Group field theory: where from, where to

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Max Planck Institute for Gravitational Physics
(Albert Einstein Institute)

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MAX-PLANCK-GESELLSCHAFT

Plan of the talk

- GFTs : what are they?
    - general formalism
        - GFTs and tensor models
        - GFT and LQG
        - GFT and spin foam models

- current research directions
    - foundations of the GFT formalism
    - GFT renormalization
    - physical applications
        - GFT condensate cosmology
        - horizon entropy via GFT methods
        - entanglement, tensor networks and holography

Part I:
the GFT formalism

a) basic QFT elements, combinatorics
and relation to tensor models

Group field theories

(Boulatov, Ooguri, De Pietri, Freidel, Krasnov, Rovelli, Perez, DO, Livine, Baratin, ……)

QFT of spacetime, not defined on spacetime

a QFT for the building blocks of (quantum) space

Quantum field theories over group manifold G (or corresponding Lie algebra) $\varphi : G^{\times d} \rightarrow \mathbb{C}$

relevant classical phase space for "GFT quanta": $(\mathcal{T}^* G)^{\times d} \simeq (\mathfrak{g} \times G)^{\times d}$

""one-particle" Hilbert space $\mathcal{H}_v = L^2 \left(G^d; d\mu_{Haar}\right)$

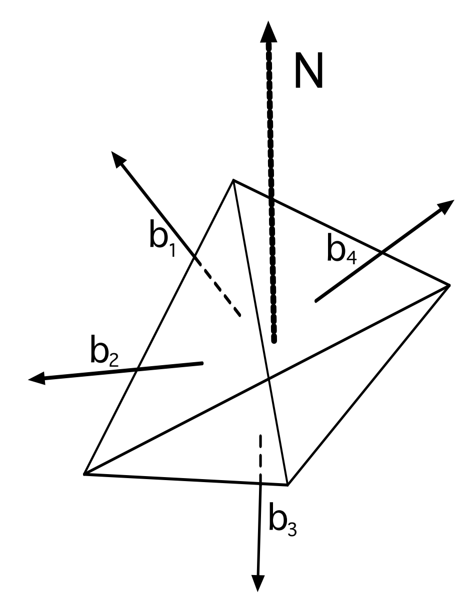
can reduce to subspaces in specific models depending on conditions on the field

d is dimension of "spacetime-to-be"; for gravity models, G = local gauge group of gravity (e.g. Lorentz group)

example: d=4 $\varphi(g_1, g_2, g_3, g_4) \leftrightarrow \varphi(b_1, b_2, b_3, b_4) \rightarrow \mathbb{C}$

arguments of GFT field: $b_i \in \mathfrak{su}(2) \quad g_i \in SU(2)$

e.g. discretised topological SU(2) BF variables (B-field and connection)



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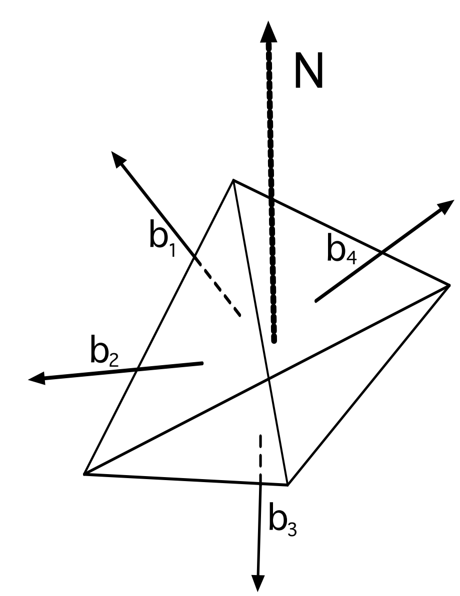

very general framework; interest rests on specific models/use
(most interesting QG models are for Lorentz group in 4d)

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$$\mathcal{F}(\mathcal{H}_v) = \bigoplus_{V=0}^{\infty} sym\left\{\left(\mathcal{H}_v^{(1)} \otimes \mathcal{H}_v^{(2)} \otimes \cdots \otimes \mathcal{H}_v^{(V)}\right)\right\}$$

boson statistics is -assumption-
(can construct, e.g., fermionic models)

Fock vacuum: “no-space” (“emptiest”) state $|\,0>$

single field “quantum”: spin network vertex or tetrahedron
(“building block of space”)

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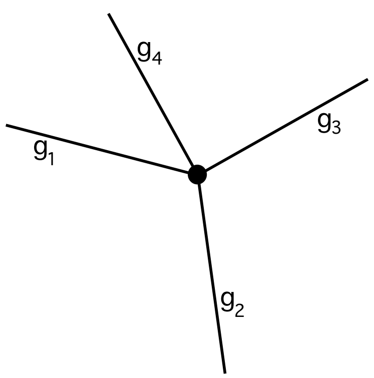
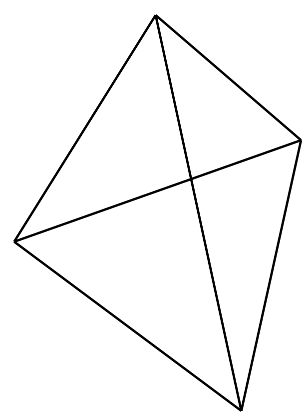
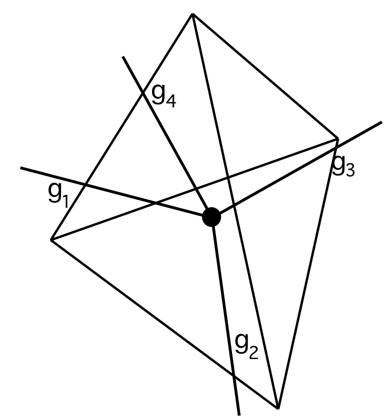
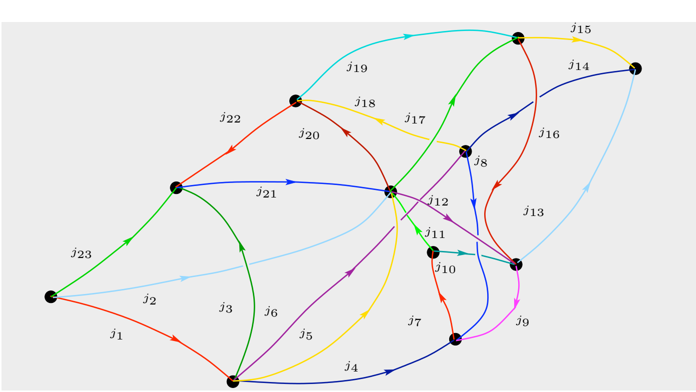


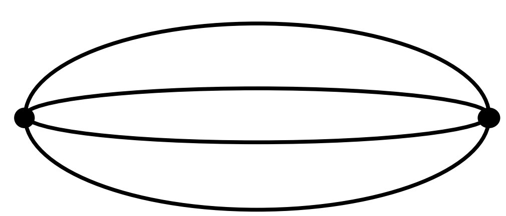


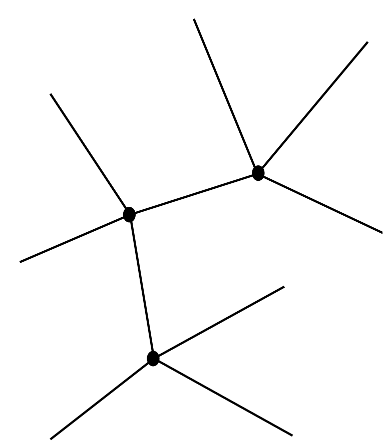


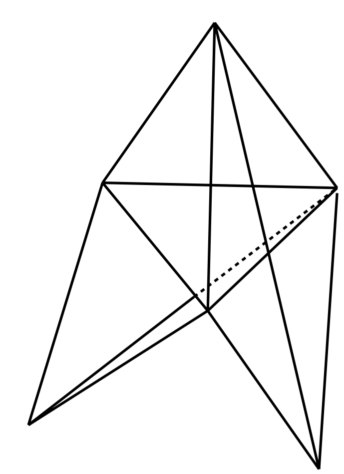
(d=4)

generic quantum state: arbitrary collection of spin network vertices (including glued ones) or tetrahedra (including glued ones)









Group field theories

a QFT for the building blocks of (quantum) space

classical action: kinetic (quadratic) term + (higher order) interaction (convolution of GFT fields)

$$S(\varphi,\overline{\varphi}) = \frac{1}{2}\int [dg_i]\overline{\varphi(g_i)}\mathcal{K}(g_i)\varphi(g_i) + \frac{\lambda}{D!}\int [dg_{ia}]\varphi(g_{i1})....\varphi(\bar{g}_{iD})\mathcal{V}(g_{ia},\bar{g}_{iD}) \quad + \quad c.c.$$

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"combinatorial non-locality"
in pairing of field arguments

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specific combinatorics depends on model

simplest example (case d=4): simplicial setting

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combinatorics of field arguments in interaction: gluing of 5 tetrahedra across common triangles, to form 4-simplex ("building block of spacetime")

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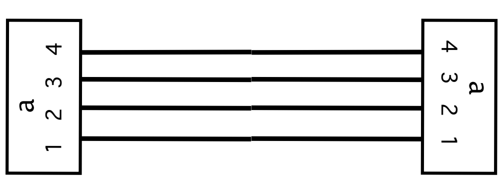
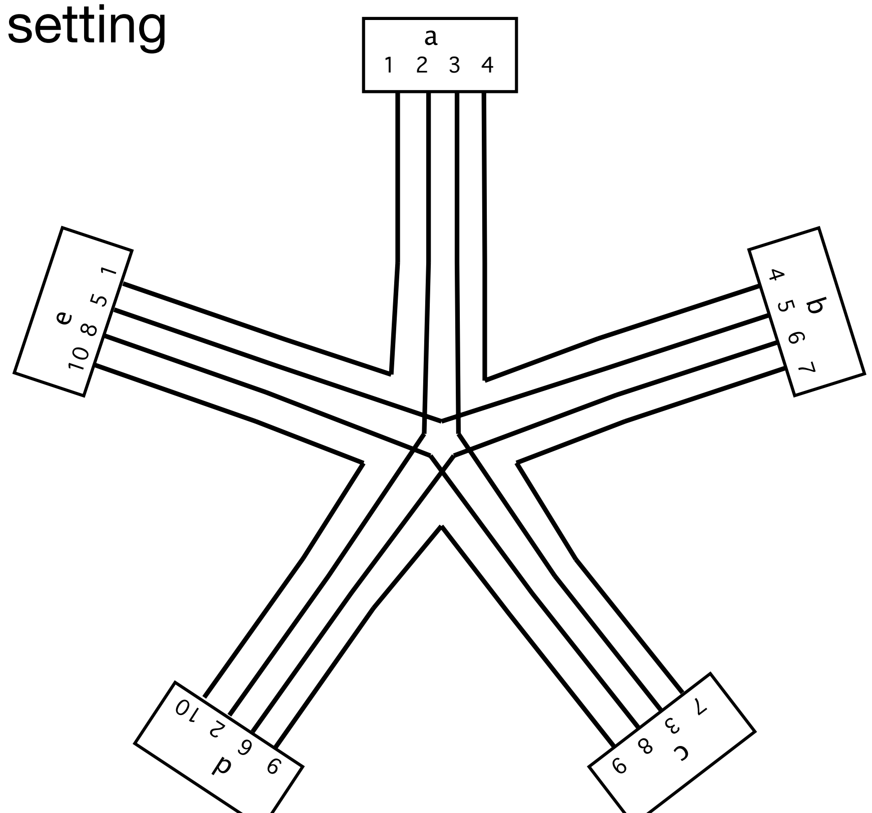
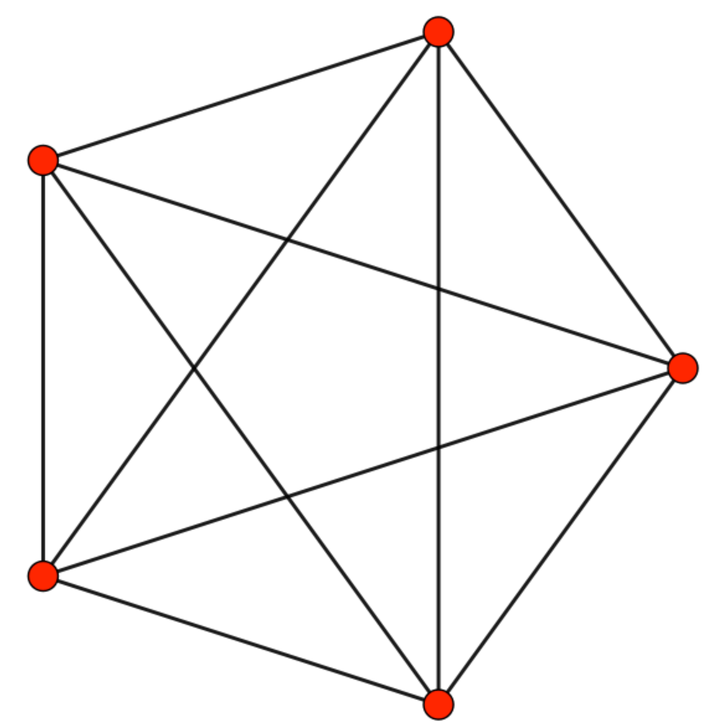






Group field theories

a QFT for the building blocks of (quantum) space

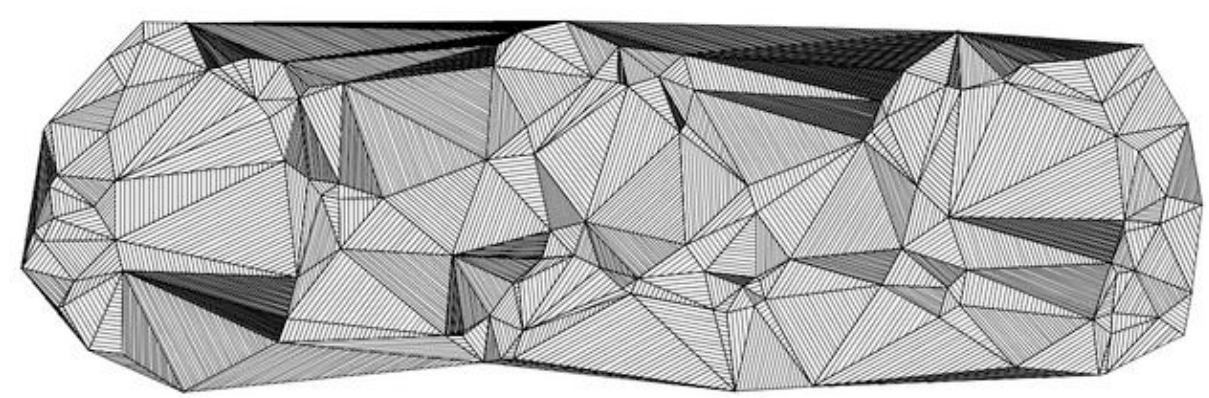
Feynman perturbative expansion around trivial vacuum

$$\mathcal{Z} = \int \mathcal{D}\varphi \mathcal{D}\overline{\varphi}\, e^{i\, S_\lambda(\varphi,\overline{\varphi})} \quad = \quad \sum_\Gamma \frac{\lambda^{N_\Gamma}}{sym(\Gamma)}\, \mathcal{A}_\Gamma$$

Feynman diagrams (obtained by convoluting propagators with interaction kernels) =

= stranded diagrams dual to cellular complexes of arbitrary topology

(simplicial case: simplicial complexes obtained by gluing d-simplices)



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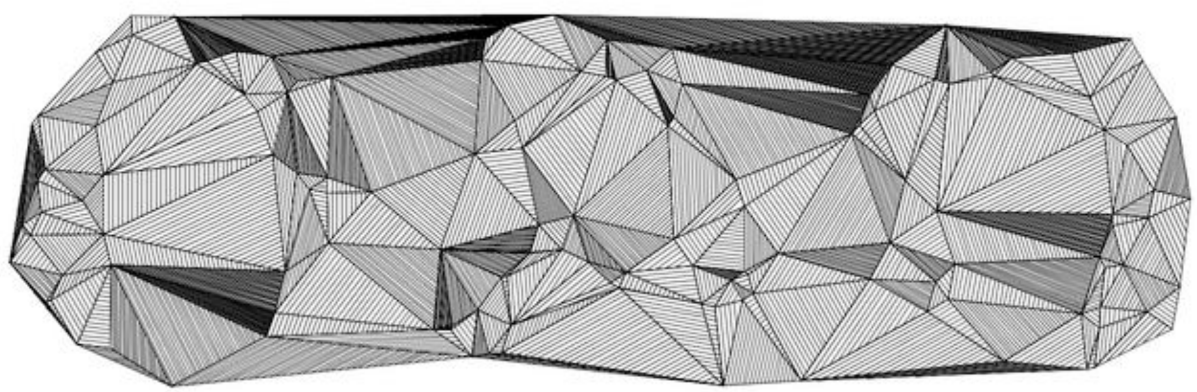


model-dependent:

- class of complexes summed over
- expression of Feynman amplitudes

sum over triangulations/complex

amplitude for each triangulation/complex

GFTs and tensor models

(Ambjorn, Durhuus, Sasakura, …, Gurau, Rivasseau, Bonzom, Ryan, …..)

same combinatorics (of states/observables and histories/Feynman diagrams), no group-theoretic data
purely combinatorial amplitudes ~ lattice gravity path integrals on equilateral triangulations

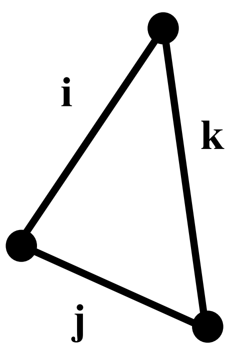
example: d=3

dropping group/algebra data
(or restricting to finite group)

$$\varphi(g_1, g_2, g_3) : G^{\times 3} \rightarrow \mathbb{C} \longrightarrow \begin{array}{l} T_{ijk} : \mathbb{Z}_N^{\times 3} \rightarrow \mathbb{C} \\ T_{ijk} : X^{\times 3} \rightarrow \mathbb{C} \end{array}$$

$$X = 1, 2, ..., N$$

$$S(T) = \frac{1}{2} \sum_{i,j,k} T_{ijk} T_{kji} - \frac{\lambda}{4! \sqrt{N^3}} \sum_{ijklmn} T_{ijk} T_{klm} T_{mjn} T_{nli}$$



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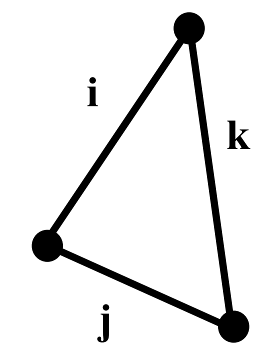
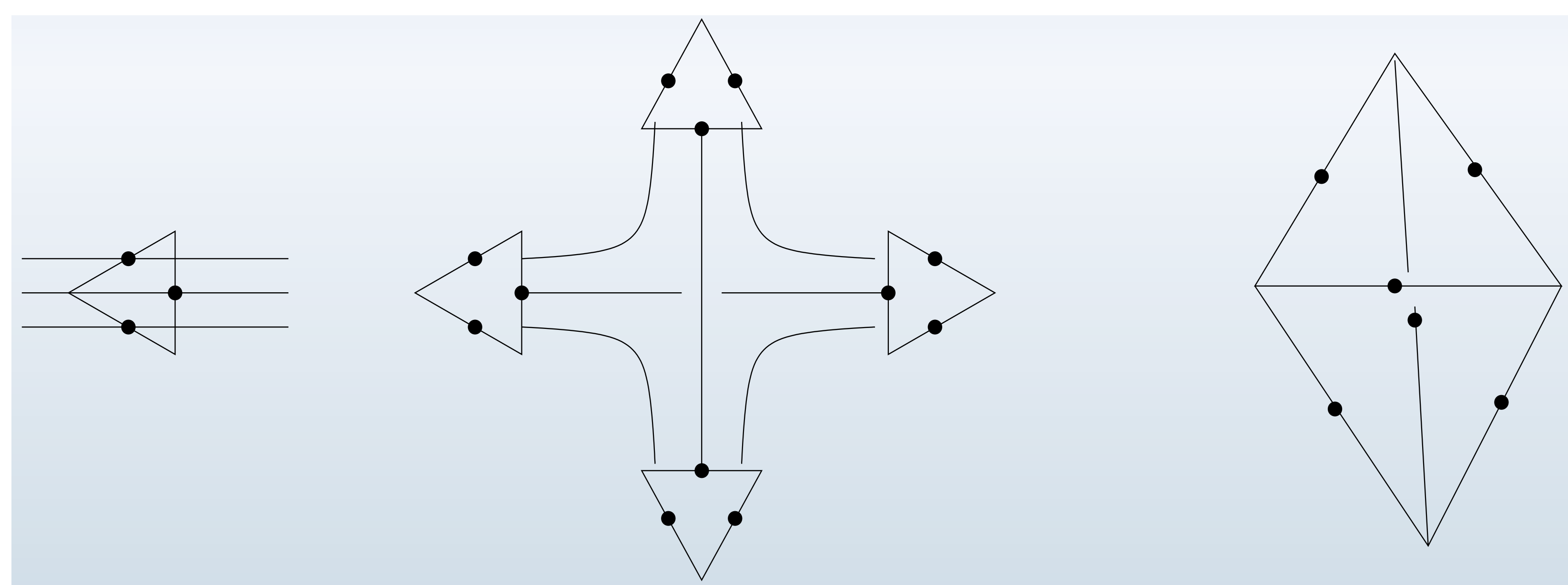


Feynman diagrams are stranded graphs dual to 3d simplicial complexes



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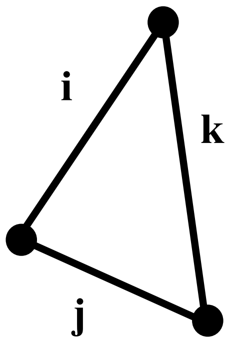

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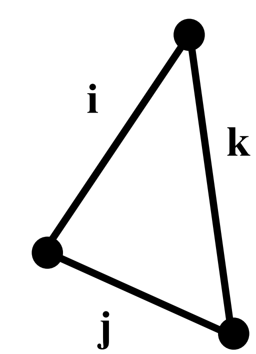

Quantum dynamics:

$$Z = \int \mathcal{D}T \, e^{-S(T,\lambda)} = \sum_\Gamma \frac{\lambda^{V_\Gamma}}{sym(\Gamma)} Z_\Gamma = \sum_\Gamma \frac{\lambda^{V_\Gamma}}{sym(\Gamma)} N^{F_\Gamma - \frac{3}{2} V_\Gamma}$$

can be recast in terms of Regge action for gravity discretised on equilateral triangulation

purely combinatorial definition of quantum gravity

GFTs and tensor models

(Ambjorn, Durhuus, Sasakura, …, Gurau, Rivasseau, Bonzom, Ryan, …..)

crucial issue: analytical control on sum over graphs/complexes ⟶ key tool: colouring

Every PL D-pseudomanifold M can be represented by a (D+1)-colored graph G

two (equivalent) implementations:

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1) colouring tensors(GFT fields)

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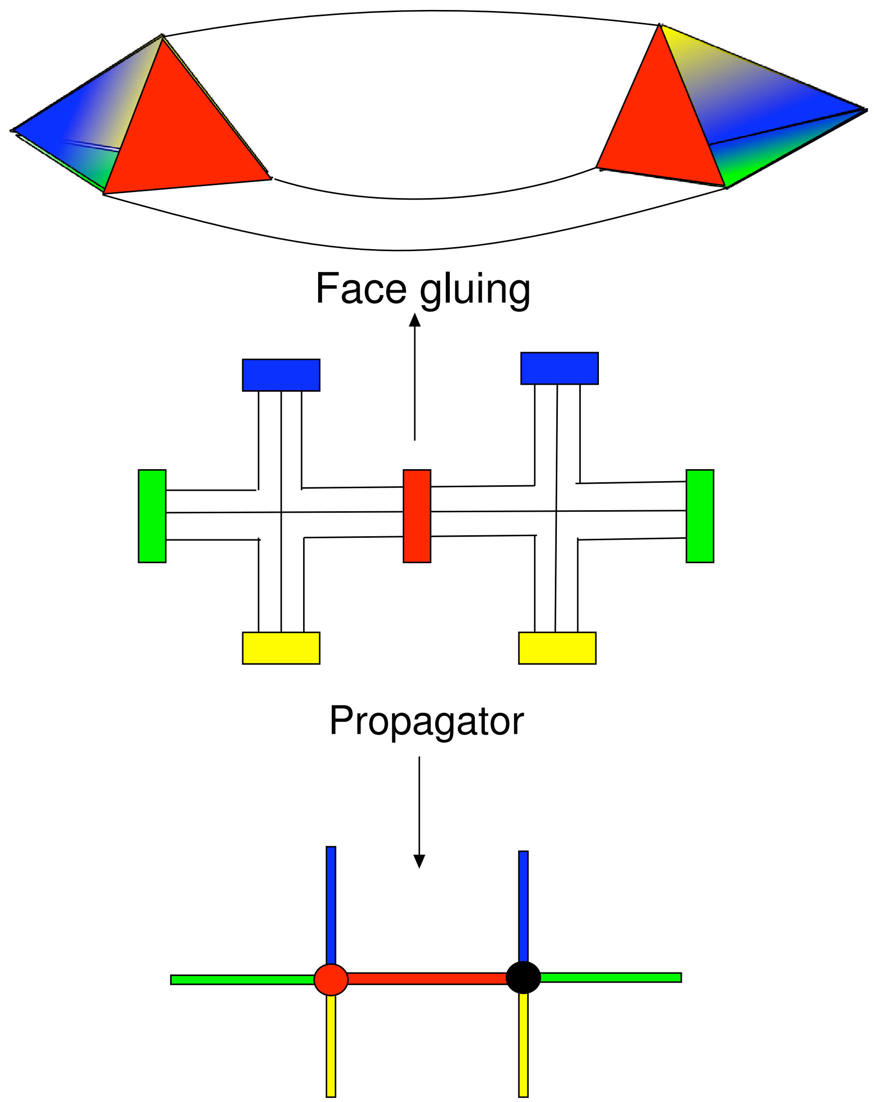

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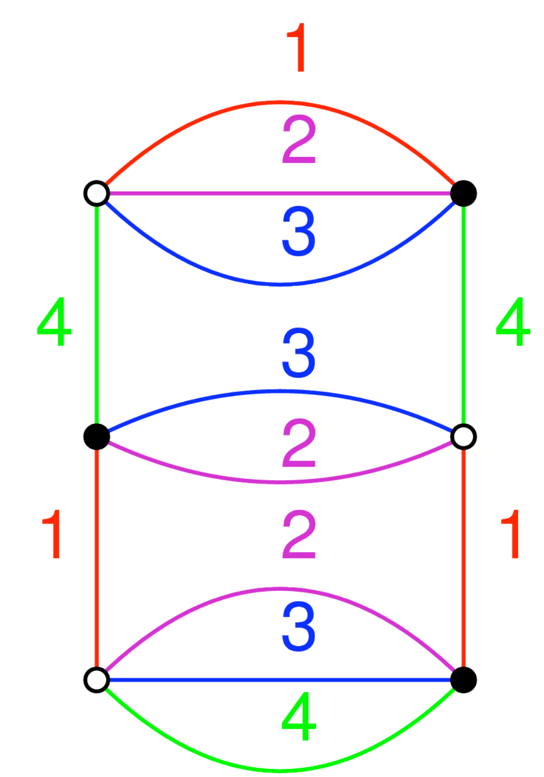
two (equivalent) implementations:

2) colouring indices (field arguments) - new U(N)^d symmetry

interaction associated to coloured bubbles:



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two (equivalent) implementations:

key results (2010 - ….):

1/N expansions

double scaling limits

phase transitions

……………………..

Part I:
the GFT formalism

b) quantum geometry and relation with LQG (and spin foam models)

Group field theories and Loop Quantum Gravity

DO, ‘13

kinematics: Hilbert space close to LQG one - same spin network dofs, but organised differently

LQG: $\mathcal{H} = \lim_{\gamma} \frac{\bigcup_{\gamma} \mathcal{H}_{\gamma}}{\approx} = L^2\left(\bar{\mathcal{A}}\right)$ $\quad$ $\mathcal{H}_{\gamma} = L^2\left(G^E/G^V, d\mu = \prod_{e=1}^{E} d\mu_e^{Haar}\right)$

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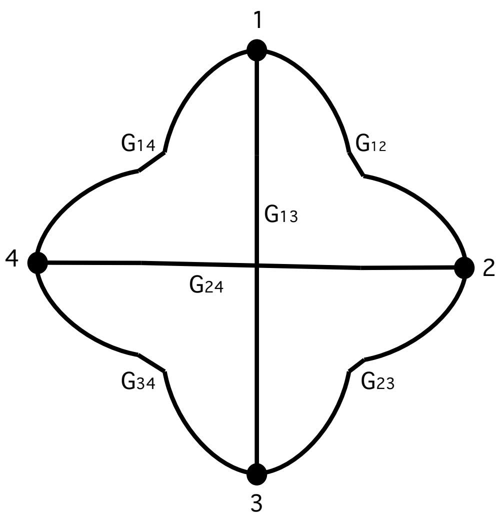
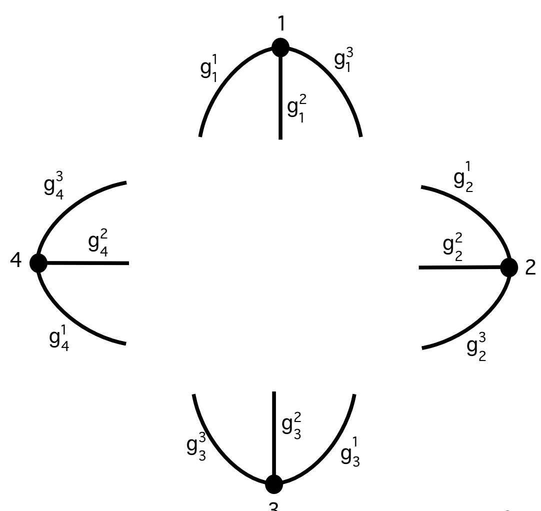
in GFT, spin networks (cylindrical functions) decomposed into building blocks, and re-organized into Fock space





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$$\mathcal{H}_v = L^2\left(G^{\times d}/G\right)$$

$$\mathcal{H}_{\gamma} \subset \mathcal{H}_V \qquad \Psi_{\gamma}(G_{ij}^{ab}) = \prod_{[(ia),(jb)]} \int_G d\alpha_{ij}^{ab}\, \phi_V(\ldots, g_{ia}\,\alpha_{ij}^{ab}, \ldots, g_{jb}\alpha_{ij}^{ab}, \ldots) = \Psi_{\gamma}(g_{ia}(g_{jb})^{-1})$$

spin networks as many-body systems and 2nd quantisation —-> GFT Fock space

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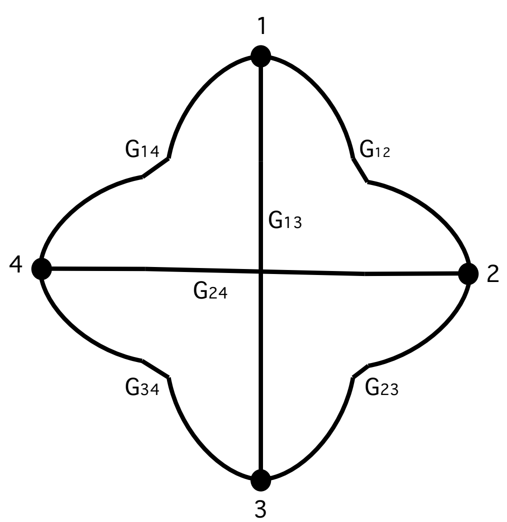

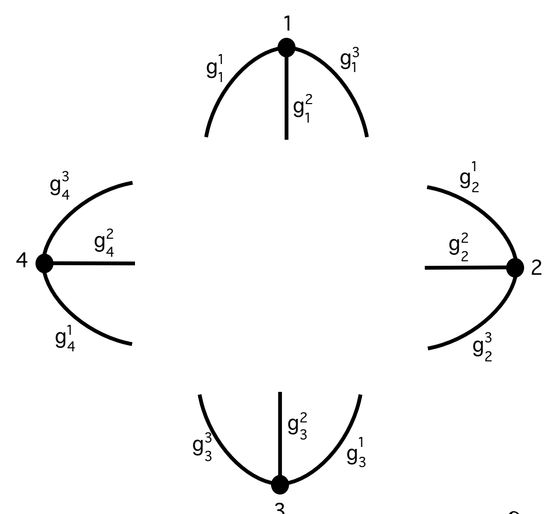

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spin networks as many-body systems and 2nd quantisation —-> GFT Fock space

- for any canonical observable (incl. Hamiltonian constraint) ——> GFT observable in 2nd quantisation

$$\widehat{\mathcal{O}_{n,m}} \rightarrow \langle \vec{\chi}_1, \ldots, \vec{\chi}_m | \widehat{\mathcal{O}_{n,m}} | \vec{\chi}_1', \ldots, \vec{\chi}_n' \rangle = \mathcal{O}_{n,m}\left(\vec{\chi}_1, \ldots, \vec{\chi}_m, \vec{\chi}_1', \ldots, \vec{\chi}_n'\right) \rightarrow$$

$$\rightarrow \widehat{\mathcal{O}_{n,m}}\left(\hat{\varphi}, \hat{\varphi}^\dagger\right) = \int [d\vec{g}_i][d\vec{g}_j'] \,\widehat{\varphi}^\dagger(\vec{g}_1)..\widehat{\varphi}^\dagger(\vec{g}_m)\mathcal{O}_{n,m}\left(\vec{g}_1, .., \vec{g}_m, \vec{g}_1', .., \vec{g}_n'\right) \widehat{\varphi}(\vec{g}_1')..\widehat{\varphi}(\vec{g}_n')$$

Group field theories and Loop Quantum Gravity

DO, '13

GFT dynamics <———> "canonical" (or "1st quantized") dynamics (see also L. Freidel, '06)

start from operator dynamics = projector operator equation: $\widehat{P}\,|\Psi\rangle \;=\; |\Psi\rangle$

Group field theories and Loop Quantum Gravity

DO, '13

GFT dynamics <———> "canonical" (or "1st quantized") dynamics (see also L. Freidel, '06)

start from operator dynamics = projector operator equation: $\widehat{P}\,|\Psi\rangle \;=\; |\Psi\rangle$

constraint as operator on Fock space + compute matrix elements + convolute with field operators

"2nd quantised" GFT observable:

$$\widehat{F}|\Psi\rangle \equiv \sum_{n,m}^{\infty} \lambda_{n,m} \left[\sum_{\{\vec{\chi},\vec{\chi}'\}} \hat{\varphi}^{\dagger}_{\vec{\chi}_1} ... \hat{\varphi}^{\dagger}_{\vec{\chi}_m} \, P_{n,m}\left(\vec{\chi}_1, ..., \vec{\chi}_m, \vec{\chi}'_1, ..., \vec{\chi}'_n\right) \; \hat{\varphi}_{\vec{\chi}'_1} ... \hat{\varphi}_{\vec{\chi}'_n} \; - \; \sum_{\vec{\chi}} \hat{\varphi}^{\dagger}_{\vec{\chi}} \hat{\varphi}_{\vec{\chi}} \right] |\Psi\rangle = \; 0$$

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GFT partition function from "grand-canonical ensemble for spin networks":

$$Z_g = \sum_s \langle s|e^{-\left(\widehat{F}-\mu\widehat{N}\right)}|s\rangle \;=\; \int \mathcal{D}\varphi\mathcal{D}\overline{\varphi}\, e^{-|\varphi|^2}\,\langle\varphi|\,e^{-\left(\widehat{F}-\mu\widehat{N}\right)}\,|\varphi\rangle$$

with effective amplitude: $e^{-|\varphi|^2}\,\langle\varphi|\,e^{-\left(\widehat{F}-\mu\widehat{N}\right)}\,|\varphi\rangle \;\equiv\; e^{-S_{eff}}$

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with effective amplitude: $\quad e^{-|\varphi|^2} \,\langle \varphi|\, e^{-\left(\widehat{F} - \mu \widehat{N}\right)}\, |\varphi\rangle \;\equiv\; e^{-S_{eff}}$

up to quantum corrections (or for normal ordering): GFT model with action:

$$S\left(\varphi, \varphi^{\dagger}\right) \;=\; m^2 \int d\vec{g}\; \varphi^{\dagger}(\vec{g})\, \varphi(\vec{g}) \;-$$

$$- \quad \sum_{n,m} \lambda_{n+m} \left[\int [d\vec{g}_i]\, [d\vec{g}'_j]\; \varphi^{\dagger}(\vec{g}_1) ... \varphi^{\dagger}(\vec{g}_m)\; V_{n+m}\left(\vec{g}_1, ..., \vec{g}_m, \vec{g}'_1, ..., \vec{g}'_n\right) \varphi(\vec{g}'_1) ... \varphi(\vec{g}'_n) \right] \quad = \frac{\langle \varphi | \widehat{F} | \varphi \rangle}{\langle \varphi | \varphi \rangle}$$

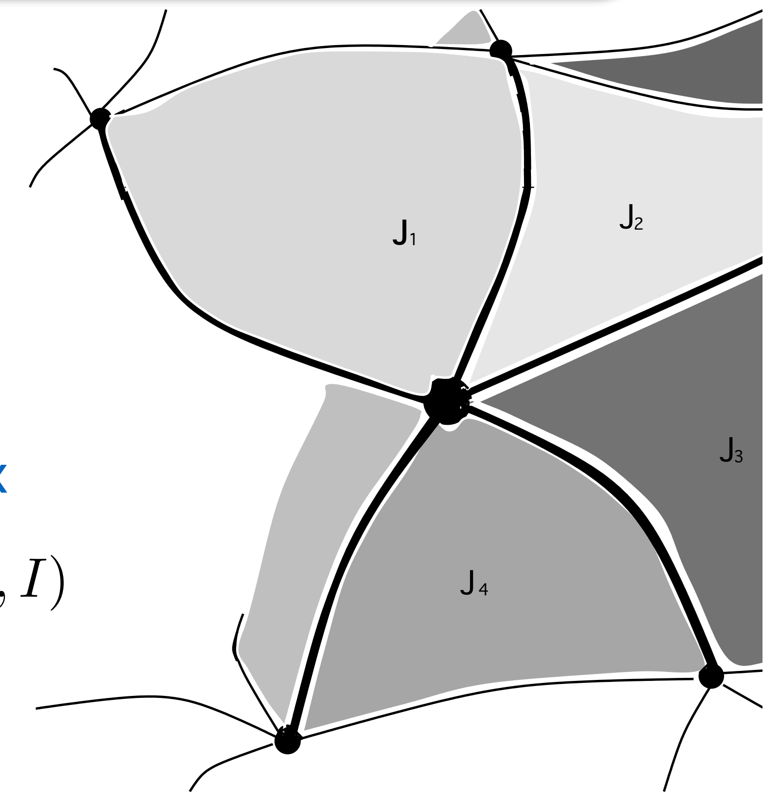
$$V_{n+m}\left(\vec{g}_1, ..., \vec{g}_m, \vec{g}'_1, ..., \vec{g}'_n\right) = P_{n+m}\left(\vec{g}_1, ..., \vec{g}_m, \vec{g}'_1, ..., \vec{g}'_n\right)$$

Group field theories and spin foam models

spin foam model: sum over histories for LQG-like states (spin networks ~ quantum simplicial geometries)

quantum history = spin foam (complex with algebraic data)

need to specify: class of complexes + type of algebraic data



quantum amplitude for spin foam —-> quantum amplitude for spin foam complex

$$\{\,\Gamma\,\} \qquad Z(\Gamma) = \sum_{\{J\},\{I\}|j,j',i,i'} \prod_f A_f(J,I) \prod_e A_e(J,I) \prod_v A_v(J,I)$$

Reisenberger, Rovelli, Baez, Barrett, Crane, Perez, DO, ……

complete (formal) definition of SF model:

set of all quantum amplitudes for all spin foam complexes (in the chosen class) + organization principle

different prescriptions available for “organization principle”

Group field theories and spin foam models

the GFT proposal:

Boulatov, Ooguri, Freidel, Rovelli, Reisenberger, Perez, Oriti, Baratin, Livine, ,.......

spin foam model with sum over complexes as perturbative expansion of GFT (valid for any SF model)

$$Z(\Gamma) \leftrightarrow \begin{cases} A_f(J) \\ A_e(J,I) \\ A_v(J,I) \end{cases} \quad \Longleftrightarrow \quad \begin{cases} \mathcal{K}(J,I) \sim \mathcal{K}(g) \\ \mathcal{V}(J,I) \sim \mathcal{V}(g) \end{cases} \leftrightarrow \; S(\varphi,\bar{\varphi})$$

$$S(\varphi,\overline{\varphi}) = \frac{1}{2}\int [dg_i]\overline{\varphi(g_i)}\mathcal{K}(g_i)\varphi(g_i) + \frac{\lambda}{D!}\int [dg_{ia}]\varphi(g_{i1})....\varphi(\bar{g}_{iD})\mathcal{V}(g_{ia},\bar{g}_{iD}) \quad + \quad c.c.$$

$$\Updownarrow$$

$$\mathcal{Z} = \int \mathcal{D}\varphi\mathcal{D}\overline{\varphi}\; e^{i\,S_\lambda(\varphi,\overline{\varphi})} \quad = \quad \sum_\Gamma \frac{\lambda^{N_\Gamma}}{sym(\Gamma)}\mathcal{A}_\Gamma \qquad Z(\Gamma) \equiv \mathcal{A}_\Gamma$$

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construction ambiguities and computational difficulties remain, (see talks by Speziale, Dona', Finocchiaro)

but several advantages:

- precise and constrained prescription for combinatorial weights + way to parametrize SF ambiguities
- QFT re-interpretation and techniques + ways to go beyond spin foams themselves

Group field theories, LQG and spin foam models

$$\mathcal{Z} = \int \mathcal{D}\varphi \mathcal{D}\overline{\varphi}\; e^{i\, S_\lambda(\varphi,\overline{\varphi})} \quad = \quad \sum_\Gamma \frac{\lambda^{N_\Gamma}}{sym(\Gamma)}\, \mathcal{A}_\Gamma$$

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Feynman amplitudes (model-dependent):

equivalently:

- spin foam models (sum-over-histories of spin networks ~ covariant LQG)
  Reisenberger,Rovelli, ’00
- lattice gravity path integrals (with group+Lie algebra variables)
  A. Baratin, DO, ‘11

GFT as lattice quantum gravity:

dynamical triangulations + quantum Regge calculus

Group field theories, LQG and spin foam models

$$\mathcal{Z} = \int \mathcal{D}\varphi \mathcal{D}\overline{\varphi} \; e^{i\, S_\lambda(\varphi,\overline{\varphi})} \quad = \quad \sum_{\Gamma} \frac{\lambda^{N_\Gamma}}{sym(\Gamma)} \, \mathcal{A}_\Gamma$$

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with GFT action:

$$S\left(\varphi, \varphi^\dagger\right) = m^2 \int d\vec{g}\, \varphi^\dagger(\vec{g})\, \varphi(\vec{g}) \; -$$

$$- \sum_{n,m} \lambda_{n+m} \left[\int [d\vec{g}_i]\, [d\vec{g}'_j]\, \varphi^\dagger(\vec{g}_1)...\varphi^\dagger(\vec{g}_m)\, V_{n+m}\left(\vec{g}_1, ..., \vec{g}_m, \vec{g}'_1, ..., \vec{g}'_n\right) \varphi(\vec{g}'_1)...\varphi(\vec{g}'_n) \right] = \frac{\langle \varphi | \widehat{F} | \varphi \rangle}{\langle \varphi | \varphi \rangle}$$

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spin foam vertex amplitude

direct path: canonical <——> covariant LQG

Part II:
research directions

QG as GFT: new research directions

One possible concrete, and complete realisation of QG based on spin networks/simplicial geometry

Translating Quantum Gravity questions (e.g. from LQG or spin foams) into GFT offers new perspective

New tools from QFT become available thanks GFT embedding

in particular, it may become possible/easier to:

- define rigorously quantum statistical mechanics for QG degrees of freedom
- relate nicely canonical and covariant formulations
- study inequivalent representations and symmetries of QG system
- constrain quantum ambiguities via renormalizability
- define continuum limit and control macroscopic phase diagram
- extract effective continuum physics (GR + QFT + corrections)
- make contact with QG phenomenology

Part II:
research directions

a) foundations of the GFT formalism

Foundations of group field theories

quantum statistical mechanics of spin networks (within GFT Fock space)

give precise meaning to: $$Z_g = \sum_s \langle s | e^{-\left(\widehat{F} - \mu \widehat{N}\right)} | s \rangle$$

N.B. related to problem of "generally covariant statistical mechanics", including gravitational field

it entails understanding more rigorously:

- identification and encoding of QG (GFT) thermodynamic potentials
- statistical implementation of quantum dynamical constraints   G. Chirco, I. Kotecha, M. Laudato, F. Mele, DO, in prep.
- QG notion of "equilibrium" (e.g. via KMS condition on abstract QG algebra)
  I. Kotecha, DO, to appear
- deparametrization wrt internal clock and recovering of standard quantum statistical mechanics

see talk by Kotecha

needed for rigorous relation between canonical (operator) and covariant (path integral) QG formulations

Foundations of group field theories

symmetries, conservation laws and symmetry breaking

GFTs are non-local quantum field theories

$$S(\varphi,\overline{\varphi}) = \frac{1}{2}\int [dg_i]\overline{\varphi(g_i)}\mathcal{K}(g_i)\varphi(g_i) + \frac{\lambda}{D!}\int [dg_{ia}]\varphi(g_{i1})....\varphi(\bar{g}_{iD})\mathcal{V}(g_{ia},\bar{g}_{iD}) \quad + \quad c.c.$$

“combinatorial non-locality” in pairing of field arguments

(at classical and quantum level) need to:

- generalise Noether framework
- identify symmetries and conservation laws for interesting models
- adapt theory of symmetry breaking (to have more control over non-perturbative sector)

A. Kegeles, DO, ’15, ‘16

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inequivalent representations of quantum GFT

see talk by Kegeles (and Geiller for LQG perspective)

like any (infinite-dimensional) quantum system, GFTs (as LQG) may have inequivalent realisations

need to:

- define carefully thermodynamic & continuum limits
- identify inequivalent representations (possibly corresponding to different phases of continuum theory)

A. Kegeles, DO, to appear

Part II:
research directions

b) GFT renormalization:
consistency and continuum limit

GFT renormalization: constraining ambiguities

$$\mathcal{Z} = \int \mathcal{D}\varphi \mathcal{D}\overline{\varphi}\, e^{i\, S_\lambda(\varphi,\overline{\varphi})} \quad = \quad \sum_\Gamma \frac{\lambda^{N_\Gamma}}{sym(\Gamma)} \mathcal{A}_\Gamma$$

Issue 1:

many construction and quantisation ambiguities in definition of GFT model (thus, many models) (LQG canonical constraint, spin foam amplitudes) -

background independent counterpart of issue of renormalizability in perturbative QG

Perez, '07

EPRL,'07, Freidel-Krasnov, '07, Baratin-Oriti, '11, Dupuis-Livine, '11 Finocchiaro, DO, to appear

- exact way of imposing simplicity constraints in spin foam models
- generalisations at combinatorial level (which complexes?)
- quantisation ambiguities (choice of quantisation map) Alexandrov, '10; Ding, Han, Rovelli, '10; Guedes, DO, Raasakka, '12
- quantum corrections and stability of spin foam amplitudes, divergences
- "measure" terms
- ……

see talk by Finocchiaro

GFT renormalization: constraining ambiguities

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Alexandrov, '10; Ding, Han, Rovelli, '10; Guedes, DO, Raasakka, '12

see talk by Finocchiaro

translating in QFT perspective:

- GFT perturbative renormalization

see talk by Carrozza

—-> renormalizability of GFT model (spin foam amplitudes)

= existence of consistent dynamics for (at least) a wide range of scales

GFT renormalization: continuum limit

$$\mathcal{Z} = \int \mathcal{D}\varphi \mathcal{D}\overline{\varphi}\; e^{i\, S_\lambda(\varphi,\overline{\varphi})} \quad = \quad \sum_{\Gamma} \frac{\lambda^{N_\Gamma}}{sym(\Gamma)} \mathcal{A}_\Gamma$$

Issue 2: controlling quantum dynamics of more and more interacting QG degrees of freedom

control quantum dynamics for boundary states involving (superpositions of) large graphs

compute spin foam amplitudes for finer complexes and corresponding sum over complexes

up to infinite refinement (infinite number of degrees of freedom), at least in simple approximations

need control over theory space

expect different phases and phase transitions as result of quantum dynamics

(what are the phases of LQG?) Koslowski, '07; DO, '07

see talk by Dittrich for lattice spin foam perspective

GFT renormalization: continuum limit

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translating in QFT perspective:

- GFT non-perturbative renormalization

——> computing RG flow of quantum dynamics

——> defining full GFT partition function (spin foam amplitudes) without cut-offs

= definition of full continuum theory and identification of macroscopic phases

see talk by Carrozza

GFT renormalisation - general scheme

$$\mathcal{Z} = \int \mathcal{D}\varphi\mathcal{D}\overline{\varphi}\; e^{i\,S_\lambda(\varphi,\overline{\varphi})} \quad = \quad \sum_\Gamma \frac{\lambda^{N_\Gamma}}{sym(\Gamma)}\,\mathcal{A}_\Gamma$$

$$S(\varphi,\overline{\varphi}) = \frac{1}{2}\int [dg_i]\overline{\varphi(g_i)}\mathcal{K}(g_i)\varphi(g_i) + \frac{\lambda}{D!}\int [dg_{ia}]\varphi(g_{i1})....\varphi(\bar{g}_{iD})\mathcal{V}(g_{ia},\bar{g}_{iD}) \quad + \quad c.c.$$

general strategy:

treat GFTs as ordinary QFTs defined on Lie group manifold

use group structures (Killing form, topology, etc) to define notion of scale and to set up mode integration

subtleties of quantum gravity context at the level of interpretation

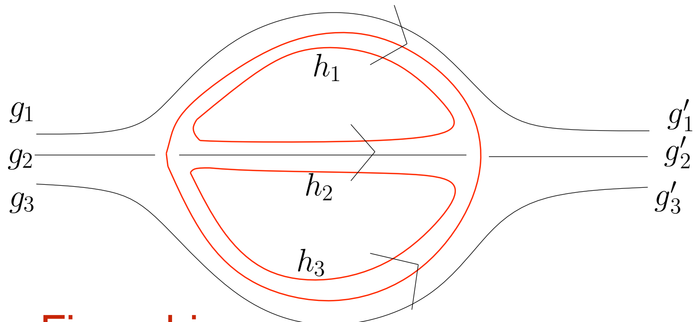
scales:

defined by propagator: e.g. spectrum of Laplacian on G = indexed by group representations

- need to have control over "theory space" (e.g. via symmetries) A. Kegeles, DO, '15,'16
- main difficulty:
  controlling the combinatorics of GFT Feynman diagrams and interactions to control RG flow and divergences
  need to adapt/redefine many QFT notions: connectedness, subgraph contraction, Wick ordering, …..

GFT perturbative renormalisation

recent results:



towards renormalizable 4d gravity simplicial GFT models:

- calculation of some radiative corrections

  T. Krajewski et al., '10; A. Riello, '13; Bonzom, Dittrich, '15; …. ; M. Finocchiaro, DO, '17

see talk by Finocchiaro

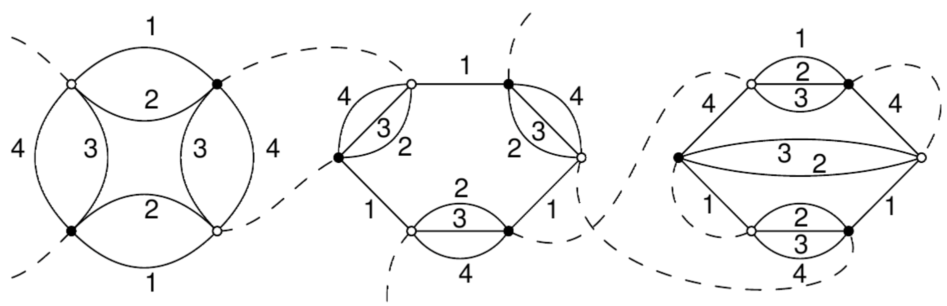
- finiteness results in 3d simplicial models (Boulatov with Laplacian kinetic term) Ben Geloun, Bonzom, '11; Ben Geloun, '13

- renormalizable TGFT models (3d, 4d, and higher) - Laplacian + tensorial interactions

  Ben Geloun, Rivasseau, '11
  Carrozza, DO, Rivasseau, '12. '13

-> with gauge invariance

$$S(\varphi,\overline{\varphi}) = \sum_{b\in\mathcal{B}} t_b I_b(\varphi,\overline{\varphi})$$



—> non-abelian (SU(2))

— —> non-abelian SU(2) model beyond melonic sector

— — —> SO(4) or SO(3,1) with simplicity constraints: first steps Lahoche, DO, '15; Carrozza, Lahoche, DO, '17

— — — —> generic asymptotic freedom/safety

Ben Geloun, '12; Carrozza, '14; Carrozza, Lahoche, '16

see talk by Carrozza

GFT non-perturbative renormalisation

two directions:

- GFT non-perturbative renormalization and "IR" fixed points (e.g. FRG analysis - e.g. a la Wetterich

  Benedetti, Ben Geloun, DO, Martini, Lahoche, Carrozza, Ousmane-Samary, Duarte, ….

- GFT constructive analysis

  Freidel, Louapre, Noui, Magnen, Smerlak, Gurau, Rivasseau, Tanasa, Dartois, Delpouve, …..

  non-perturbative resummation of perturbative (SF) series

  variety of techniques:
  - intermediate field method
  - loop-vertex expansion
  - Borel summability

+

many results in simpler tensor models

see talk by Carrozza

GFT non-perturbative renormalisation

recent results: see talk by Carrozza

FRG for (tensorial) GFT models (similar to matrix/tensor models but distinctively field-theoretic)

Eichhorn, Koslowski, '14

- Polchinski formulation based on SD equations Krajewski, Toriumi, '14
- general set-up for Wetterich formulation based on effective action
  - analysis of TGFT on compact U(1)^d
    - RG flow and phase diagram established
  - analysis of TGFT on non-compact R^d
    - RG flow and phase diagram established
  - analysis of TGFT on non-compact R^d with gauge invariance
    - RG flow and phase diagram established
  - analysis of TGFT on SU(2)^3 Carrozza, Lahoche, '16

Benedetti, Ben Geloun, DO, '14 ; Ben Geloun, Martini, DO, '15, '16, Benedetti, Lahoche, '15; Ben Geloun, Duarte, Koslowsk, DO, to appear

Carrozza, Lahoche, DO, '17

genericaly (so far):

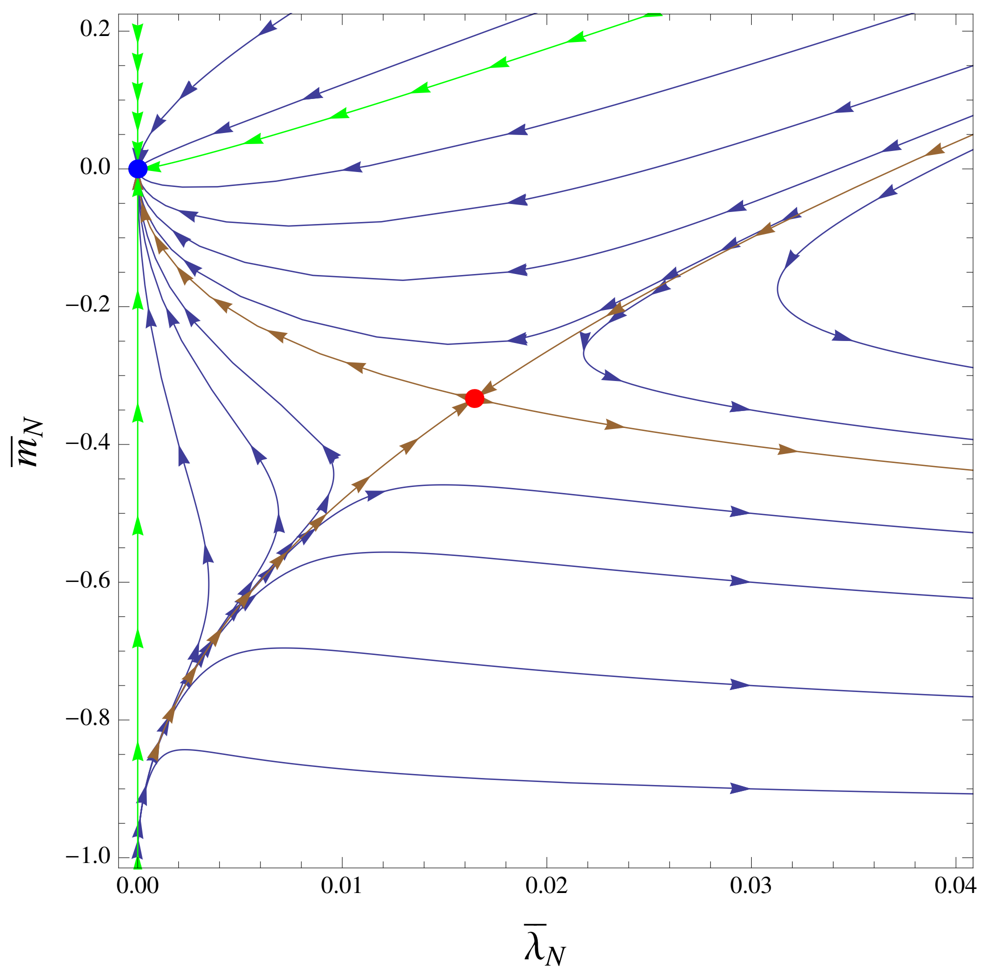
two FPs (Gaussian-UV, Wilson-Fisher-IR)

asymptotic freedom

one symmetric phase

one broken or condensate phase

(non-trivial minimum of classical potential)



Part II:
research directions

c) extracting effective continuum physics

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research directions

c) extracting effective continuum physics

advantages of GFT formalism:

- control over (large) superpositions of spin network states via 2nd quantised formalism
- new (QFT) analytic tools for control over sum over complexes in quantum dynamics
- bypassing (some) conceptual issues by adapting QFT tools

GFT condensate cosmology

Cosmology from QG perspective

see talk by Wilson-Ewing for LQC perspective

- few “macroscopic” observables, of “global” nature (understood as suitably defined averages over fundamental degrees of freedom, e.g. inhomogeneities, microscopic dofs, …)
- close to equilibrium
- insensitive to (or not too much affected by) microstructure

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hydrodynamics regime!

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Quantum Gravity hydrodynamics

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↓

hydrodynamics regime!

→ cosmology as

Quantum Gravity hydrodynamics

what could be the relevant hydrodynamic observables in QG?

simple averages of "one-body" observables, extensive in the "number of atoms of space"

e.g. the total volume V of space, if each "atom of space" gives a contribution to it

what would key hydrodynamic quantities look like in QG?

one key hydrodynamic quantity would be reduced "one-body" density,
i.e. some function on the space of data associated with a single "atom of space"

cosmology is (non-linear) dynamics for such density and for geometric (global) observables computed from it

GFT condensate cosmology

see talk by Gielen

S. Gielen, DO, L. Sindoni, PRL, arXiv:1303.3576 [gr-qc]; JHEP, arXiv:1311.1238 [gr-qc] ....

start with fundamental (Fock) space of GFT states (arbitrary collections of tetrahedra labelled by SU(2) data

problem 1:
identify quantum states in fundamental theory with continuum spacetime interpretation

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Quantum GFT condensates are continuum homogeneous (quantum) spaces

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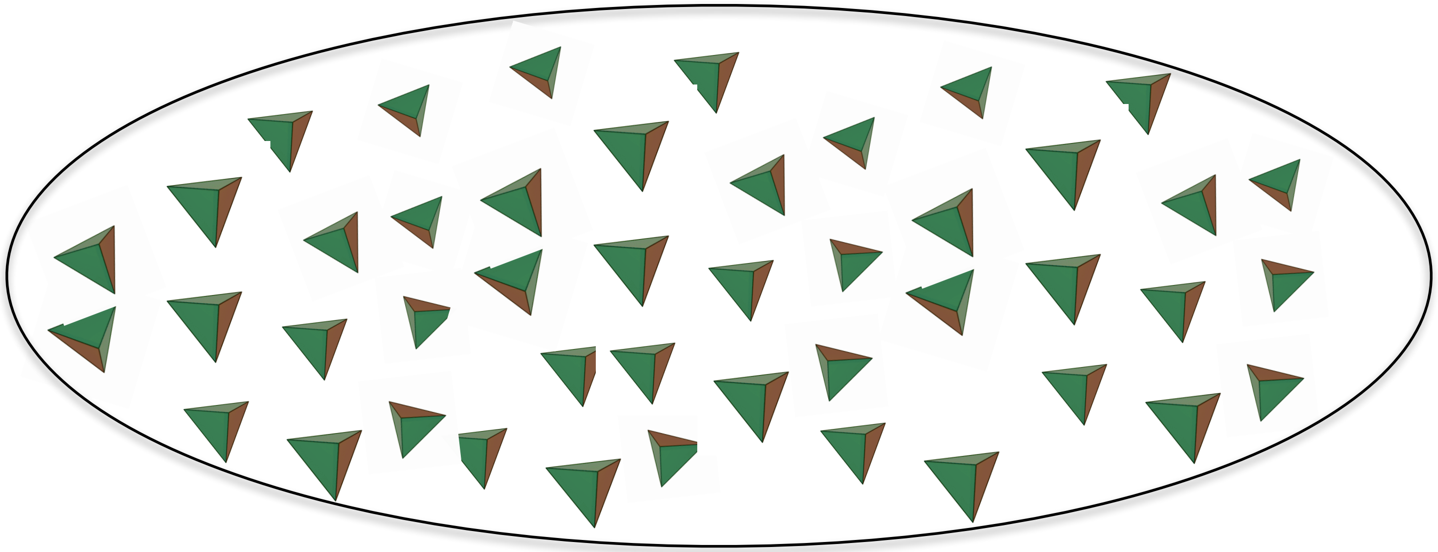
Quantum GFT condensates are continuum homogeneous (quantum) spaces

e.g. (simplest): GFT field coherent state

$$|\sigma\rangle := \exp\left(\hat{\sigma}\right)|0\rangle$$

$$\hat{\sigma} := \int d^4g\, \sigma(g_I)\hat{\varphi}^\dagger(g_I) \qquad \sigma(g_I k) = \sigma(g_I)$$



superposition of infinitely many spin networks dofs,
“gas”of tetrahedra, all associated with same state

GFT condensate cosmology

see talk by Gielen

S. Gielen, DO, L. Sindoni, PRL, arXiv:1303.3576 [gr-qc]; JHEP, arXiv:1311.1238 [gr-qc] ....

start with fundamental (Fock) space of GFT states (arbitrary collections of tetrahedra labelled by SU(2) data

problem 1:
identify quantum states in fundamental theory with continuum spacetime interpretation

Quantum GFT condensates are continuum homogeneous (quantum) spaces

e.g. (simplest): GFT field coherent state

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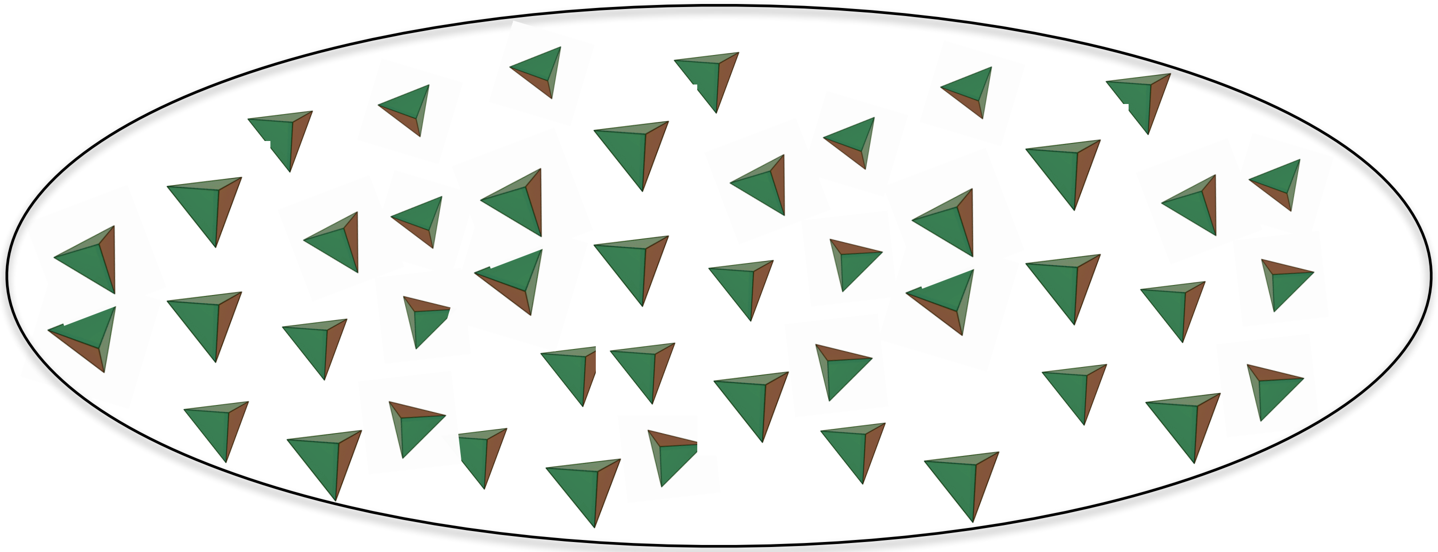

special states with (plausible) continuum geometric interpretation:

infinite dofs, such that, if one tries to reconstruct continuum geometry from them, one obtains same geometric data at each "point", i.e. homogeneous spatial (quantum) geometry (still, fully diffeo-invariant)

GFT condensate cosmology

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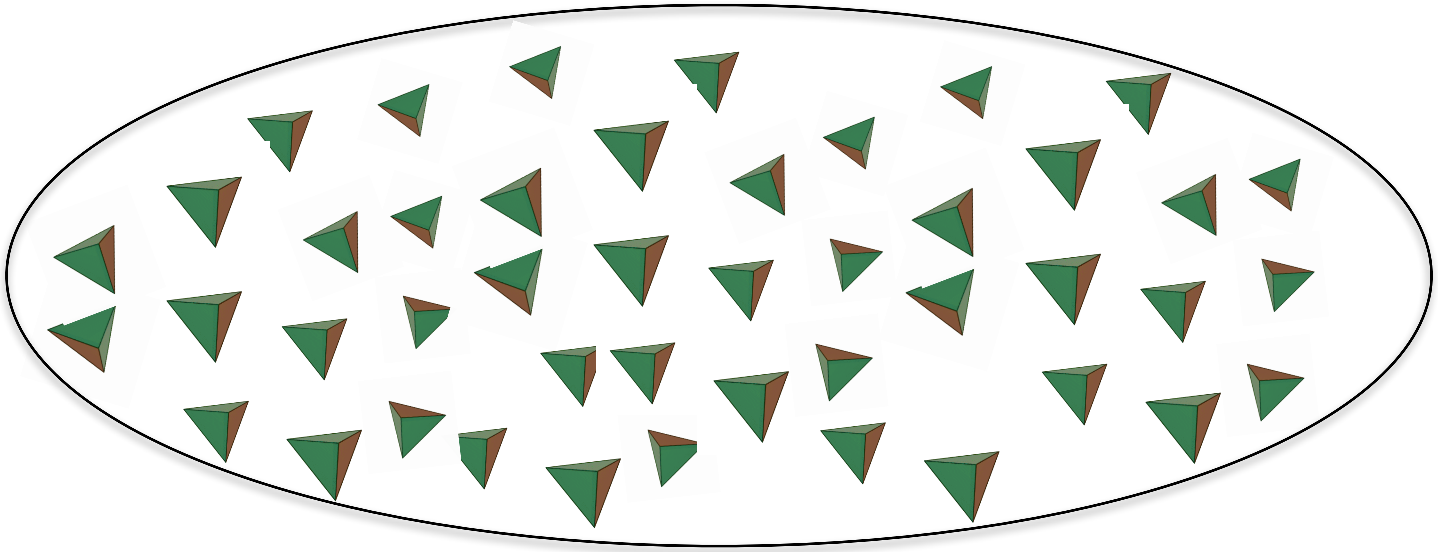

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superposition of infinitely many spin networks dofs,
“gas”of tetrahedra, all associated with same state

described by single collective wave function
(depending on homogeneous anisotropic geometric data)

$$\sigma\left(\mathcal{D}\right) \qquad \mathcal{D} \simeq \{\text{geometries of tetrahedron}\} \simeq$$
$$\simeq \{\text{continuum spatial geometries at a point}\} \simeq$$
$$\simeq \text{minisuperspace of homogeneous geometries}$$

GFT condensate cosmology

see talk by Gielen

S. Gielen, DO, L. Sindoni, PRL, arXiv:1303.3576 [gr-qc]; JHEP, arXiv:1311.1238 [gr-qc]

problem 1:
identify quantum states in fundamental theory with continuum spacetime interpretation

Quantum GFT condensates are continuum homogeneous (quantum) spaces

described by single collective wave function (1-particle density)
(depending on homogeneous anisotropic geometric data)

problem 2:
extract from fundamental theory an effective macroscopic dynamics for such states

equation for “condensate wave function”:

$$\int [dg'_i]\, \tilde{\mathcal{K}}(g_i, g'_i)\sigma(g'_i) + \lambda \frac{\delta \tilde{\mathcal{V}}}{\delta\varphi(g_i)}|_{\varphi\equiv\sigma} = 0$$

infinite superposition of Feynman diagrams
(infinite sum over discrete “spacetime” lattices)

non-linear and non-local extension of quantum cosmology-like equation for “collective wave function”

QG (GFT) analogue of Gross-Pitaevskii hydrodynamic equation in BECs

similar to quantum cosmology, but: no Hilbert space structure (no superposition of “states of universe”, no “collapse of wavefunction”) - “statistical nature” of wavefunction; still, fluctuations of geometric quantities

GFT condensate cosmology

see talk by Gielen

S. Gielen, DO, L. Sindoni, PRL, arXiv:1303.3576 [gr-qc]; JHEP, arXiv:1311.1238 [gr-qc]

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QG (GFT) analogue of Gross-Pitaevskii hydrodynamic equation in BECs
is
non-linear extension of quantum cosmology equation for collective wave function

GFT condensate cosmology

see talk by Gielen

S. Gielen, DO, L. Sindoni, PRL, arXiv:1303.3576 [gr-qc]; JHEP, arXiv:1311.1238 [gr-qc]

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QG (GFT) analogue of Gross-Pitaevskii hydrodynamic equation in BECs
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non-linear extension of quantum cosmology equation for collective wave function

cosmology as QG hydrodynamics!!!

GFT condensate cosmology

summary of recent results:

see talk by Gielen

- general scheme, geometric interpretation and effective dynamics S. Gielen, DO, L. Sindoni, '13
- generalised condensate states (also for spherical black holes) DO, D. Pranzetti, J. Ryan, L. Sindoni, '15; DO, D. Pranzetti, L. Sindoni, '15
- lattice refinement and GFT cosmological observables S. Gielen, DO, '14
- relation with LQC S. Gielen, '14, '15, '16; G. Calcagni, '14
- effective cosmological dynamics from EPRL model DO, L. Sindoni, E. Wilson-Ewing, '16
  - generalised Friedmann equations
  - generic big bounce resolution of classical singularity
  - reduction to LQC dynamics

see talks by:

De Cesare, Pithis, Wilson-Ewing

- effect of GFT interaction in emergent cosmological dynamics
  - long-lasting acceleration after bounce (no inflation) M. De Cesare, A. Pithis, M. Sakellariadou, '16
  - non-normalisable condensate states (hints of GFT phase transition?) A. Pithis, M. Sakellariadou, P. Tomov, '16
  - first analysis of dynamics of anisotropies A. Pithis, M. Sakellariadou, '16; M. De Cesare, DO, A. Pithis, M. Sakellariadou, to appear
- cosmological perturbations S. Gielen, '14, '15; F. Gerhardt, DO, E. Wilson-Ewing, to appear; S. Gielen, DO, to appear

Quantum horizons in full QG via GFT

see talk by Pranzetti

DO, D. Pranzetti, L. Sindoni, PRL, '15

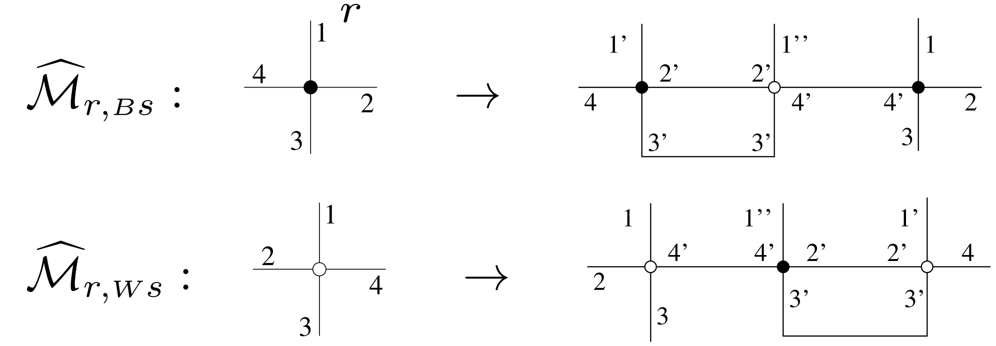
GFT allows to go beyond (symmetry-reduced) models and control continuum states involving large superpositions of spin networks (also using tensorial techniques, e.g. colouring and dipole moves)

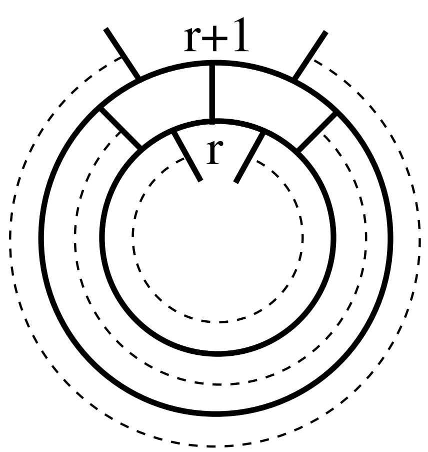
generalised GFT condensate states for arbitrary topology:

- obtained from initial "seed" graph by action of "refinement operators"

$$|\Psi\rangle = \prod_r f_r(\widehat{\mathcal{M}}_{r,B}, \widehat{\mathcal{M}}_{r,W})\, |seed\rangle$$

- homogeneous shell (two spherical boundaries)
- spherical symmetry (by gluing shells along boundaries)





(special) quantum states for spherical horizons:

- quantum states dependent on one condensate wave function for each shell
- conditions on quantum states, to have consistent interpretation as spherical horizons
- reduced density matrix associated to horizon shows holographic properties

horizon entropy:

- holography: entanglement entropy equal to its Boltzmann entropy
- entropy computed by counting number of possible horizon graphs
- assuming maximal entropy, $S(\mathcal{A}_H, \lambda) \sim 2\lambda \mathcal{A}_H + \log\left(\mathcal{A}_H / a\right)$

constant fixed by thermodynamic consistency

a = <area> for each puncture

no dependence on Immirzi parameter

crucial role of GFT number operator

logarithmic corrections depend on combinatorics

Holography and entanglement

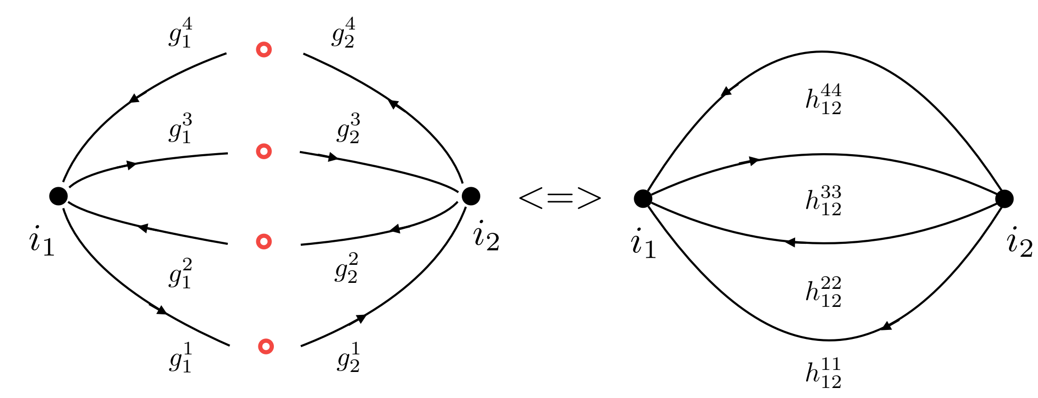
entanglement in spin networks via GFT

several studies of entanglement properties of spin networks

see talks by Livine, Riello

gluing of GFT quanta ~ connectivity of space ~ entanglement

gauge invariant gluing ~ maximal entanglement



G. Chirco, F. Mele, DO, P. Vitale, '17

various measures of entanglement, e.g. geometric entanglement via Fisher metric

GFT states, spin networks and tensor networks

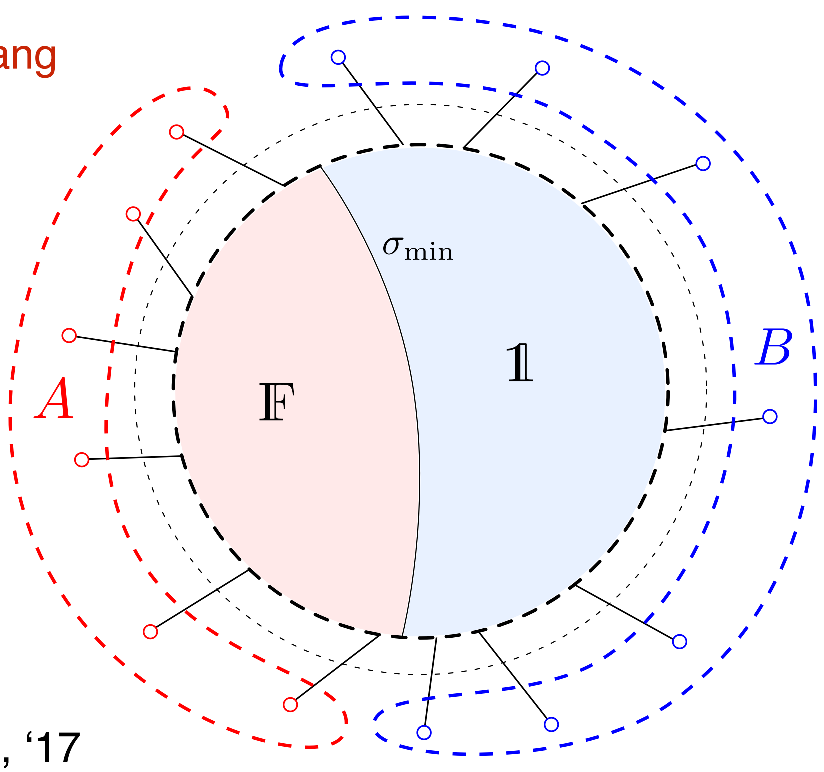
see talks by Chirco, Zhang

GFT states are generalised (random) tensor network states

- tensor at node generalised to 1-particle GFT wavefunction
- linking info in gluing kernels between GFT quanta

GFT dynamics provides measure on random tensor networks

QG analogue of Ryu-Takanayagi entropy formula via GFT techniques



G. Chirco, DO, M. Zhang, '17

Thank you for your attention!