Black Hole Bounce

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Based on works with Alex Corichi, Gaurav Khanna, Javier Olmedo, Sahil Saini and Alec Yonica

Outline

- How much do we understand singularity resolution in loop quantum black hole spacetimes and the resulting physics in contrast to cosmological spacetimes (LQC)?
- Classical setting of Schwarzschild interior and different quantization strategies.
- A quantization of the interior, free of various problems.
- First results from stability and numerical simulations.
- Phenomenology from a concrete quantization of black hole interior. Consequences of Weyl causing classical singularity and the resulting asymmetric bounce.
- Can we engineer a symmetric bounce?
- Summary

Progress so far in Loop Quantum Cosmology

- Rigorous quantization of various spacetimes performed. Detailed understanding of different quantization prescriptions. Improved dynamics is the only viable choice in isotropic models.

- Stability of quantum Hamiltonian difference eq well understood.

- Quantum bounce in isotropic FRW in presence of massless scalar, radiation, $\pm\Lambda$ and spatial curvature. Ongoing attempts to include potentials. Inhomogenities included in hybrid approach.

- Quantum bounces established rigorously in vacuum Bianchi-I spacetime using high performance computing. (Pawlowski's talk)

- Effective dynamics verified extensively. Rich phenomenology. Insights on generic singularity resolution. Signatures in CMB explored. (Talks by Martineau, Olmedo, Wang, Wilson-Ewing)

(Agullo, Ashtekar, Barrau, Brizuela, Bodendorfer, Bojowald, Calcagni, Cartin, Campiglia, Chiou, Corichi, Craig, Dapor, Date, Diener, Engle, Fleischhack, Grain, Gupt, Hanusch, Henderson, Hossain, Joe, Kaminski, Kagan, Karami, Koslowski, Khanna, Lewandowski, Ma, Maartens, Martin-Benito, Martin-de Blas, Megevand, Mena Marugan, Mielczarek, Montoya, Nelson, Olmedo, Pawlowski, PS, Puchta, Rovelli, Sakellariadou, Sahlmann, Saini, Sloan, Szulc, Taveras, Thiemann, Tsujikawa, Vandersloot, Varadarajan, Vidotto, Willis, Wilson-Ewing, ...)

Progress so far on loop quantization of black holes

Many complexities going beyond cosmological setting.
Various careful constructions show that central singularities in black holes can be eliminated. Interesting implications for black hole evaporation, Hawking radiation, shell collapse etc. (Gambini's talk)

(Ashtekar, Bojowald (06); Modesto (06); Bojowald, Swirdeski (06); Boehmer, Vandersloot (07); Campiglia, Gambini, Pullin, Olmedo, Rastgoo (07-16)); Corichi, PS (16))

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• Singularity resolution argued via properties of difference equation. No numerical simulations and so far no evidence of bounce.

• Stability of quantum difference equation not well understood.

• Consistent viable quantization?

• Not much known on bounces even in effective dynamics.

• Connection to phenomenology from related works remains unexplored (mass gap, Choptuik scaling, observable signatures)

(Hajiceck, Kiefer (01); Bojowald, Goswami, Maartens, PS (05); Husain, Winkler (06); Goswami, Joshi, PS (06); Ziprick, Kunstatter (10); Kreienbuehl, Husain, Seahra (12); Tavakoli, Marto, Dapor (14); Barcelo, Carballo-Rubio, Garay, Jannes (11-15); Barrau, Bolliet, Christodoulou, Haggard, Perez, Rovelli, Speziale, Vidotto,... (14-16))

Questions we are interested in this talk

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• Schwarzschild interior is Kantowski-Sachs vacuum. Highly anisotropic singularity implies in general asymmetric bounce. In some studies probing QG effects in collapse, such as Planck stars, a symmetric bounce is considered as in isotropic models with a Ricci (instead of Weyl) dominated singularity.
Is a symmetric bounce possible? **(Olmedo, Saini, PS (appearing soon))**

Classical aspects

Schwarzschild interior can be described by a vacuum Kantowski-Sachs spacetime. Spatial manifold: $\mathbb{R} \times \mathbb{S}^2$, with a fiducial metric:

$$\mathrm{d}s_o^2 = \mathrm{d}x^2 + r_o^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2)$$

To define symplectic structure, restrict the non-compact x coordinate by L_o. Fiducial volume of the cell $V_o = 4\pi r_o^2 L_o$.

Using the symmetries, and imposing Gauss constraint, the connection and triads become:

$$A_a^i\,\tau_i\,\mathrm{d}x^a \;=\; \frac{c}{L_o}\,\tau_3\,\mathrm{d}x + b\,\tau_2\mathrm{d}\theta - b\tau_1\sin\theta\,\mathrm{d}\phi + \tau_3\cos\theta\,\mathrm{d}\phi$$

$$E_i^a\,\tau^i\,\frac{\partial}{\partial x^a} \;=\; p_c\,\tau_3\,\sin\theta\,\frac{\partial}{\partial x} + \frac{p_b}{L_o}\,\tau_2\,\sin\theta\,\frac{\partial}{\partial\theta} - \frac{p_b}{L_o}\,\tau_1\,\frac{\partial}{\partial\phi}$$

The connection and triad components are invariant under freedom to rescale coordinates and satisfy: $\{c, p_c\} = 2G\gamma, \;\; \{b, p_b\} = G\gamma$

Under rescaling of fiducial length $L_o \to \xi L_o$: $c \to \xi c$, $p_b \to \xi p_b$. p_c and b invariant. Physical predictions must be independent of ξ.

Classical aspects

Spacetime metric:

$$\mathrm{d}s^2 = -N^2\mathrm{d}t^2 + \frac{p_b^2}{|p_c|L_o^2}\,\mathrm{d}x^2 + |p_c|\,(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2)$$

where $$\frac{p_b^2}{|p_c|L_o^2} = (2m/t - 1), \qquad |p_c| = t^2, \qquad m = GM$$

Classical dynamics:

$$\mathcal{H}_{\text{class}} = -\frac{N\mathrm{sgn}(p_c)}{2G\gamma^2}\left((b^2+\gamma^2)\frac{p_b}{\sqrt{|p_c|}} + 2bc|p_c|^{1/2}\right)$$

For a black hole of mass m:

$$b(t) = \pm\gamma\,\sqrt{(2m-t)/t}\,, \quad p_b(t) = L_o\,\sqrt{t(2m-t)}$$

$$c(t) = \mp\gamma\frac{mL_o}{t^2}, \qquad \text{and} \quad p_c(t) = \pm t^2$$

Singularity at $p_b = 0$ and $p_c = 0$. Horizon at $p_b = 0$, $p_c = 4m^2$.

Quantization

$$C_{\rm Ham} = -\int \mathrm{d}^3x\, e^{-1}\varepsilon_{ijk}E^{ai}E^{bj}(\gamma^{-2}\,F^k_{ab} - \Omega^k_{ab})$$

F^i_{ab} expressed in terms of holonomies over loops in $x-\theta$, $x-\phi$ and $\theta-\phi$ planes. Edge length of the loops along x direction and along $\mathbb{S}^2$ parameterized by δ_c and δ_b respectively.

Choice 1: **(Ashtekar, Bojowald; Modesto (05))** Minimum area of all the loops equal to the same constant value (early LQC idea).

Choice 2: **(Boehmer, Vandersloot (07))** Loops with triad dependent areas motivated by improved dynamics in LQC **(Ashtekar, Pawlowski, PS (06))**

Choice 3: **(Corichi, PS (16))** A more careful implementation of fixed area loops considering underlying geometry.

All choices yield non-singular quantum Hamiltonian constraint. This does not mean all of them are viable. Independence from fiducial length and correct infra-red behavior necessary.

Quantization prescriptions: comparison

Quantum constraint consists of $\sin(\delta_b b)/\delta_b$ and $\sin(\delta_c c)/\delta_c$.
Departures from classical theory when $\delta_b b$ and $\delta_c c$ large.

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Choice 2: **(Boehmer, Vandersloot (07))** Mimics improved dynamics: $\delta_b = \sqrt{\frac{\Delta}{p_c}}$, $\delta_c = \sqrt{\frac{p_c \Delta}{p_b}}$. $\sin(\delta_c c)$ and $\sin(\delta_b b)$ terms do not suffer with fiducial length rescaling. But, $\delta_c c$ diverges at horizon implying large "Planck scale effects" at small spacetime curvature. Spacetime after bounce not a white hole **(Dadhich, Joe, PS (15))**.

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Choice 3: **(Corichi, PS (16))** No fiducial cell dependence. Right infra-red behavior. Min. area of $\square_{x-\theta}$ and $\square_{x-\phi}$: $\delta_b r_o \delta_c L_o = \Delta$
Min. area of $\square_{\theta-\phi}$: $(\delta_b r_o)^2 = \Delta$

(open loop, generalization possible **(Olmedo, Saini, PS (to appear))**)

Quantum Hamiltonian constraint

Quantization results in an anisotropic difference equation with unequal spacings in volume $V = 4\pi|p_b||p_c|^{1/2}$:

$$
\begin{aligned}
\hat{C}\Psi(\mu,\tau) \quad = \quad & \Big[\left(V_{\mu+\delta_b,\tau} - V_{\mu-\delta_b,\tau} + V_{\mu+3\delta_b,\tau+2\delta_c} - V_{\mu+\delta_b,\tau+2\delta_c}\right) \Psi(\mu+2\delta_b,\tau+2\delta_c) \\
& + \left(V_{\mu-\delta_b,\tau} - V_{\mu+\delta_b,\tau} + V_{\mu+\delta_b,\tau-2\delta_c} - V_{\mu+3\delta_b,\tau-2\delta_c}\right) \Psi(\mu+2\delta_b,\tau-2\delta_c) \\
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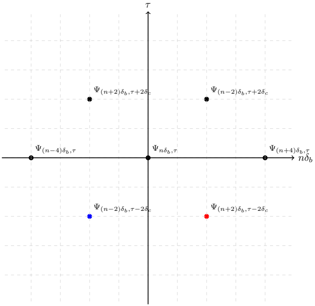
& + \left(V_{\mu+\delta_b,\tau} - V_{\mu-\delta_b,\tau} + V_{\mu-\delta_b,\tau-2\delta_c} - V_{\mu-3\delta_b,\tau-2\delta_c}\right) \Psi(\mu-2\delta_b,\tau-2\delta_c) \\
& + \frac{1}{2}\Big[\left(V_{\mu,\tau+\delta_c} - V_{\mu,\tau-\delta_c} + V_{\mu+4\delta_b,\tau+\delta_c} - V_{\mu+4\delta_b,\tau-\delta_c}\right) \Psi(\mu+4\delta_b,\tau) \\
& + \left(V_{\mu,\tau+\delta_c} - V_{\mu,\tau-\delta_c} + V_{\mu-4\delta_b,\tau+\delta_c} - V_{\mu-4\delta_b,\tau-\delta_c}\right) \Psi(\mu-4\delta_b,\tau)\Big] \\
& + 2(1+2\gamma^2\delta_b^2)(V_{\mu,\tau-\delta_c} - V_{\mu,\tau+\delta_c})\Psi(\mu,\tau)\Big] / (2\gamma^3\delta_b^2\delta_c l_{\mathrm{Pl}}^2)
\end{aligned}
$$

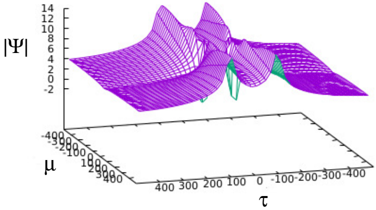
($\delta_b = \sqrt{\Delta}/2m$ and $\delta_c = \sqrt{\Delta}/L_o$)

At classical scales, yields the corresponding Wheeler-DeWitt equation. Correct classical limit at the horizon.

Preliminary results from numerical simulations

(Yonica, Khanna, PS (in progress)) For black holes with masses much larger compared to Planck mass, difference equation turns out to be von-Neumann stable. Starting from initial data, stable evolution in either τ or μ as clock can be obtained.





Non-singular numerical evolution across the central singularity at $\tau = 0$ achieved. Relative fluctuations grow near the bounce but have very similar features across the bounce. Agreement with classical trajectories far away from bounce.

Physics from effective dynamics

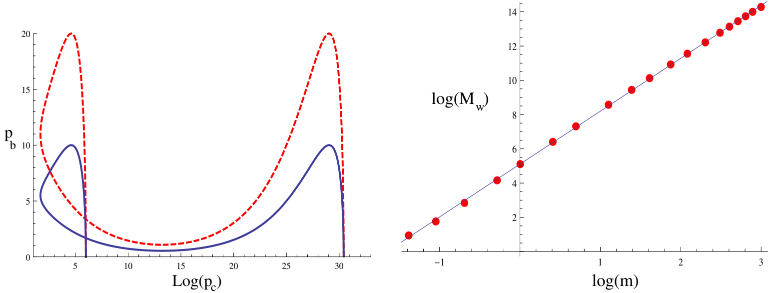
$$\mathcal{H}_{\text{eff}} = -\frac{N \text{sgn}(p_c)}{2G\gamma^2}\left[2\frac{\sin(\delta_c c)}{\delta_c}\frac{\sin(\delta_b b)}{\delta_b}|p_c|^{1/2} + \left(\frac{\sin^2(\delta_b b)}{\delta_b^2} + \gamma^2\right) p_b |p_c|^{-1/2}\right]$$

Very similar to Bianchi-I spacetime where effective dynamics validated recently

(Diener, Megevand, Joe, PS (17))

Key features:

- Unlike earlier quantizations, white hole mass independent of fiducial length L_o.
- Bounce highly asymmetric due to anisotropic shear. WH mass is approximately a quartic power of the initial BH mass.



Some details of quantum gravitational regimes

(Corichi, PS (to appear))

- Generally there are two distinct quantum gravitational regimes. The departures of $\sin(\delta_c c)/\delta_c$ from classical connection c begin and end quickly in comparison to departures of $\sin(\delta_b b)/\delta_b$ from classical connection b.
- For some time, the effective geometry is a mixture of "black hole" and "white hole" geometries. In this period, quantum regime in c has passed and that in b is yet to begin.
- Quantum regime in b very asymmetric in proper time. Very short regime in the quantum black hole geometry, but a very long regime in quantum white hole geometry.

For a black hole of mass $m = 50$:
Time to cross quantum regime in c: ≈ 3.6 Planck seconds
Time to cross quantum regime in b: $\approx$ 37000 Planck seconds
(Only 126 Planck seconds to reach bounce from black hole horizon)
Time to white hole horizon: $\approx 1.36 \times 10^{12}$ Planck seconds

Different time steps to white hole formation

- Bounce time to cross quantum regime in c.
- Bounce time to cross quantum regime in b, into white hole geometry. Dominates quantum regime.
- Time to form the white hole horizon. (Delivery time from the parent black hole to the child white hole)

Results from numerics:

Proper time for bounce scales exactly as m for large black holes with $m \geq 10$ (universal relationship).

Scales roughly as m^2 for Planck size black holes $(m \sim 0.7 - 5)$.

Size matters for bounce time because quantum geometry explicit.

Delivery time scales exactly as m^5 for black holes of all masses.

These are not the transition times seen by external observers.
How do these translate to relevant scaling for external observers?

Are symmetric bounces possible in loop quantum Schwarzschild interior?

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(Olmedo, Saini, PS (to appear)) Asymmetry in bounce occurs because p_b and p_c bounce at different times and the lack of reflection symmetry at the bounce times.

Demanding bounce to be symmetric in the effective dynamics leads to

$$\frac{\operatorname{arctanh}\left[\frac{1}{\sqrt{1+\gamma^2\delta_b^2}}\right]}{\sqrt{1+\gamma^2\delta_b^2}} = \frac{1}{4}\log\left(\frac{8GM}{\gamma L_o \delta_c}\right)$$

This condition can not be satisfied in the CS quantization for any real value of M.

The constraint on the edge lengths of the loops δ_b and δ_c can be fulfilled by assuming generalization in loop construction via some constants α and β:

$$\delta_b^2 r_o^2 = \alpha^2 \Delta, \quad \delta_b r_o \delta_c L_o = \alpha\beta\Delta$$

Symmetric bounces

One such generalization allows keeping the closed loop intact, but exploits the freedom in the open loop as

$$\delta_b r_o \delta_c L_o = \Delta, \quad \delta_b^2 r_o^2 = \alpha^2 \Delta$$

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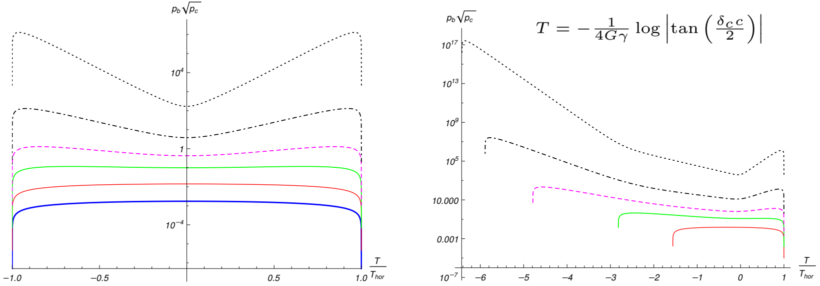

In contrast to CS model (right), symmetric bounces can be obtained by altering the construction of loops over which holonomies are constructed. Different quantizations possible which yield a symmeric bounce, albeit with a delicate loop construction.

Summary/open issues

- Is there a consistent viable loop quantization of Schwarzschild interior? Hopefully, but various checks still remaining.
  Quantization prescription strictly tied to Schwarzschild spacetime. Horizon plays a key role in this choice. Problems with improved dynamics motivated prescription occur because of the coordinate singularity!
  (But, gives generic resolution of strong singularities in Kantowski-Sachs cosmology! **(Saini, PS (16))**). Surprising (but related) results found in Milne and flat Kasner spacetimes in LQC **(Garriga, Vilenkin, Zhang (13); PS (16))**.
  Important to have an understanding of LQG effects in empty spacetimes.

- In Schwarzschild interior bounces are highly asymmetric.
  This is in general true whenever Weyl dictates singularity.

- Bounces can be constructed to be symmetric by "tuning" the quantization prescription. Viable? Desired mass scaling? Relation with ideas in QRLG (Alesci's talk) or coherent states (Liegener's talk)?

- Concrete quantum gravitational model available, detailed phenomenological implications can be studied. So far only first steps taken. Some ideas can be useful for Planck stars.