Diffeomorphism invariance and the flat space limit

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WD & S. B. Giddings, Phys. Rev. D 93, 024030 (2016), arXiv:1507.07921
WD & S. B. Giddings, Phys. Rev. D 94, 104038 (2016), arXiv:1607.01025
WD & S. B. Giddings, arxiv:1706.03104

Local quantum field theory

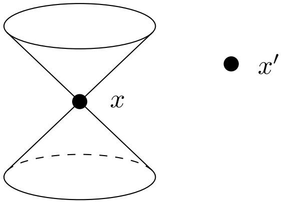
Quantum gravity should have a flat space limit: in this limit it should reproduce local quantum field theory.

Quantum field theory is a theory of **gauge invariant**, **local** operators $\mathcal{O}$.
e.g. ϕ, $\bar{\psi}\psi$, $F_{\mu\nu}$, $F^a_{\mu\nu}F^{a\mu\nu}$, $\mathrm{tr}\,\mathcal{P}e^{i\int_\gamma A}$, ...

Locality in this theory is encoded by the axiom of **microcausality**:

When x and x' are spacelike separated,

$$[\mathcal{O}(x), \mathcal{O}(x')] = 0$$



Observables in a region form an algebra, these define **local subsystems**.

Diffeomorphism-invariant observables

In gravity, observables must be diffeomorphism invariant.

Exact diffeomorphism-invariant observables are hard to construct.

- Dressed operators in QED [Dirac].
- Using a reference frame of dust [Brown & Kuchař].
- GPS observables [Rovelli].
- Perturbative observables [Dittrich & Tambornino].
- Observables in Gaußian normal coordinates [Bodendorfer, Duch, Lewandowski, & Świeżewski].

Observables are specified relative to some reference such as matter fields, or an asymptotic region.

Questions

- What operators in quantum gravity reduce to $\phi(x)$ as $G \to 0$?
- What are corrections to microcausality when $G > 0$?
- How can we define locality and subsystems in quantum gravity?

Outline

- Construction of gravitationally dressed observables perturbatively in asymptotically flat spacetime.
- Corrections to microcausality.
- A bound on locality: the Dressing Theorem.
- Implications for local information and subsystems.

Perturbative gravity

Consider perturbative gravity coupled to a real scalar field of mass m:

$$\mathcal{L} = \frac{2}{\kappa^2} R - \frac{1}{2}(\nabla\phi)^2 - \frac{1}{2}m^2\phi^2 + \mathcal{L}_{\text{gauge fixing}}.$$

Expand the metric about flat spacetime,

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}.$$

We expand $\sqrt{g}\mathcal{L}$ in $\kappa = \sqrt{32\pi G}$, keeping the matter-gravity coupling.

Commutators of ϕ and of $h_{\mu\nu}$ are causal.

The fields transform under a linearized diffeomorphism $\kappa\xi^\mu$ as:

$$\phi \to \phi - \kappa\xi^\mu\partial_\mu\phi,$$
$$h_{\mu\nu} \to h_{\mu\nu} - \partial_\mu\xi_\nu - \partial_\nu\xi_\mu.$$

Since $\phi(x)$ is not invariant we have to construct a **dressed** operator.

A note on Dirac brackets

One common approach to defining observables involves Dirac brackets.

- First, fix the gauge completely.
- Reduce the phase space (solve the constraints & gauge conditions).
- Now everything is gauge invariant.
- Replace Poisson brackets with *nonlocal* Dirac brackets.

Dirac brackets are doing two things at once:

- Implicitly replacing operators with dressed versions.
- Calculating Poisson brackets of the dressed operators.

Instead we will follow a more transparent approach:

- Construct manifestly diffeomorphism-invariant dressed operators.
- Calculate their (causal) Poisson brackets.

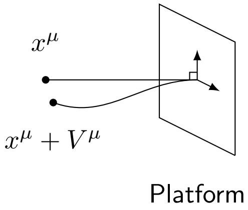
Gravitational Wilson line

To define an invariant observable, start at a fixed "platform" $z = Z$.
Shoot a geodesic from $(x_\perp, Z)$ a proper distance $Z - z$.
Measure ϕ at the endpoint.

This prescription defines a diffeomorphism-invariant dressing of $\phi(x)$.

Solving the geodesic equation perturbatively:

$$\Phi_W(x) = \phi(x) + V_W^\mu(x)\partial_\mu \phi(x),$$
$$V_W^\mu(x) = -\int_0^\infty ds\ s\ \Gamma^\mu_{zz}(x + s\hat{z})$$



This is invariant under diffeomorphisms ξ
such that $\xi^\mu = 0$, $\partial_\nu \xi^\mu = 0$ at the platform.

This observable is very singular, and not sufficiently invariant.

Gravitational Coulomb dressing

To make a symmetric dressing, we can average over all directions:

$$\Phi_C(x) = \phi(x) + V_C^\mu(x)\partial_\mu\phi(x),$$
$$V_C^\mu(x) = -\frac{1}{4\pi}\int d^3x' \frac{1}{|x-x'|}\,\Gamma^\mu_{\alpha\beta}(x')\hat{r}^\alpha\hat{r}^\beta$$

This operator is more well-behaved. What does it do?

To find the gravitational field created, consider the commutator:

$$[h_{\mu\nu}(x'), \Phi_C(x)] = [h_{\mu\nu}(x'), V_C^\lambda(x)]\partial_\lambda\phi(x)$$

The metric depends on derivatives of ϕ; gravity couples to momentum.

Φ_C creates a ϕ particle plus its quantum gravitational field:
it creates a superposition of particles with different momenta,
entangled with a superposition of different gravitational fields.

Microcausality

How does the nonlocal dressing affect microcausality?

Consider the equal-time commutator for $x \neq x'$:

$$[\Phi_C(x), \dot{\Phi}_C(x')] = [V_C^\mu(x), \dot{V}_C^\nu(x')]\partial_\mu\phi(x)\partial_\nu\phi(x') + O(\kappa^3)$$

In the nonrelativistic limit, we can replace $\partial_\mu\phi(x) \to \delta_\mu^0 im\phi$

$$\begin{aligned}[\Phi_C(x), \dot{\Phi}_C(x')] &\sim [V_C^0(x), \dot{V}_C^0(x')]m^2\phi(x)\phi(x') \\ &= \frac{Gm^2}{|x - x'|}\phi(x)\phi(x')\end{aligned}$$

Corrections to microcausality are related to the Newtonian potential.

The Dressing Theorem

Theorem [WD & Giddings 2016]
Let $\mathcal{O}$ be a diffeomorphism-invariant operator, with κ expansion

$$\mathcal{O} = \mathcal{O}^{(0)} + \kappa\mathcal{O}^{(1)} + \cdots$$

If $O^{(0)}$ has a nonzero commutator with a spacetime translation generator, then the dressing falls off no faster than a monopole:

$$\frac{\delta\mathcal{O}^{(1)}}{\delta g_{\mu\nu}} \sim \frac{1}{r}.$$

Note: Any compactly supported operator must have nonzero commutator with P^i.

Proof: Consider commutator of $\mathcal{O}$ with the Poincaré generators.

Poincaré charges

In gravity, the conserved charges are the 10 Poincaré generators:

$$
\begin{aligned}
P^0 &= \quad \frac{2}{\kappa}\oint_S dA\,\hat{r}^i\left[\partial_j h_{ij} - \partial_i h_{jj}\right], \\
P^i &= -\frac{2}{\kappa}\oint_S dA\,\hat{r}^j\left[\partial_0 h_{ij} - \delta_{ij}\partial_0 h_{kk} + \partial_i h_{0j} - \partial_j h_{0i}\right], \\
L^{ij} &= -\frac{2}{\kappa}\oint_S dA\,\hat{r}^k\left[x^i(\partial_0 h_{jk} - \partial_k h_{0j}) + h_{0j}\delta_{ik}\right] - (i \leftrightarrow j), \\
K^i &= \quad \frac{2}{\kappa}\oint_S dA\,\hat{r}^j\left[x^i(\partial_k h_{jk} - \partial_j h_{kk}) - h_{ij} + h_{kk}\delta_{ij}\right]
\end{aligned}
$$

These are the **energy**, **momentum**, **angular momentum**
and the **Beig-O'Murchadha-Regge-Teitelboim center of mass**.

These all take the form of integrals of h over spatial infinity.

The Dressing Theorem

Proof: Let $\mathcal{O} = \mathcal{O}^{(0)} + \kappa\mathcal{O}^{(1)} + \dots$ be diffeomorphism invariant.
We can write the 4-momentum as a boundary term on-shell:

$$P^\mu := \int_\Sigma \epsilon_\Sigma T^\mu{}_\nu n^\nu = \frac{1}{\kappa}\oint_S B^{\mu\lambda\alpha\beta}\partial_\lambda h_{\alpha\beta} + \text{constraints}.$$

Since $\mathcal{O}$ is diffeomorphism invariant, it commutes with the constraints:

$$[P^\mu, \mathcal{O}] = \frac{1}{\kappa}\oint_S B^{\mu\lambda\alpha\beta}\,[\partial_\lambda h_{\alpha\beta}, \mathcal{O}]$$

Now consider the leading term at order κ^0:

$$[P^\mu, \mathcal{O}^{(0)}] = \oint_S B^{\mu\lambda\alpha\beta}[\partial_\lambda h_{\alpha\beta}, \mathcal{O}^{(1)}]$$

$[P^\mu, \mathcal{O}^{(0)}]$ is nonzero, so $\mathcal{O}^{(1)}$ must depend on the asymptotic metric.

To make the integral nonzero, the dependence of the dressing $\mathcal{O}^{(1)}$ on the metric cannot decay faster than $1/r$. □

Localized information and subsystems

Gauge-invariant degrees of freedom are nonlocal.

Algebraic definition of subsystems fails even perturbatively.

What do we do?

- Abandon locality (S-matrix, AdS/CFT) c.f. talk by **Jamie Sully**.
- Refine our notion of subsystems.

How much independent information is encoded in a region of space?

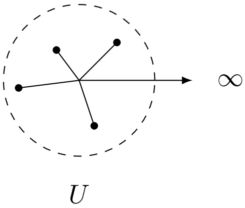
How much information is accessible outside?

Local classical information

Claim: Given a classical matter distribution $T_{\mu\nu}$ with compact support U, there is a solution of the linearized constraints whose gravitational field outside U only depends on the Poincaré charges.

Proof (sketch):
For simplicity, consider point sources at positions r^i_A with momenta p^μ_A. Dress them with gravitational Wilson lines starting from the origin:

$$\tilde{h}_{\mu\nu} = \sum_A [h_{\mu\nu}, P^\lambda_A V_\lambda(r^i_A)]$$



$\tilde{h}_{\mu\nu}$ solves the constraints except at the origin, where the constraints are proportional to the Poincaré charges.

We can carry these charges away with a Wilson line from the origin to ∞.

Classically, we can screen higher multipoles: all we can learn about a matter distribution from outside are the Poincaré charges.

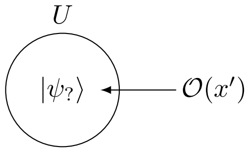
Translation argument

Could we have two states $|\psi_1\rangle, |\psi_2\rangle$ that give the same expectation values for all observables localized outside a compact region U, but different expectation values for some operator $\mathcal{O}(x)$ inside the region?

The translation generator P is an operator outside U. So given the operator $\mathcal{O}(x')$ supported outside U, we can consider

$$\langle\psi_1| e^{-i(x-x')\cdot P}\mathcal{O}(x')e^{i(x-x')\cdot P}|\psi_1\rangle = \langle\psi_1|\mathcal{O}(x)|\psi_1\rangle$$

Any information in U can be extracted by translation.



Note: This is really a nonperturbative argument. Translating in this way requires $\mathcal{O}$ to be dressed to all orders, since e^{iP} is not analytic in κ.

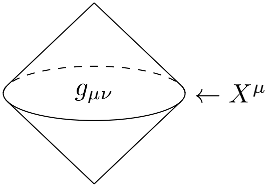
Covariant definition of subsystems

There appears to be no suitable *invariant* definition of local subsystems. However, we can introduce a *covariant* definition [WD & Freidel 2016].

Define a phase space with variables $(g_{\mu\nu}, X^\mu)$:

- $g_{\mu\nu}$ is a solution to Einstein's equation on a domain in M.
- $X^\mu : S^2 \to M$ gives the location of its codimension-2 boundary.

The boundary X^μ transforms covariantly under diffeomorphisms.
X^μ defines a reference frame with which we can define observables.



Subsystems include edge mode degrees of freedom, new symmetries ...
See talks by **Freidel, Pranzetti, Hopfmüller**.

Conclusion

- In quantum gravity, local operators must be gravitationally dressed.
- This dressing leads to violations of microcausality at order G.
- Gravitational dressing of compactly supported operators must extend to infinity, and cannot decay faster than $1/r$.
- Classically, the only information accessible outside a region are the Poincaré charges.
- In the quantum theory, we can access information nonlocally through translations.
- This suggests that local subsystems are not an *invariant*, but a *covariant* concept.

Thank you!

(See you at Loops 2027)