli, Vidotto; Rennert, Sloan; ...],
- Group field theory condensate cosmology [Gielen, Oriti, Sindoni, WE; de Cesare, Pithis, Sakellariadou; ...],
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Despite remaining open questions, it's important to understand how to test LQC as these tests can presumably later be adapted to full LQG. If there are any additional quantum gravity effects not captured by LQC, these can be added in later studies.

Cosmological Observables

In cosmological perturbation theory, one studies small departures from homogeneity in the metric and the matter field. The temperature anisotropies observed in the CMB can be related to perturbations $\mathcal{R}$ in the spatial curvature ${}^{(3)}R$. Its power spectrum

$$\Delta^2_{\mathcal{R}}(k) = \frac{k^3}{2\pi^2}|\mathcal{R}_k|^2 = A_s \left(\frac{k}{k_\star}\right)^{n_s-1},$$

has been measured and is parametrized in terms of its amplitude A_s and the scalar index n_s. Scale-invariance means that $\Delta^2_{\mathcal{R}}(k) \sim k^0$, which implies $\mathcal{R}_k \sim k^{-3/2}$ and $n_s = 1$.

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Observations indicate that [Planck2015+BICEP2/Keck]

$$A_s = (2.14 \pm 0.05) \times 10^{-9} \quad (68\%), \qquad n_s = 0.968 \pm 0.006 \quad (68\%),$$

$$r = \frac{A_t}{A_s} < 0.09 \quad (95\%).$$

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These frameworks rely on different approximations/truncations, and so their regimes of validity will be different.

Predictions of LQC Effects

It is important to keep in mind that the dynamics of LQC (just like general relativity) depend on the matter content. As a result, the predictions of LQC will strongly depend on what the dominant matter field (radiation, inflaton, ekpyrotic field, . . .) is during the bounce.

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There are three main cosmological scenarios—each of which predict a nearly scale-invariant spectrum of primordial perturbations, as observed—that have been considered in LQC in order to determine whether it may be possible to test LQC via observations of the CMB:

- **Inflation**,
- **Matter bounce**,
- **Ekpyrosis**.

Inflation

In slow-roll inflation, the universe expands in a near-exponential fashion, typically driven by a scalar field ϕ slowly rolling down its potential $V(\phi)$. Importantly, during inflation quantum vacuum fluctuations in perturbations $\mathcal{R}$ of the curvature become nearly scale-invariant.

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One difficulty is that for a sufficient amount of inflation to occur, ϕ must start quite far away from the minimum of the potential. This difficulty vanishes in LQC, since the pre-bounce dynamics give a lot of kinetic energy to ϕ which will then typically be driven far from its minimal energy [Ashtekar, Sloan; Corichi, Karami; Linsefors, Barrau; ...].

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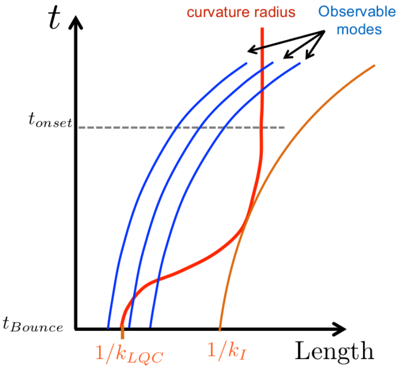
In addition, the fact that ϕ is kinetic-dominated during the bounce is important: any LQC effects in inflation will be independent of the choice of $V(\phi)$; see, e.g., studies of $V = \frac{1}{2}m^2\phi^2$ [Agulló, Ashtekar, Nelson] and potentials with a plateau at large ϕ [Bonga, Gupt].

LQC Effects in Inflation

Observable effects can arise due to perturbations feeling the LQC modifications to the background dynamics.

Specifically, long-wavelength modes will be excited by the LQC-corrected space-time curvature during the bounce.



[Agulló]

If there is a 'minimal' amount of inflation ($\sim$ 60 e-folds), then it is possible for the LQC bounce to leave an imprint on the CMB at large scales. (If there is more inflation, these effects will be outside the horizon.)

Challenges

There is some freedom in the **choice of the vacuum state**. Different vacua have been proposed in the context of LQC:

- Adiabatic vacuum which approaches the Minkowski vacuum at a polynomial rate as the wavelength becomes much smaller than the curvature radius [Agulló, Ashtekar, Nelson],
- Vacuum that minimizes the variation in the amplitude of the perturbations during the bounce [Martín-de Blas, Olmedo],
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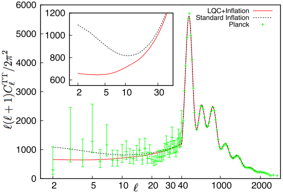
There is also a **trans-Planckian problem**: perturbations with a wavelength smaller than $\ell_{\rm Pl}$ should feel the quantum discreteness. How should this be taken into account? Do modes with $\lambda < \ell_{\rm Pl}$ exist?

Some Results from the Hybrid Quantization

Considering different vacua for long-wavelength modes:

For the minimal oscillations vacuum and for the vacuum motivated by Penrose's hypothesis, there is less power at large scales, which improves the fit to observations [Martín-de Blas, Olmedo; Ashtekar, Gupt]. Same effect predicted for polarization.



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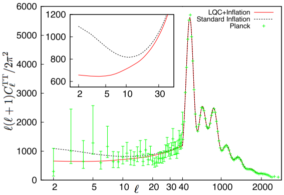

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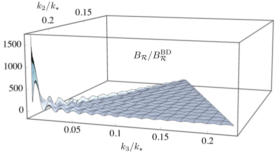
For an adiabatic vacuum, non-Gaussianities at large scales are enhanced compared to the standard inflationary predictions, this could be tested by future observations [Agulló].



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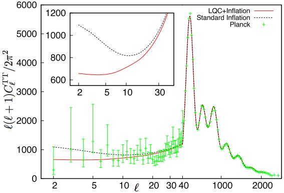

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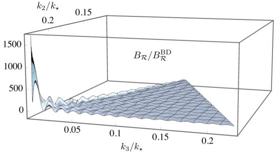
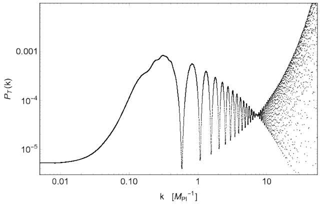


[Agulló]

⇒ There could be observational signatures if the universe does not expand by too many e-folds during inflation.

Results from Anomaly-Free Effective Equations

For inflation, the anomaly-freedom framework for perturbations in LQC predicts a large amplification of perturbations at short scales. This effect is ruled out by observations [Bolliet, Grain, Stahl, Linsefors, Barrau; Bolliet, Barrau, Grain, Schander].



[Bolliet, Barrau, Grain, Schander]

This is because these effective equations break down when quantum fluctuations are important [Bojowald; Rovelli, WE; ...], and quantum fluctuations are large for short-wavelength modes during and following the bounce.

The effective equations should only be used for long-wavelength modes, their predictions cannot be trusted (and in fact are ruled out for inflation) when applied to short-wavelength modes.

Matter Bounce

In a contracting matter-dominated universe (i.e., $P = 0$), quantum vacuum perturbations become scale-invariant and if the contracting phase can be joined to our expanding universe via a bounce, these scale-invariant perturbations can explain the CMB [Wands; Finelli, Brandenberger].

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Importantly, it is possible to differentiate between the matter bounce scenario and inflationary models by measuring the running of the scalar index $\alpha_s = \mathrm{d}n_s/\mathrm{d}\ln k$, since the matter bounce typically predicts $\alpha_s > 0$ while inflation typically predicts $\alpha_s < 0$ [Lehners, WE].

In fact, observational constraints on α_s can be used to rule out some realizations of the matter bounce [Cai, Marcianò, Wang, WE].

One of the challenges facing the matter bounce scenario is that anisotropies grow in a contracting phase, and this effect could change the predictions [Levy].

LQC Predictions for Matter Bounce

Clearly, LQC can provide the bounce that is required. But do the perturbations safely travel the bounce, or is their spectrum modified by the bounce?

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Using the effective equations coming from the loop quantization of long-wavelength perturbations, it is possible to explicitly calculate the evolution of the perturbations through an isotropic bounce, with the result that they remain scale-invariant [WE].

In addition, LQC effects during the bounce suppress the tensor-to-scalar ratio for an equation of state $\omega = P/\rho$ in the range $-1/3 < \omega < 1$ [WE],

$$r_+ = \frac{4\,\omega^2}{(1+\omega)^2} r_- \quad \text{for } -1 < \omega \leq 1, \qquad r_+ = r_- \quad \text{for } \omega \geq 1.$$

This effect is a potential observational test for LQC and its amplitude depends on ω during the bounce.

Ekpyrosis

A contracting space-time with an ultrastiff fluid ($P > \rho$) generates scale-invariant entropy perturbations from vacuum fluctuations, which in turn can source scale-invariant curvature perturbations [Khoury, Ovrut, Steinhardt, Turok].

Such a scenario of course requires a bounce to go from the contracting space-time to an expanding one; LQC automatically provides the bounce to satisfy this requirement [Cailleteau, Singh, Vandersloot].

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The evolution of the perturbations through the LQC bounce can be calculated analytically, showing they remain scale-invariant [WE].

However, while this provides a viable cosmological scenario, there seems to be no imprint from LQC in observable quantities that could differentiate LQC from other types of bounces.

Conclusions

In loop quantum cosmology quantum gravity effects resolve the big-bang singularity and replace it by a bounce.

Using perturbation theory, one can look for quantum gravity effects that could leave an imprint in the CMB. In LQC, some potential predictions include:

- Suppression of power at large scales in inflation,
- Amplification of non-Gaussianities at large scales in inflation,
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- Initial conditions for perturbations in inflation,
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Thank you for your attention!