Group Field Theory and Tensor Networks: holographic entanglement entropy in full quantum gravity

LOOPS'17

Warsaw - 07.07.2017

Goffredo Chirco - AEI

GC, D.Oriti, M.Zhang arXiv:1701.01383v2

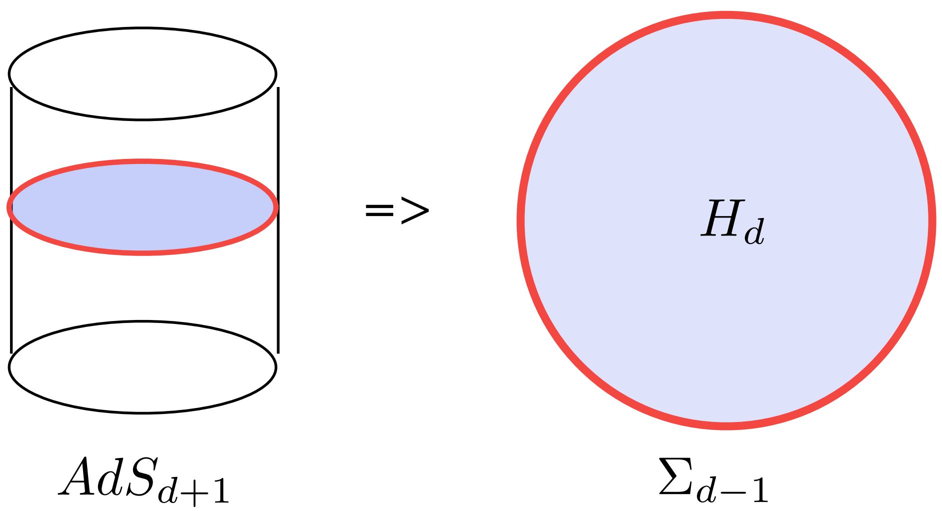
A new common framework in Quantum Gravity

major advances in our understanding of quantum gravity come from insights and techniques from quantum information theory and quantum statistical mechanics

Holographic principle

Bekenstein 81,t'Hooft 93, Susskind 94

=> gauge field theory/gravity duality



String theory: AdS/CFT duality

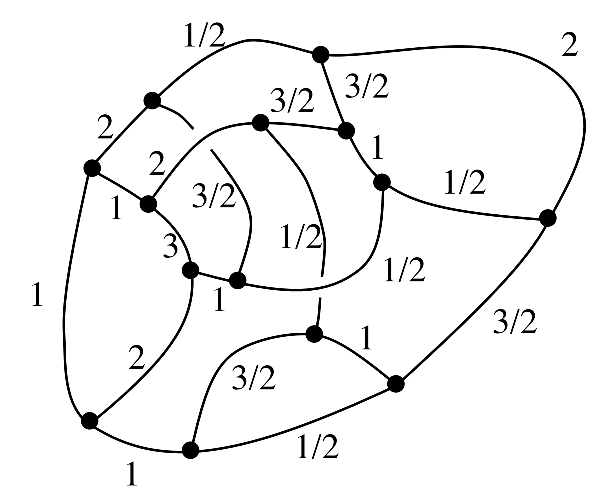
Equivalence principle

Einstein => Penrose 71, Rovelli & Smolin 95

=> relational texture of space-time

ST dynamical record of frames transformations

Spin-Networks



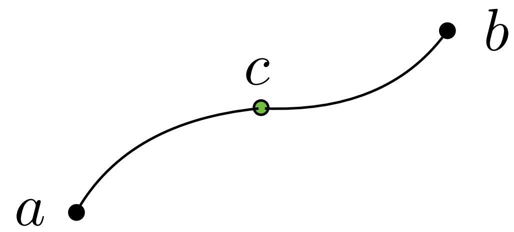
LQG, SF, GFTs: quantum space-(time) geometry described by discrete, pre-geometric degrees of freedom, of combinatorial and algebraic nature

in both scenarios quantum entanglement becomes a tool to characterise the quantum texture of space-time in terms of the structure of correlations of some microscopic states as well as the emergence of a continuum geometric description for space-time geometry

LQG: space(time) from entangled states of quantum geometry

gravity as a lattice gauge theory on a superposition of SU(2)/SL(2,C) spin-network graphs

- LQG structural level:



$$= \frac{1}{\sqrt{2j+1}} \sum_{c=1}^{2j+1} \langle U|\gamma_1, j, a, c\rangle \, \langle U|\gamma_2, j, c, b\rangle$$

maximally mixed state

Donnelly 2012

=> space geometry from pre-geometry, ent & coarse graining

(study of continuum limit)

Girelli Livine 05, Livine Terno 2005-08
Dittrich, Bahr, Steinhaus, Martin-Benito...
Charles Livine 2016, GC Mele, Vitale, Oriti

=> diffeos compatible definition of entanglement: localisation and boundary charges — holographic dualities?

Freidel Donnelly 16 Freidel Perez Pranzetti 16
Delcamp Dittrich Riello, Geiller 16-17

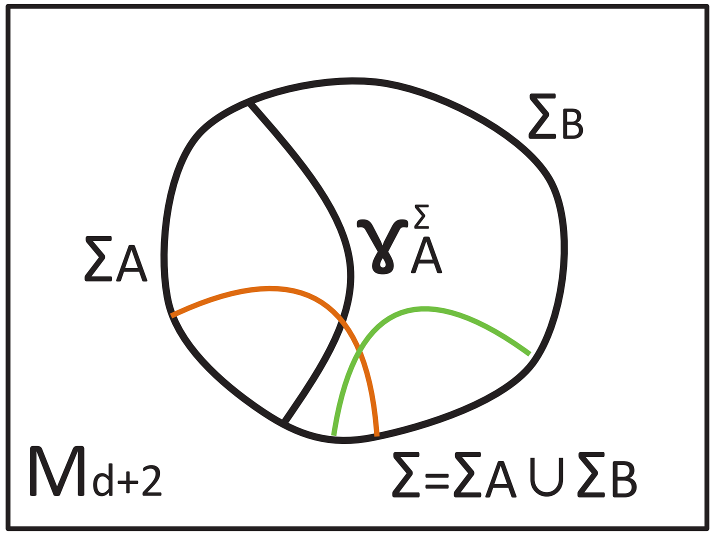
- Area law for entanglement entropy as a signature of good semiclassical behaviour:

Bianchi Myers 2012
Bianchi Guglielmon Hackll Yokomizo 16
GC Rovelli Haggard Riello Ruggiero 14-15, Hamma Hung Marciano Zhang 15
GC Anzà 16, Han et al. 16

& BH entropy: Rovelli, Perez, de Lorenzo, Smerlak, Husain, Bodendorfer, Oriti, Pranzetti Sindoni … \infty similar structural behaviour in AdS/CFT:

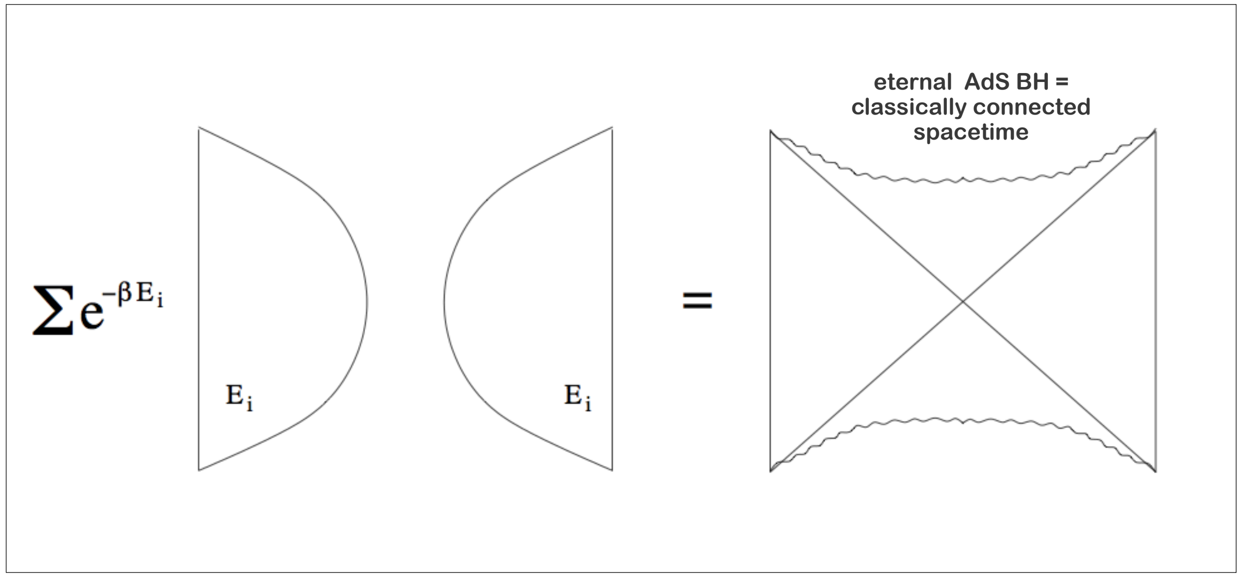
gravitational theories are equivalent to non-gravitational theories defined as quantum many-body systems or quantum field theories: a dual non-gravitational theory lives on the boundary of its original gravitational spacetime

- holographic entanglement entropy $$S_A = \frac{\mathrm{Area}(\gamma_A)}{4G_N}$$



=> a series of attempts of space-time bulk geometry reconstruction from the structure of correlations of the boundary state

Ryu–Takayanagi 2015] … [Hubeny, Rangamani]



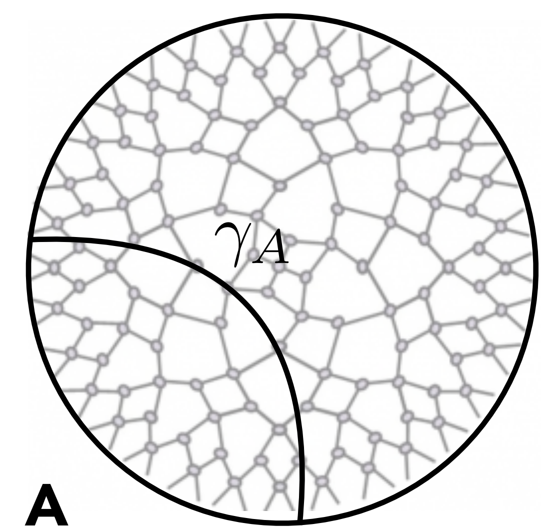
[Van Raamsdonk 2009] [Cao Carroll Michalakis 2016]

- QIT toy models for the bulk/boundary correspondence: holographic quantum error-correcting codes

[Pastawski, Yoshida, Harlow Preskill]

one of main direct interests for our community:

- again key role played by networks: CMT Tensor Network techniques to express quantum wave functions in terms of network diagrams => help understanding holography via geometrization of algebraically complicated quantum states



e.g. MERA states: particular example of TN states designed to support the entanglement of a CFT with geometrical interpretation (hyperbolic space)

conjecture: AdS/CFT correspondence to be interpreted as a Multiscale Entanglement Renormalisation Algorithm

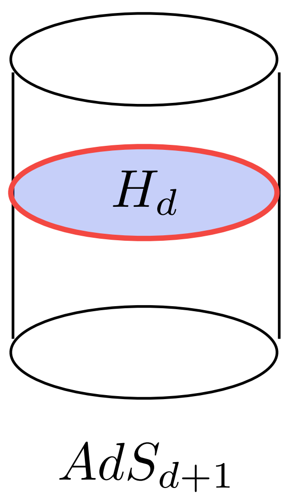
[Vidal 06, Swingle 09]

generalisation Miyaji–Takayanagi 2015]

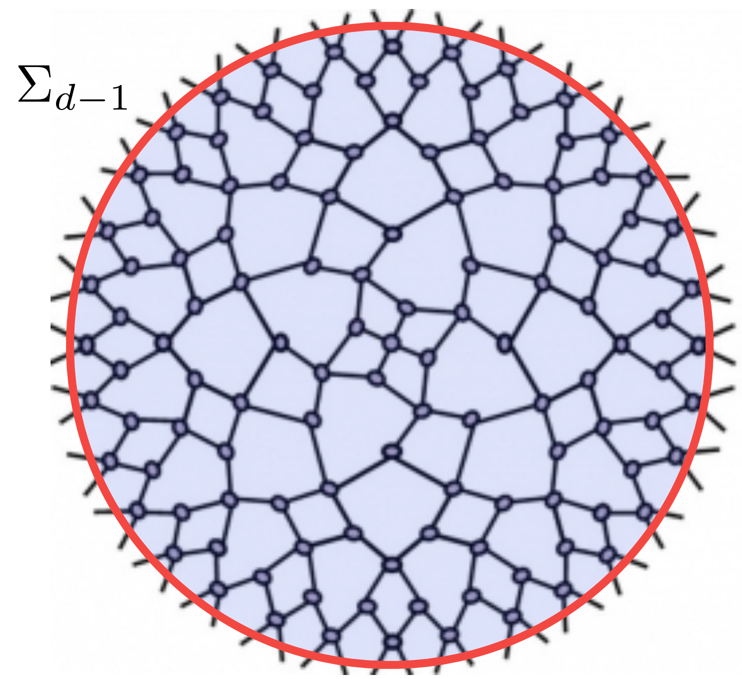
surface/state correspondence: in any spacetime described by Einstein gravity, each codim-2 convex surface corresponds to a quantum state in the dual theory. This largely extends the holographic principle as it can be applied to gravitational spacetime without any boundaries

Freidel, Donnelly, Pranzetti, Dittrich,……

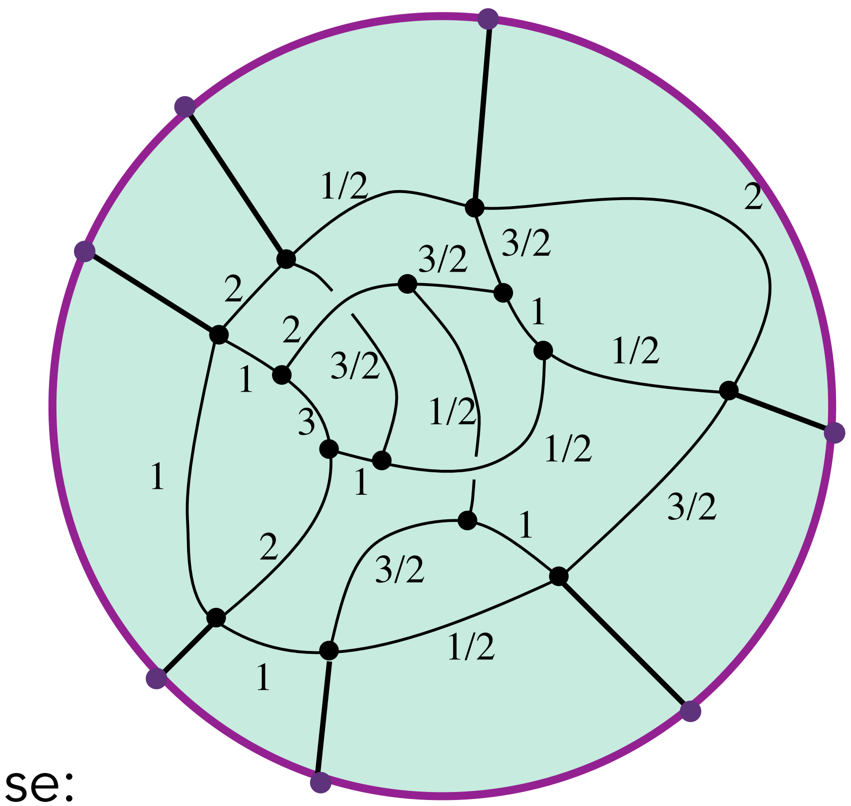
Frame a common field in Quantum Gravity



=>



<=>



LQG-wise:

surface/state correspondence

continuum limit through renormalisation of general random cellular complexes dual to discrete space w/ possible QFT duals on boundaries

think of TND path integral definition of boundary states through sums over spin-foam 'open graph states'

sketch a concrete realisation of this scenario by means of the generalised GFT formalism:

=> define a GFT Tensor Network analogue with nice properties

=> look for the holographic behaviour of entanglement entropy

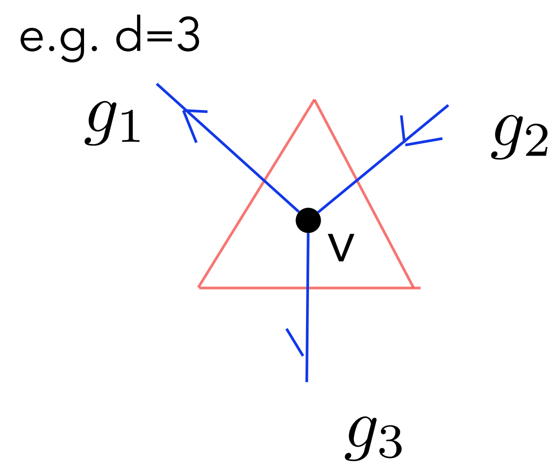
Group Field Theories (GFTs) are combinatorially non-local quantum field theories defined on a group manifold

$$\phi(g_1, ..., g_d) : G^{\times d} \to \mathbb{C}$$

random function (field)

$$\mathrm{d}\nu(\phi)/Z$$

probability measure



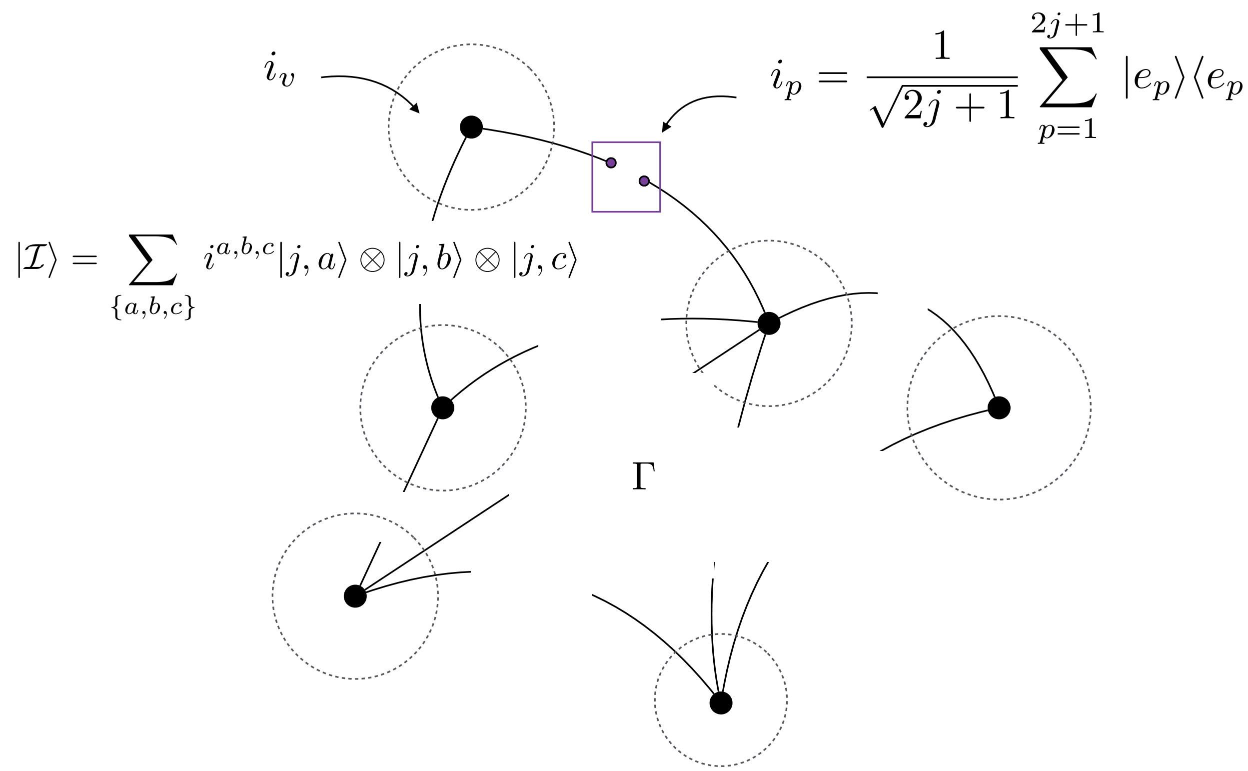
with gauge invariance at the vertex

$$\phi(g_i) = \phi(g_i\beta)$$

provide a 2n quantisation scheme for LQG:

- no embedding in a continuum manifold and no cylindrical consistency imposed on our quantum geometry wave-functionals

- Fock construction through decomposition of spin network states in terms of elementary building blocks corresponding to tensor maps associated to nodes of the spin network graphs (quantum many body system)



GFTs random lattices as simplicial geometries

- the Feynman diagrams $\mathcal{F}$ of the theory are dual to cellular complexes, and the perturbative expansion of the quantum dynamics defines a sum over random lattices of (a priori) arbitrary topology

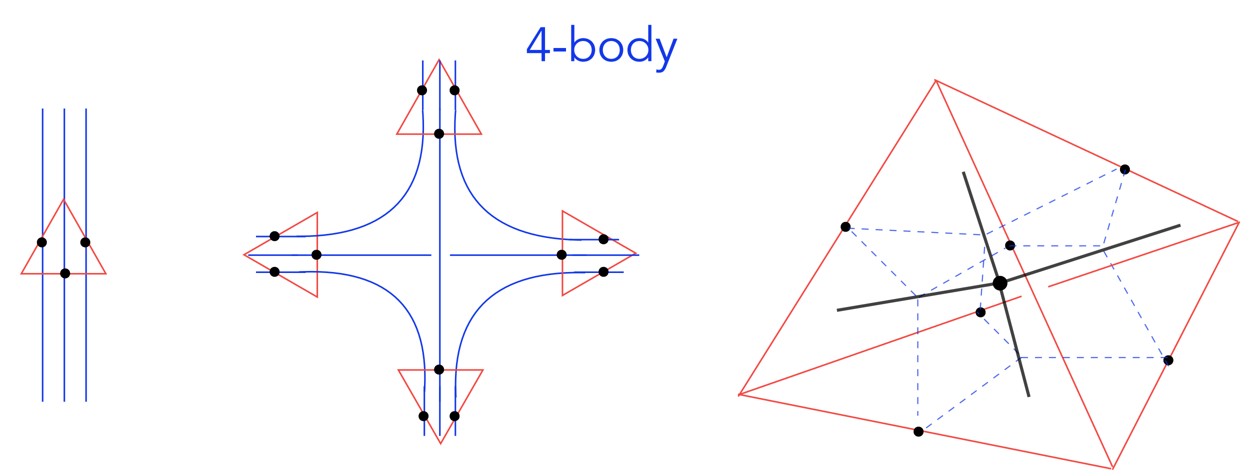
$$Z = \int \mathcal{D}\phi \mathcal{D}\bar{\phi} e^{-S[\phi,\bar{\phi}]} = \sum_{\sigma} \frac{\Pi_i (\lambda_i)^{N_i(\sigma)}}{Aut(\sigma)} \mathcal{A}_\sigma$$

- for GFT models where appropriate group theoretic data are used and specific properties are imposed on the states and quantum amplitudes, the same lattice structures can be understood in terms of simplicial geometries

$$S_d[\phi] = \int \mathrm{d}g_i \mathrm{d}g_i' \, \phi(g_i) \, \mathcal{K}(g_i g_i'^{-1}) \, \phi(g_i') \, + \, \lambda \int \prod_{i\neq j=1}^{d+1} \mathrm{d}g_{ij} \, \mathcal{V}(g_{ij} g_{ji}'^{-1}) \, \phi(g_{1j}) \cdots \phi(g_{d+1 j})$$

with

$$\boxed{\begin{aligned} \mathcal{K}(g_i, g_i') &= \int_G \mathrm{d}h \prod_i \delta(g_i g_i'^{-1} \, h), \\ \mathcal{V}(g_{ij} g_{ji}'^{-1}) &= \int_G \prod_i \mathrm{d}h_i \prod_{i<j} \delta(h_i \, g_{ij}) \end{aligned}}$$



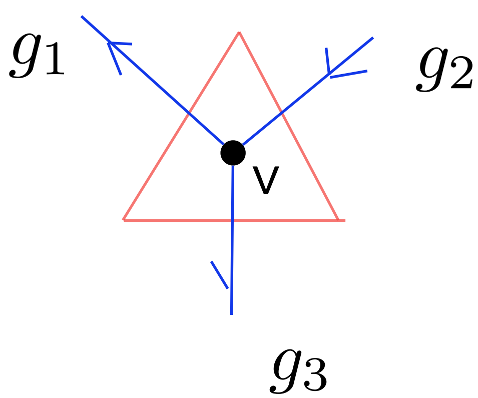
(e.g. Boulatov model => Ponzano-Regge)

emphasise: GFT fields as tensors

- the single-particle quantum state

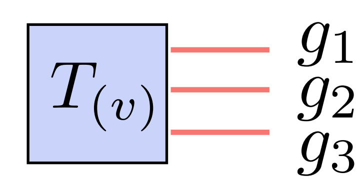
$$|\phi\rangle = \int_{G^d} \mathrm{d}g_i \; \phi(g_i)|g_i\rangle \in \mathcal{H}^{\otimes d} \quad \text{for} \quad |g_i\rangle \in \mathcal{H} \simeq L^2[G]$$

behaves as generalised rank-d tensor states, i.d. multi dimensional arrays of c-numbers



$$\boxed{\phi(\vec{g}) \;\; <=> \;\; T_{\lambda_1,\lambda_2 \ldots,\lambda_d} \equiv T(\vec{\lambda})}$$

or



where generally $\quad T : X \to \mathbb{C} \;$ with $\; X = \{\vec{\lambda}|\vec{\lambda} = (\lambda_1, ..., \lambda_d)\}$

(exploit the combinatorially tensorial nature)

- a V-particle states can be decomposed into products of elements of single-particle space

$$|\Phi\rangle = \int \prod_{j=1}^{V} \mathrm{d}g_i^j \, \Phi(g_i^j) \, |\mathbf{g}^1\rangle \ldots |\mathbf{g}^V\rangle \in \mathcal{H}^{d \otimes V} \simeq L^2[G^{d\times V}/G^V]$$

with $\quad \Phi(g_i^j) = \Phi(g_1^1, g_2^1, ..., g_d^1, g_1^V, ... g_d^V)$

can be seen as a Tensor Network state

In the tensor network methods, a quantum state $|\Psi\rangle$ is described in terms of a set of tensors. Consider a lattice **L** made of **N** sites, where each site is described by a complex vector space $\mathbb{V}$ of finite dimension **d**.

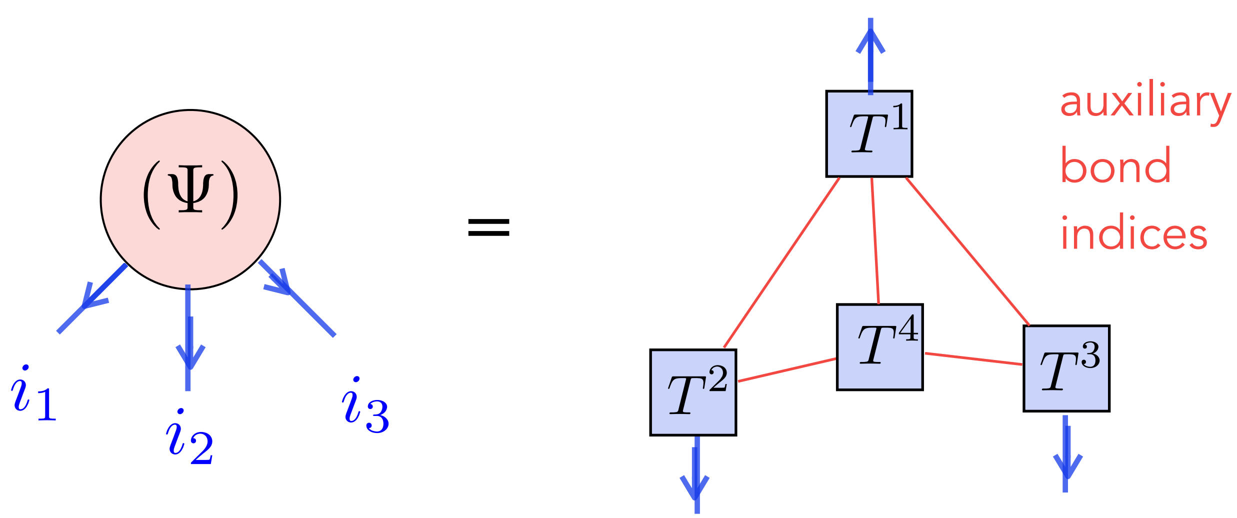
[Vidal]

- a pure state $|\Psi\rangle \in \mathbb{V}^{\otimes N}$ of the lattice can be expanded as

$$|\Psi\rangle = \sum_{i_1,i_2,\ldots,i_N=1}^{d} (\psi)_{i_1,i_2,\ldots,i_N} |i_1, i_2, \ldots, i_N\rangle$$

where $|i_s\rangle$ denotes a basis of $\mathbb{V}$ for site s in L

- a TN decomposition for $|\Psi\rangle$ consists of a set of tensors $T^{(v)}$ and a network pattern or graph characterised by a set of vertices and a set of directed edges



e.g. N=3

$$\boxed{(\psi)_{i_1,i_2,\ldots,i_N} = \mathrm{tTr}\left(\bigotimes_v T^{(v)}\right)}$$

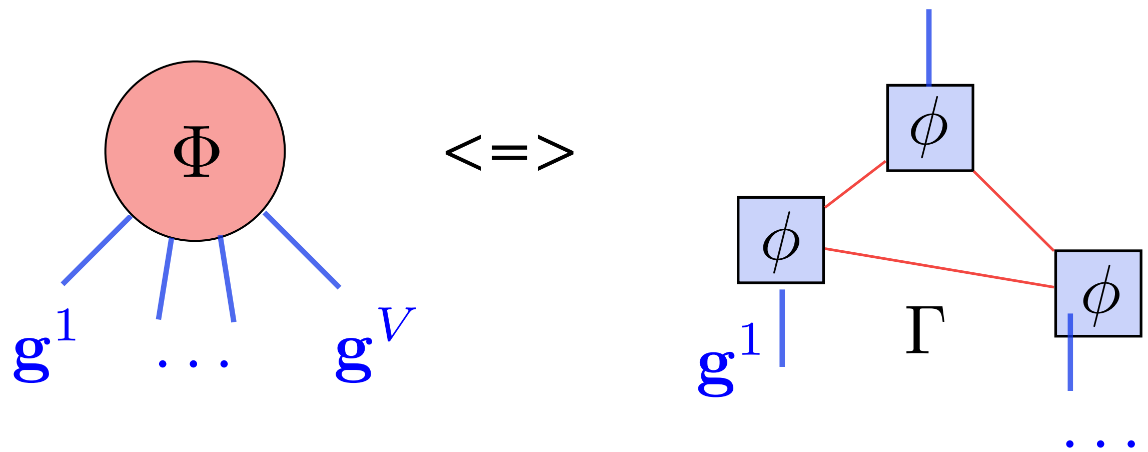
the tensor trace contracts all bond indices, leaving only the physical indices

Multiparticle state as a tensor network state

- we can understand the wave-function on an open graph of V vertices or their dual polyhedra as a tensor network encoding the entanglement structure of the multi-particle state

$$\Phi(g_i^j) \quad <=> \quad (\Phi)_{\mathbf{g}^1,\mathbf{g}^2,\ldots,\mathbf{g}^V} = \mathrm{tTr}\left(\bigotimes_{v=1}^{V} \phi(g_i)_v\right)$$

- construct a representation with auxiliary group fields



glued by links convolution functions Mij $\quad |M_\ell\rangle = M_{ij}\,|g_i\rangle \otimes |g_j\rangle \in \mathcal{H}^{\otimes 2}$

- a V-particle states can be then decomposed as

$$\boxed{|\Phi_\Gamma\rangle \equiv \bigotimes_{\ell\in\Gamma} \langle M_\ell| \bigotimes_{v}^{V} |\phi_v\rangle}$$

Dictionary: GFT states (many body wave-functions) as tensors networks

Table A	Group Fields	Tensors	
quantum	$\|\vec{g}\,\rangle \in \mathbb{H}^{\otimes d} \simeq L^2[G^d]$	$\|\lambda_i\rangle$, $i = 1, \ldots, d_{\|\lambda\|} = D$ in $\mathbb{H}_D$	
one particle state	$\|\varphi\rangle = \varphi(\vec{g})\,\|\vec{g}\,\rangle$	$\|T_n\rangle = T_{\lambda_1\cdots\lambda_d}\|\vec{\lambda}\rangle \in \mathbb{H}_n = \mathbb{H}_D^{\otimes d}$	tensor state
gluing functional	$\langle M_{g_\ell}\| =$ $\int \mathrm{d}g_1\mathrm{d}g_2\ M(g_1^\dagger g_\ell g_2)\ \langle g_1\|\ \langle g_2\|$ $\in \mathbb{H}^{*\otimes 2}$	$\|M\rangle = M_{\lambda_1\lambda_2}\,\|\lambda_1\rangle \otimes \|\lambda_2\rangle \in$ $\mathbb{H}_\ell = \mathbb{H}_D^{\otimes 2}$	link state
multiparticle state	$\|\Phi_\Gamma\rangle \in \mathbb{H}_V \simeq L^2[G^{d\times V}/G^V]$	$\|\Psi_\mathcal{N}\rangle$	tensor network state
product state convolution	$\|\Phi_\Gamma^{g_\ell}\rangle \equiv \bigotimes_{\ell\in\Gamma}\langle M_{g_\ell}\|\bigotimes_n^V\|\varphi_n\rangle$ $= \int \mathrm{d}g_\partial\,\Phi_\Gamma(g_\ell, g_\partial)\,\|g_\partial\rangle$	$\|\Psi_\mathcal{N}\rangle \equiv \bigotimes_\ell^L\langle M_\ell\|\bigotimes_n^N\|T_n\rangle \in$ $\mathbb{H}_{\partial\mathcal{N}}$	tensor network decomposition
randomness	$\frac{1}{Z}d\nu(\varphi)$ field theory probability measure	$T_\mu^U \equiv (UT^0)_\mu$ $T_\mu^0 \equiv T^0_{\lambda_1\cdots\lambda_d} \in \mathbb{H}_T,$ $U \in \mathrm{U}(\dim(\mathbb{H}_T))$	random tensor state

Dictionary: Spin-networks as gauge invariant tensors networks

Table B	GFT network	Spin Tensor Network	Tensor Network
node	$\varphi(\vec{g})$ $\equiv \varphi(g_1, g_2, g_3, g_4)$	$\varphi^{\mathbf{j}}_{\{m\}}$ $\propto \sum_{\{k\}} \hat{\varphi}^{\mathbf{j}}_{\{m\}\{k\}}\, i^{\mathbf{j}\{k\}}$	$T_{\{\mu\}}$
link	$M(g_1^\dagger g_\ell g_2)$	M^j_{mn}	$M_{\lambda_1\lambda_2}$
sym	$\varphi(h\vec{g}) = \varphi(\vec{g})$	$\prod_s^v D^j_{m_s m'_s}(g) i^i_{m'_1\cdots m'_v}$ $= i^i_{m_1\cdots m_v}$	$\prod_s^v U_{\mu_s\mu'_s} T_{\mu'_1\cdots\mu'_v} =$ $T_{\mu_1\cdots\mu_v}$
state	$\left\vert\Phi^{g_\ell}_\Gamma\right\rangle \equiv$ $\bigotimes_\ell \langle M_{g_\ell}\vert \bigotimes_n \vert\psi_n\rangle$	$\vert\Psi^{\mathbf{ji}}_\Gamma\rangle \equiv$ $\bigotimes_\ell \langle M^{j_\ell}\vert \bigotimes_n \vert\phi_n^{\mathbf{j}_n i_n}\rangle$	$\vert\Psi_{\mathcal{N}}\rangle \equiv$ $\bigotimes_\ell^L \langle M_\ell\vert \bigotimes_n^N \vert T_n\rangle$
indices	$g_i \in G$, $\vert g_i\rangle \in \mathbb{H} \simeq L^2[G]$	$m_i \in \mathbb{H}_j$, SU(2) spin-j irrep.	$\mu_i \in \mathbb{Z}_n$, nth cyclic group
dim	∞	$\dim \mathbb{H}_j = 2j+1$	$\dim \mathbb{Z}_n = n$

being tensor field theories, GFTS reproduce the structure of very interesting TN states: random tensor networks (RTS)

$$|\Psi\rangle \equiv \bigotimes_{<ij>} \langle M_{ij}| \bigotimes_{v}^{N} |T_v\rangle$$

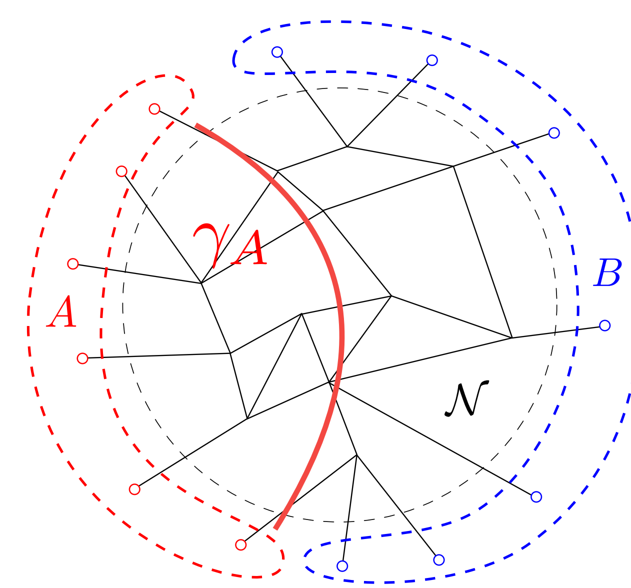
1 - maximally entangled link states $\quad |M\rangle = \frac{1}{\sqrt{D}} \delta_{\lambda_1 \lambda_2} |\lambda_1\rangle \otimes |\lambda_2\rangle$

2 - tensors T_v are unit vectors chosen independently at random from their respective Hilbert spaces. the unique "uniform" unitarily invariant distribution is induced by the Haar measure on the unitary group by acting on an arbitrarily chosen generating vector:

(for arbitrary reference state $|0_v\rangle$ define $|T_v\rangle = U|0_v\rangle$ with U unitary)

3 - In the large bond dimension limit, RTS saturate the TN entropy bound, reproducing the holographic Ryu Takanayagi entropy formula

$$S(A) \simeq \log(D)|\gamma_A|$$



Hayden et al.arXiv:1601.01694v1 F. Pastawski, B. Yoshida, D. Harlow and J. Preskill

A relevant example: Random Tensor Networks

=> holographic behaviour: random tensors networks provide explicit toy examples for the surface/state correspondence

key features of the random character:

- the random average of an arbitrary function f (T_v) of the the state | T_v > is equivalent to an integration over U according to the Haar probability measure
- averages enters linear traces operation, hence simplifying the derivation of the non-trivial entanglement properties of the states, induced as usual by partial tracing
- as the system dimension becomes large, random states have typical behaviour

Hayden et al.arXiv:1601.01694v1

=> we expect to find a similar behaviour in our GFT setting along with Hayden's statistical approach

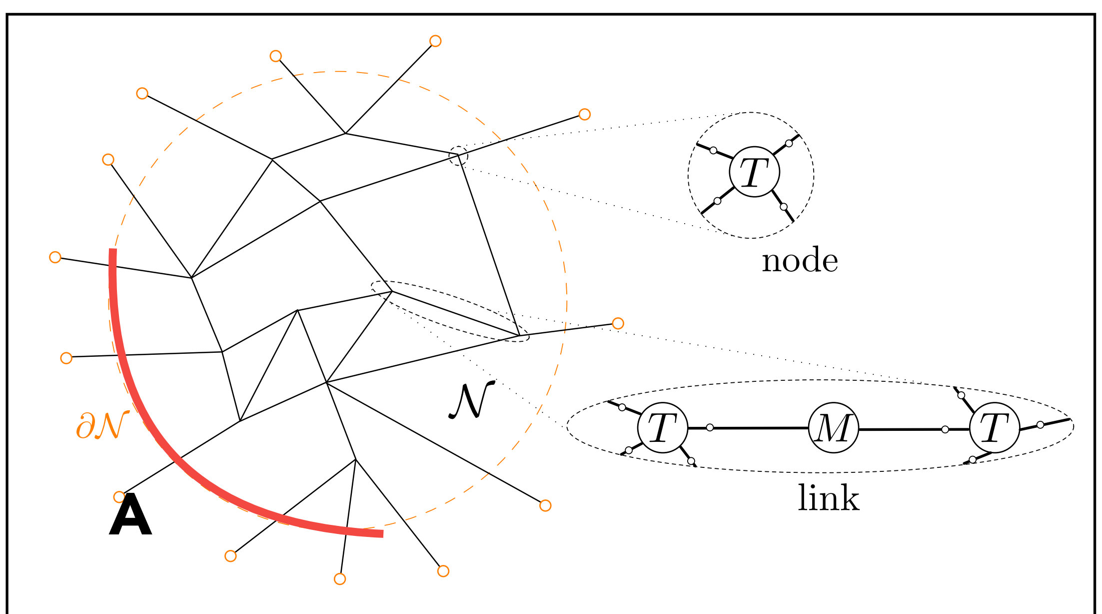
Let's consider the boundary state associated to the open spin network graph $\mathcal{N}$

$$|\Phi_{\mathcal{N}}\rangle \equiv \bigotimes_{\ell\in\mathcal{N}} \langle M_\ell| \bigotimes_{n}^{V} |\phi_n\rangle \in \bigotimes_{\ell\in\partial\mathcal{N}} \mathcal{H}_\ell$$

- the boundary density operator is a linear function of independent pure states of each tensor

$$\rho = \mathrm{tr}_\ell \left[\bigotimes_{\ell\in\Gamma} |M_\ell\rangle\langle M_\ell| \bigotimes_{v}^{V} |\phi_v\rangle\langle\phi_v| \right]$$

- to a subregion (A) of the boundary we associate a reduced state



$$\hat{\rho}_A = \mathrm{tr}_B[\rho]/\mathrm{tr}[\rho]$$

- we then look for the entanglement entropy of A/B:

$$S_{EE} = -\mathrm{tr}[\hat{\rho}_A \log \hat{\rho}_A] = \lim_{N\to 1} S_N(A) = \frac{1}{1-N} \log \mathrm{tr}[\hat{\rho}_A^N]$$

(Rényi via replika trick)

where $\quad e^{-S_N(A)} = \mathrm{tr}[\rho_A^N]/(\mathrm{tr}[\rho])^N \equiv Z_A/Z_0$

- KEY 1: calculating the Rényi entropy is hard, however we can use the random character of the field to calculate the expectation value of the Rényi: expand in the fluctuation

$$\overline{S_N(A)} = -\overline{\log \frac{\overline{Z_A} + \delta Z_A}{\overline{Z_0} + \delta Z_0}} = -\log \frac{\overline{Z_A}}{\overline{Z_0}} + \left(\sum_n^{\infty} \frac{(-1)^{n-1}}{n} \frac{\overline{\delta Z_0^n}}{\overline{Z_0^n}} - \frac{\overline{\delta Z_A^n}}{\overline{Z_A^n}} \right)$$

- KEY 2: fluctuations are suppressed in the limit of large bond dimension

random states in high-dimensional bipartite systems: “concentration of measure” phenomenon applies, meaning that on a large-probability set macroscopic parameters are close to their expectation values (bond/group dimension, => continuum limit)

- we then look for the entanglement entropy of A/B:

$$S_{EE} = -\mathrm{tr}[\hat{\rho}_A \log \hat{\rho}_A] = \lim_{N\to 1} S_N(A) = \frac{1}{1-N} \log \mathrm{tr}[\hat{\rho}_A^N]$$

(Rényi via replika trick)

where $e^{-S_N(A)} = \mathrm{tr}[\rho_A^N]/(\mathrm{tr}[\rho])^N \equiv Z_A/Z_0$

- KEY 1: calculating the Rényi entropy is hard, however we can use the random character of the field to calculate the expectation value of the Rényi: expand in the fluctuation

$$\boxed{\overline{S_N(A)} = -\overline{\log \frac{\overline{Z_A} + \delta Z_A}{\overline{Z_0} + \delta Z_0}} = -\log \frac{\overline{Z_A}}{\overline{Z_0}} + \simeq S_N(A)}$$

- KEY 2: fluctuations are suppressed in the limit of large bond dimension

random states in high-dimensional bipartite systems: “concentration of measure” phenomenon applies, meaning that on a large-probability set macroscopic parameters are close to their expectation values (bond/group dimension, => continuum limit)

permutation operator
acting on the states in A

- since $$\frac{\overline{Z_A}}{\overline{Z_0}} = \frac{\mathbb{E}(\mathrm{tr}\rho_A^N)}{\mathbb{E}(\mathrm{tr}\rho)^N} = \frac{\mathbb{E}\mathrm{tr}[\rho^{\otimes N}\mathbf{P}(\pi_A^0; N, d)]}{\mathbb{E}(\mathrm{tr}\rho)^N}$$

+ assuming factorised state $$= \frac{\mathrm{tr}\left[\bigotimes_\ell \rho_\ell^N \bigotimes_n \mathbb{E}(\rho_n^N)\mathbf{P}(\pi_A^0; N, d)\right]}{\mathrm{tr}\left[\bigotimes_\ell \rho_\ell^N \bigotimes_n \mathbb{E}(\rho_n^N)\right]} \simeq S_N(A)$$

- we can get $S_N(A)$ by computing the expectation values:

$$\mathbb{E}(\rho_n^N) = \mathbb{E}[(|\phi_n\rangle\langle\phi_n)^N] = \mathbb{E}\left[\left(\int \prod_a^N \mathrm{d}\mathbf{g}_a \mathrm{d}\mathbf{g'}_a \, \phi_n(\mathbf{g}_a)\overline{\phi_n(\mathbf{g'}_a)}|\mathbf{g}_a\rangle\langle\mathbf{g'}_a|\right)\right]$$

- in the standard field theory formalism we define the averaging via the path integral of some GFT model

$$\mathbb{E}\left[f[\phi,\overline{\phi}]\right] \equiv \int [\mathcal{D}\phi][\mathcal{D}\overline{\phi}] \, f[\phi,\overline{\phi}] \, e^{-S[\phi,\overline{\phi}]}$$

- the average over the N-replica of the wave functions (generalised tensors) associated to each network vertex can be interpreted as a GFT N-point correlation function

Path integral averaging for the free theory

- we take the case $S[\phi,\overline{\phi}] = \int \mathrm{d}\mathbf{g}\mathrm{d}\mathbf{g}' \, \overline{\phi(\mathbf{g})}\mathcal{K}(\mathbf{g},\mathbf{g}')\phi(\mathbf{g}') + \lambda S_{\mathrm{int}}[\phi,\overline{\phi}] + cc$

  with $\mathcal{K}(\mathbf{g},\mathbf{g}') = \delta(\mathbf{g}^\dagger\mathbf{g}')$

- λ <<1 and consider a perturbative expansion of the path integral in powers of λ

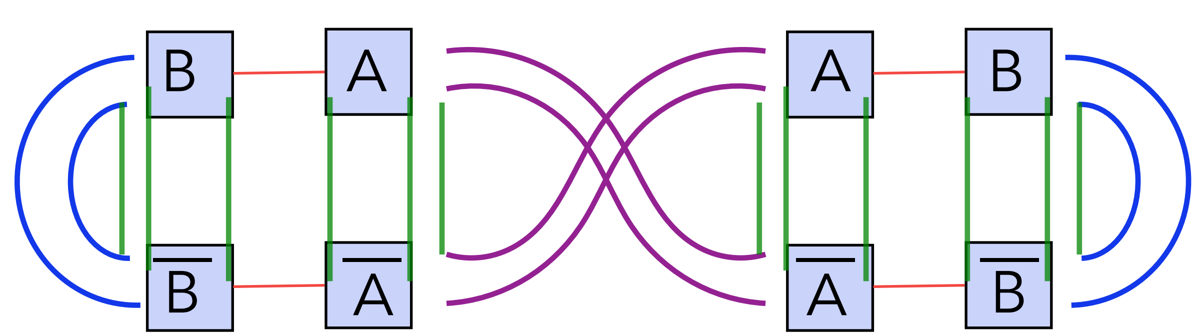
$$\mathbb{E}\left[\prod_a^N \phi(\mathbf{g}_a)\overline{\phi(\mathbf{g'}_a)}\right] \equiv \mathbb{E}_0\left[\prod_a^N \phi(\mathbf{g}_a)\overline{\phi(\mathbf{g'}_a)}\right] + \mathcal{O}(\lambda)$$

Wick theorem

$$\mathcal{C}\sum_{\pi\in\mathcal{S}_N}\mathbb{P}_{\mathbf{h}_n}(\pi_n) \quad \text{with} \quad \mathbb{P}_{\mathbf{h}}(\pi) = \prod_a^N \delta\left(h_a\mathbf{g}_a\mathbf{g}^\dagger_{\pi(a)}\right)$$

the free theory N points correlation function translates into a sum over all permutations among the group elements attached at each node

$$\mathrm{tr}[\rho_A^2] = \mathrm{tr}[\rho\otimes\rho\,\mathbf{P_A}]$$



Path integral averaging for the free theory

- Z_A and Z_0 correspond to summations of the combinatorial networks $N_A(h_n, \pi_n)$ and $N_0(h_n,\pi_n)$

$$Z_A \approx \mathcal{C}^{V_\Gamma} \sum_{\boldsymbol{\pi_n}\in\mathcal{S}_N} \int \prod_n \mathrm{d}\mathbf{h}_n \,\mathrm{Tr}\left[\bigotimes_\ell \rho_\ell^N \bigotimes_n \mathbb{P}_{\mathbf{h}_n}(\pi_n)\mathbb{P}(\pi_A^0; N, d)\right]$$

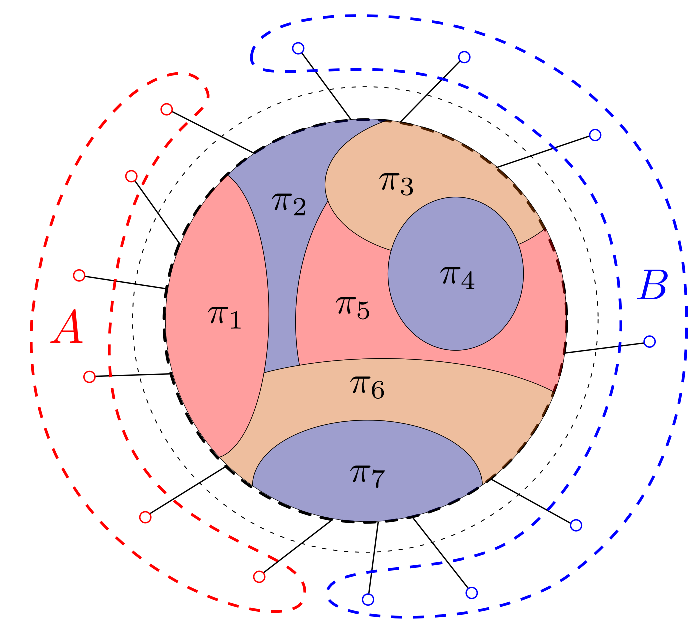
$$\equiv \mathcal{C}V_\Gamma \sum_{\boldsymbol{\pi_n}\in\mathcal{S}_N} \int \prod_n \mathrm{d}\mathbf{h}_n \,\mathcal{N}_A(\mathbf{h_n}, \boldsymbol{\pi_n})$$

at each node n we have a contribution $P_{h_n}(\pi_n)$

$$Z_0 = \mathcal{C}^{V_\Gamma} \sum_{\boldsymbol{\pi_n}\in\mathcal{S}_N} \int \prod_n \mathrm{d}\mathbf{h}_n \,\mathrm{Tr}\left[\bigotimes_\ell \rho_\ell^N \bigotimes_n \mathbb{P}_{\mathbf{h}_n}(\pi_n)\right]$$

$$\equiv \mathcal{C}^{V_\Gamma} \sum_{\boldsymbol{\pi_n}\in\mathcal{S}_N} \int \prod_n \mathrm{d}\mathbf{h}_n \,\mathcal{N}_0(\mathbf{h_n}, \boldsymbol{\pi_n})$$

links contribution



- the (Feynman)networks get divided into several regions, with same π_n and h_n. The links which connect different regions identify boundaries between each pair of different regions or domain walls.

- for different domain walls and different assignments of permutation groups to each region, we have different patterns for given network

=> Z_A and Z_0 are sum of BF amplitudes with different 2-complexes remark

this simple form of the various functions entering the calculation of the entropy, with the emergence of BF-like amplitudes, is not generic: it follows from the choice of GFT kinetic term, from the approximation used in the calculation of expectation values (neglecting GFT interactions) and from the special type of GFT tensor network chosen

- we are interested in is the leading term of Z_A and Z_0, while taking the dimension D of the bond Hilbert space much larger than 1. This leads us to seek the most divergent term of the amplitudes (bubble divergences)

=> the divergence degree of BF amplitudes discretised on a lattice has been the subject of a number of works, both in the spin foam an GFT literature

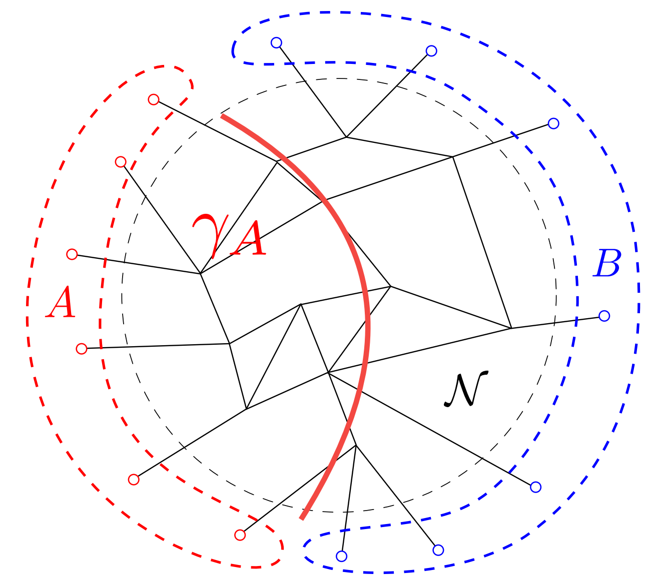
Freidel, Gurau, Oriti 09, Bonzom and Smerlak 10-12

- the Nth Rényi entropy S_N is associated to patterns with only one domain wall and h_n=1

$$\boxed{\mathrm{e}^{(1-N)S_N} = \frac{Z_N}{Z_0^N} = [\delta(\mathbb{1})]^{(1-N)\min(\#_{\ell\in\partial_{AB}})}\left[1+\mathcal{O}(\delta^{-1}(\mathbb{1}))+\mathcal{O}(\lambda)\right]}$$

When N goes to 1, S_N becomes the entanglement entropy S_{EE}. The leading term of the entanglement entropy S_{EE} is

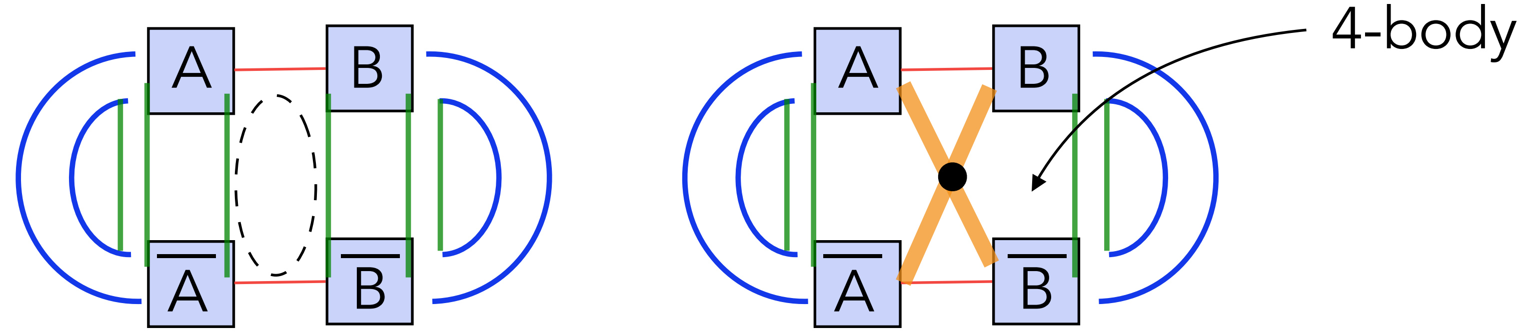
$$S_{\mathrm{EE}} = \min(\#_{\ell \in \partial_{AB}}) \ln \delta(\mathbb{1})$$



which can be understood as the Ryu-Takayanagi formula in a GFT context, with the same interpretation for the area of the minimal surface that we have mentioned in the previous section, concerning the tensor network techniques.

the interaction kernel will generally lead to non-trivial bulk corrections!

$\mathrm{tr}[\rho]^2$



Mingyi's talk later!

- establish a precise dictionary between GFT states and (generalized) random tensor networks. Such a dictionary also implies, under different restrictions on the GFT states, a correspondence between LQG spin network states and tensor networks, and a correspondence between random tensors models and tensor networks.

- compute the Rényi entropy and derived the RT entropy formula:

  - using directly GFT and spin network techniques, first using a simple approximation to a complete definition of a random tensor network evaluation seen as a GFT correlation function, but still using a truly generalized tensor network seen as a GFT state, and then considering directly a spin network state as a random tensor network. This elucidates further the correspondence and its potential

- the result shows that the same formalism allows to compute non-perturbative quantum gravity corrections to the Ryu-Takayanagi formula, by including the contributions from the GFT interaction term into the amplitude (as well as considering different choices for the GFT kinetic term).

  Mingyi's talk later!

Remarks/Speculations

- AdS/MERA/CFT may be extended, beyond AdS/CFT, to a more general space/TNR/QFT correspondence: GFTs may play a role as auxiliary tensor field theories both fixing the entanglement structure of the boundary physical theory and providing a dual simplicial characterisation of the tensor network diagrams as discretised space

- dynamics induces entanglement: looks like standard multi scale renormalisation techniques may be associated to cMERA within the field theory framework of GFTs: what would play the role of the MERA-like renormalisation scale (radial dimension)?

- the structural similarity had been noted before, and also exploited, in the context of renormalization of spin foam models treated as lattice gauge theories

  G. Vidal,(2008), S. Singh, R. N. C. Pfeifer, and G. Vidal, Tensor network decompositions in the presence of a global symmetry, G. Evenbly and G. Vidal, Tensor network states and geometry; M. Han and L.-Y. Hung, Loop Quantum Gravity, Exact Holographic Mapping, and Holographic Entanglement Entropy, B. Dittrich, S. Mizera, and S. Steinhaus, Decorated tensor network renormalization for lattice gauge theories and spin foam models, C. Delcamp and B. Dittrich, Towards a phase diagram for spin foams, B. Dittrich, F. C. Eckert, and M. Martin-Benito, Coarse graining methods for spin net and spin foam models, B. Dittrich, E. Schnetter, C. J. Seth, and S. Steinhaus, Coarse graining flow of spin foam intertwiners

Thank You!

Tensor Network decomposition

TN representations of a quantum state

structure

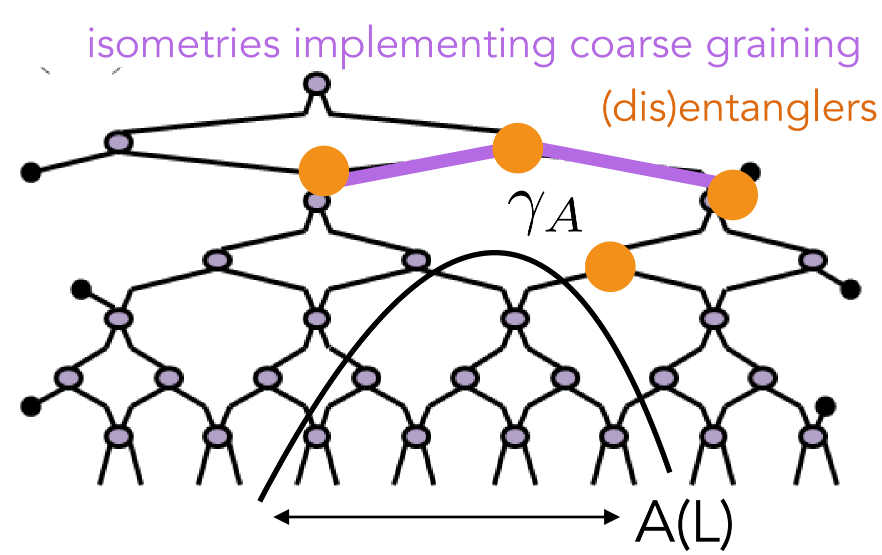
network diagram where bond indices represent unphysical, auxiliary degrees of freedom that are introduced for the purpose of efficiently writing down a ground state

variational parameters

to be optimized, within a given structure of the tensor network, to find a best tensor network state which best describes the target quantum state

e.g. Multi-scale Entanglement Renormalization Ansatz (MERA): efficient TNR for variationally estimating the ground state of a critical quantum system (CFT).

$$\min_{\gamma_A}[\#\,\text{bonds}(\gamma_A)] \sim \log L$$



network geometry resembles a discretization of spatial slices of an AdS spacetime and ``geodesics'' in the MERA reproduce the Ryu-Takayanagi formula for the entanglement entropy of a boundary region in terms of bulk properties => AdS/MERA