Cosmology from group field theory condensates

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Main References

- SG, Oriti, Sindoni, "Cosmology from Group Field Theory Formalism for Quantum Gravity" (*Phys. Rev. Lett.* **111** (2013), 031301) and "Homogeneous cosmologies as group field theory condensates" (*JHEP* **1406** (2014) 013)
- SG, Sindoni, "Quantum Cosmology from Group Field Theory Condensates: a Review" (*SIGMA* **12** (2016) 082)
- Oriti, Sindoni, Wilson-Ewing, "Emergent Friedmann dynamics with a quantum bounce from quantum gravity condensates" (*CQG* **33** (2016), 224001)
- SG, "Emergence of a low spin phase in group field theory condensates" (*Class. Quant. Grav.* **33** (2016), 224002)

Other work by Calcagni, **de Cesare, Oriti, Pithis, Pranzetti**, Sakellariadou, Sindoni, Tomov; work in progress by SG, Oriti and Adjei, SG, Wieland

Outline

1. Motivation

2. Basics of Group Field Theory (GFT)

3. GFT Condensates and Cosmology: Basic Concepts

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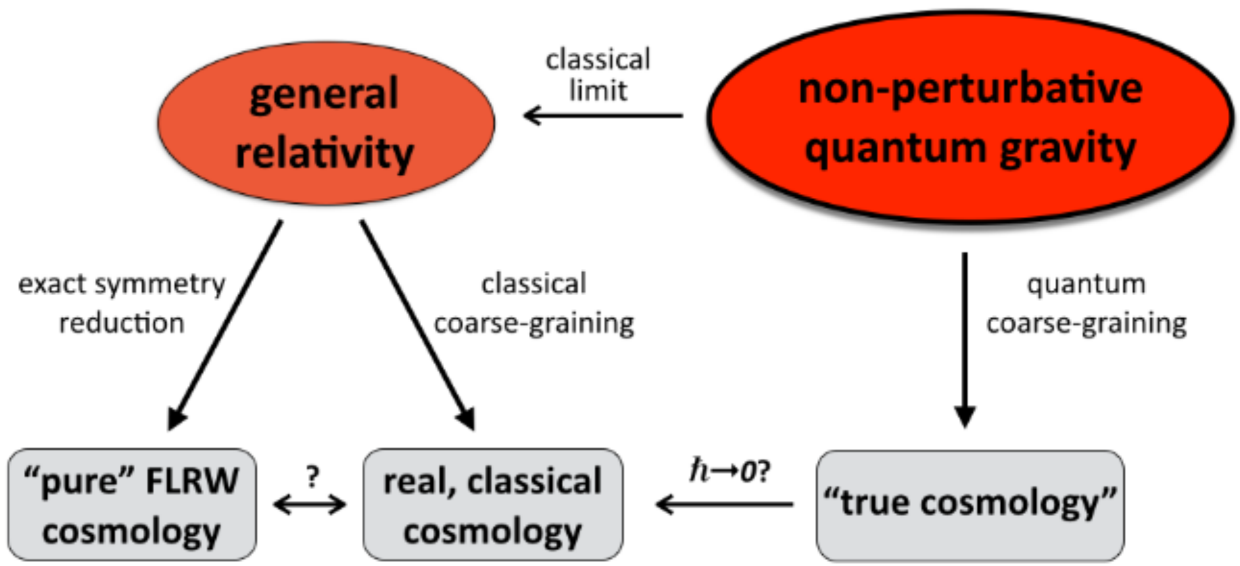
Motivation: symmetry reduction or coarse graining?

The relation between the physics of symmetry-reduced models and the dynamics of the full theory is already subtle in classical GR, let alone in quantum gravity:



(from [Glaser, Loll 2017]). In quantum theory, exact symmetry reduction seems to violate the uncertainty principle, and so has to be applied before quantisation.

Cosmology as quantum gravity hydrodynamics

Fundamentally, classical and quantum cosmology should arise from a *coarse graining* of the fundamental, microscopic dynamics, after which only information about large-scale observables remains:
"Cosmology is the hydrodynamics of quantum gravity."

Our Universe appears very special, close to exact homogeneity: a large number of (quantum) geometric degrees of freedom seem to collectively form a highly symmetric, macroscopic configuration.

In condensed matter physics, this is what happens in **Bose–Einstein condensation**, where a macroscopically large number of atoms condense into a single ground state, characterised by a single-particle wavefunction.

Suggests that the universe is indeed described by a type of condensate [Hu; Oriti; Koslowski, Sahlmann; Bojowald, . . .].

Group Field Theory (GFT) [more details in Oriti's talk, see also Carrozza]

Group field theory provides a quantum field theory language for the kinematics and dynamics of LQG. Spin foam amplitudes are generated as Feynman amplitudes in the GFT expansion in Feynman graphs.

GFT models for gravity coupled to a massless scalar can be defined by a complex scalar field

$$\varphi : G^4 \times \mathbb{R} \to \mathbb{C}\,, \quad (g_I, \phi) \mapsto \varphi(g_I, \phi)$$

where often we work in Ashtekar–Barbero variables where $G = \mathrm{SU}(2)$.
The action takes the general form

$$S[\varphi, \bar{\varphi}] = -\int \mathrm{d}^4 g\, \mathrm{d}\phi\; \bar{\varphi}(g_I, \phi) \mathcal{K} \varphi(g_I, \phi) + \mathcal{V}[\varphi, \bar{\varphi}]$$

where $\mathcal{V}$ encodes the vertex amplitude of the corresponding spin foam model.

Group Field Theory – Fock Space Formalism

One can introduce a canonical formalism in which the GFT fields satisfy

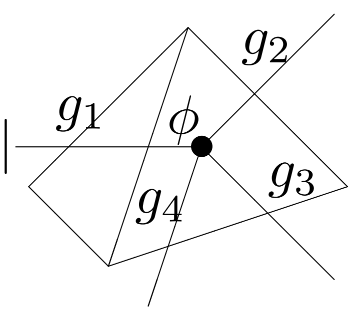
$$[\hat{\varphi}(g_I,\phi),\hat{\varphi}^\dagger(g'_I,\phi')] = \int \mathrm{d}h \prod_{I=1}^{4} \delta(g'_I h g_I^{-1})\delta(\phi-\phi')$$

where the delta function on $\mathrm{SU}(2)^4$ is compatible with gauge invariance

$$\varphi(g_I,\phi) = \varphi(g_I h,\phi) \quad \forall\, h \in \mathrm{SU}(2)\,.$$

Starting from a Fock vacuum $|\emptyset\rangle$ that corresponds to the Ashtekar–Lewandowski vacuum, with $\hat{\varphi}(g_I,\phi)|\emptyset\rangle = 0$, we can create a tetrahedron, or spin network vertex:

$$\hat{\varphi}^\dagger(g_1,g_2,g_3,g_4,\phi)|\emptyset\rangle = |\quad\rangle$$



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Defining GFT Condensates

In a condensate the field acquires a nonvanishing expectation value,

$$\langle\hat{\varphi}(g_I,\phi)\rangle =: \sigma(g_I,\phi) \neq 0\,.$$

In a **mean-field approximation**, all correlation functions can be expressed in terms of σ, with no correlations between quanta. This approximation corresponds to working with the state

$$|\sigma\rangle := \mathcal{N}(\sigma)\exp\left(\int \mathrm{d}^4g\,\mathrm{d}\phi\;\sigma(g_I,\phi)\hat{\varphi}^\dagger(g_I,\phi)\right)|\emptyset\rangle\,.$$

Such states describe weakly interacting Bose condensates. In GFT, they indeed describe a homogeneous universe in which spatial points are entirely decoupled [SG, Oriti, Sindoni 2013].

Macroscopic Observables

In cosmology we are interested in large-scale observables. For an FLRW universe, dynamics corresponds to the evolution of $V(\phi)$, the 3-volume at a given value ϕ of the relational clock.
This quantity corresponds to the expectation value of an operator on the GFT Fock space [Oriti, Sindoni, Wilson-Ewing 2016],

$$\langle\hat{V}(\phi)\rangle = \langle\int \mathrm{d}^4g\,\mathrm{d}^4g'\;\hat{\varphi}^\dagger(g_I,\phi)V(g_I,g'_I)\hat{\varphi}(g'_I,\phi)\rangle$$

where $V(g_I,g'_I)$ are matrix elements of the LQG volume operator. Another interesting quantity is the total number of quanta

$$\langle\hat{N}(\phi)\rangle = \langle\int \mathrm{d}^4g\;\hat{\varphi}^\dagger(g_I,\phi)\hat{\varphi}(g_I,\phi)\rangle = \int \mathrm{d}^4g\;|\sigma(g_I,\phi)|^2$$

where the last equality holds for the mean-field condensate coherent state.

Approximating the GFT dynamics

The simple mean-field coherent state can only be an approximate physical state of the full theory; it is required to satisfy a truncation of the full dynamics. The first truncation is to the classical equation of motion for σ,

$$\langle\sigma|\frac{\delta S[\hat{\varphi},\hat{\varphi}^\dagger]}{\delta\hat{\varphi}^\dagger(g_I,\phi)}|\sigma\rangle = \frac{\delta S[\sigma,\bar{\sigma}]}{\delta\bar{\sigma}(g_I,\phi)} = -\mathcal{K}\sigma(g_I,\phi) + \frac{\delta\mathcal{V}[\sigma,\bar{\sigma}]}{\delta\bar{\sigma}(g_I,\phi)} = 0\,.$$

This is the GFT analogue of the **Gross–Pitaevskii equation** in condensed matter physics for the mean field of a Bose–Einstein condensate.

Within these approximations, a solution $\sigma(g_I,\phi)$ to the classical GFT equation of motion defines a physical mean field configuration, for which expectation values such as $\langle\hat{V}(\phi)\rangle$ and $\langle\hat{N}(\phi)\rangle$ can be computed.
These expectation values then satisfy **effective Friedmann equations**.

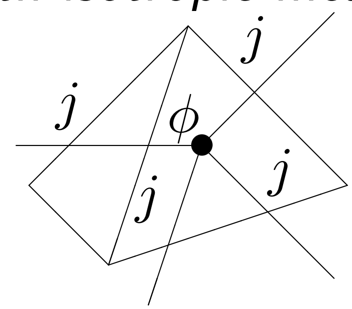
Emergent Friedmann equations with massless scalar

A GFT for a spin foam model coupled to a massless scalar is given by

$$S[\varphi,\bar\varphi] = -\int \mathrm{d}^4 g\,\mathrm{d}\phi\;\bar\varphi(g_I,\phi)\left(\mathcal{B}-\mathcal{A}\,\partial_\phi^2+\ldots\right)\varphi(g_I,\phi)+\mathcal{V}[\varphi,\bar\varphi]$$

with no explicit dependence on ϕ, and an expansion of $\mathcal{K}$ in (even) ϕ derivatives, truncated at ∂_ϕ^2 [Oriti, Sindoni, Wilson-Ewing 2016; Li, Oriti, Zhang 2017]. $\mathcal{V}$ corresponds, e.g., to the EPRL model. Choosing a condensate with *isotropic* mean field,

$$\sigma(g_I,\phi)\equiv\sum_{j=0}^{\infty}\sigma_j(\phi)\,\mathbf{D}^j(g_I)\,,$$



the GFT Gross–Pitaevskii equation then becomes

$$\langle\sigma|\frac{\delta S[\hat\varphi,\hat\varphi^\dagger]}{\delta\hat\varphi^\dagger(g_I,\phi)}|\sigma\rangle=0\quad\Rightarrow\; A_j\partial_\phi^2\sigma_j(\phi)-B_j\sigma_j(\phi)+w_j\bar\sigma_j(\phi)^4=0\,.$$

Emergent Friedmann equations with massless scalar (II)

One considers a weak-coupling approximation in which the interactions can be neglected; seems required for self-consistency but also corresponds again to spatial homogeneity. The equation for the mean field then becomes

$$A_j \partial_\phi^2 \sigma_j(\phi) - B_j \sigma_j(\phi) = 0$$

with general solution [de Cesare, Sakellariadou 2016; SG 2016]

$$\sigma_j(\phi) = \alpha_j^+ \, e^{\sqrt{B_j/A_j}\,\phi} + \alpha_j^- \, e^{-\sqrt{B_j/A_j}\,\phi} \, .$$

From this one immediately obtains the 3-volume as a function of ϕ,

$$\begin{aligned} V(\phi) = \langle \hat{V}(\phi) \rangle &= \sum_j V_j |\sigma_j(\phi)|^2 \\ &= \sum_j V_j \left(|\alpha_j^+|^2 \, e^{2\sqrt{B_j/A_j}\,\phi} + 2\Re(\overline{\alpha_j^+}\alpha_j^-) + |\alpha_j^-|^2 \, e^{-2\sqrt{B_j/A_j}\,\phi} \right) . \end{aligned}$$

Emergent Friedmann equations with massless scalar (III)

The effective LQC Friedmann equation for a flat FLRW universe can be written as

$$\left(\frac{V'(\phi)}{V(\phi)}\right)^2 = 12\pi G \left(1 - \frac{\rho}{\rho_c}\right) .$$

Assuming that only a single $j = j_0$ is excited, the GFT condensate 3-volume $V(\phi)$ satisfies [Oriti, Sindoni, Wilson-Ewing 2016]

$$\left(\frac{V'(\phi)}{V(\phi)}\right)^2 = 4\frac{B_{j_0}}{A_{j_0}}\left(1 - \frac{\rho}{\rho_c}\right) + \frac{4V_{j_0}E}{V(\phi)}$$

with $\rho_c = (B_{j_0}/A_{j_0}) \cdot (1/2V_{j_0}^2)$ and E is a (state-dependent) constant of motion.

If we identify $B_{j_0}/A_{j_0} \equiv 3\pi G$, a single-spin condensate reproduces the classical Friedmann equations at large volumes, as well as an LQC-type bounce with a bounded energy density. By assumption, here $V(\phi) = N(\phi) \cdot V_{j_0}$.

Emergence of a Low Spin Phase [SG 2016]

Can the LQC assumption that only a single spin j_0 (potentially $j_0 = \frac{1}{2}$) is excited be justified dynamically? Look at the general solution

$$\begin{aligned}\langle \hat{V}(\phi)\rangle &= \sum_j V_j |\sigma_j(\phi)|^2 \\ &= \sum_j V_j \left(|\alpha_j^+|^2 \, e^{2\sqrt{B_j/A_j}\,\phi} + 2\Re(\overline{\alpha_j^+}\alpha_j^-) + |\alpha_j^-|^2 \, e^{-2\sqrt{B_j/A_j}\,\phi} \right) .\end{aligned}$$

It is clear that, if there is a j for which $\frac{B_j}{A_j}$ is (positive and) maximal, this spin j_0 will completely dominate asymptotically: the number of quanta $N_{j_0}(\phi)$ with $j = j_0$ then grows exponentially faster than all other j (both as $\phi \to \pm\infty$).

Hence, for this (rather general) class of models, *a constant spin phase* in which a large number of quanta all have the same $j = j_0$ emerges **dynamically**. Bounce phase may depart from LQC, but late-time behaviour is always GR!

A toy model for GFT condensate dynamics [Adjei, SG, Wieland, in prep.]

Captures many features of GFT condensate cosmology. Consider the action

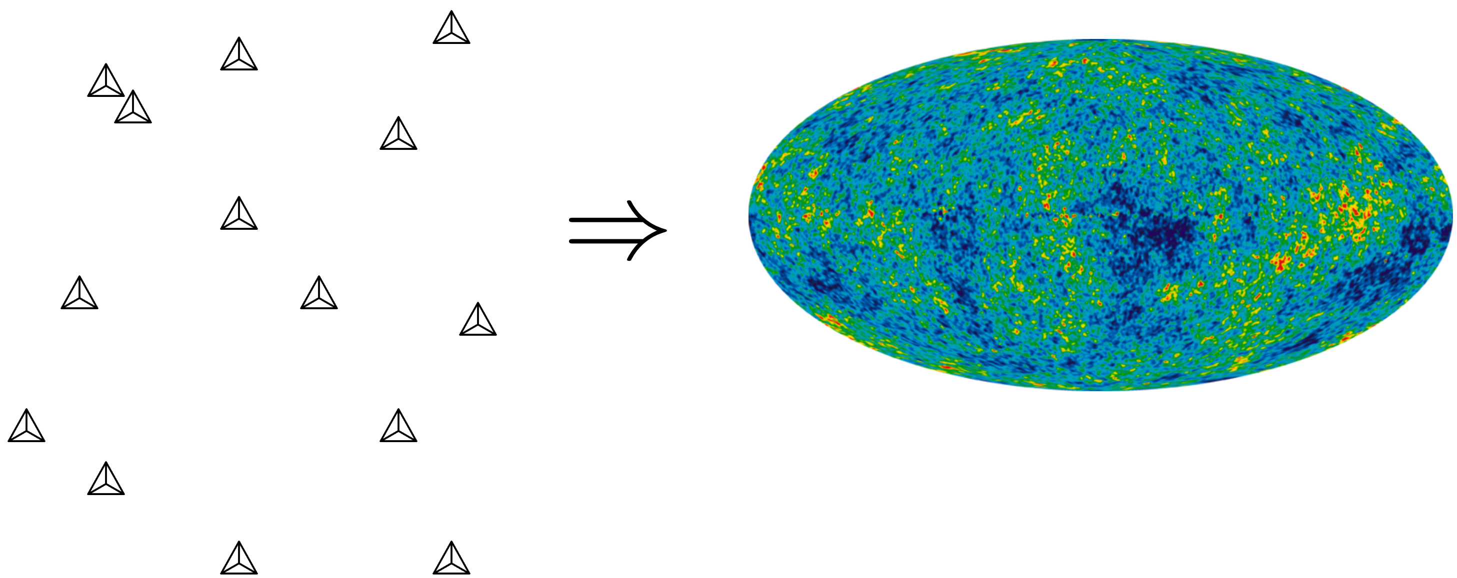
$$S = \frac{\mathrm{i}}{2}\int \mathrm{d}\phi \, \left(H^{ABCD}(\phi)\dot{H}^{\dagger}_{ABCD}(\phi) - \dot{H}^{ABCD}(\phi)H^{\dagger}_{ABCD}(\phi)\right)$$
$$-z\left(H^{ABCD}(\phi)H_{ABCD}(\phi) - (H^{ABCD})^{\dagger}(\phi)H^{\dagger}_{ABCD}(\phi)\right)$$

which defines a GFT where all excitations are only $j = 1/2$.
For this model, the dynamics is **solved** by any *squeezed state* of the form

$$|\phi\rangle = \exp\left[-\frac{z}{2}\left(H^{ABCD}H_{ABCD} - (H^{ABCD})^{\dagger}H^{\dagger}_{ABCD}\right)\phi\right]|\mathrm{initial}\rangle$$

where $|\mathrm{initial}\rangle$ can, e.g., be the Fock vacuum state.
Again, one finds that the 3-volume follows exactly the form of LQC effective dynamics (up to an offset given by the initial state).

Still, as in Loops '15, this is our ultimate goal. . .



Inhomogeneities as vacuum fluctuations [SG, Oriti, in prep.]

Inflation explains the inhomogeneities in the Universe as originating from quantum fluctuations: even a homogeneous (Bunch–Davies) vacuum state has fluctuations around homogeneity that leave an imprint in the sky [Mukhanov, Chibisov 1981].

GFT condensates do not implement symmetry reduction but excite infinitely many degrees of freedom: the same could be true here!

We couple additional "rod" scalar fields to GFT, define a relational volume element

$$\langle \hat{V}(\phi^J) \rangle = \langle \int \mathrm{d}^4 g \, \mathrm{d}^4 g' \, \hat{\varphi}^\dagger(g_I, \phi^J) V(g_I, g'_I) \hat{\varphi}(g'_I, \phi^J) \rangle$$

and find, for a "spatially homogeneous" condensate,

$$\langle \delta\hat{V}(\phi^0, k_i) \delta\hat{V}(\phi^0, K_i) \rangle = \mathcal{P}(|k|)(2\pi)^3 \delta^3(k_i + K_i)$$

where $\mathcal{P}(k) \sim 1/N$ is naturally small (but nonzero)!!

Summary & Outlook

- **Condensates** of GFT quanta (spin network vertices) are a proposal for describing a macroscopic cosmological universe in the context of LQG.

- Cosmology emerges as the hydrodynamic approximation to the quantum gravity dynamics. Focusing on macroscopic, coarse-grained observables and compute their evolution leads to effective Friedmann equations.

- Many features of the LQC effective dynamics can be reproduced, including dynamical emergence of a constant spin phase in which $V(\phi) \propto N(\phi)$. Details of the bounce are state-dependent and may depart significantly from LQC.

- Tentative possibility to explain inhomogeneities from GFT quantum fluctuations, similar to inflation!

- Need to show dynamical formation of the condensate itself more rigorously, e.g. through GFT renormalisation methods [Carrozza's talk]

Thank you!