PERIMETER PI INSTITUTE FOR THEORETICAL PHYSICS

Spin foam renormalization à la GFT: status and prospects

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Loop Quantum Gravity and Spin Foams

- Loop Quantum Gravity proposes **kinematical states** describing (spatial) quantum geometry [Ashtekar, Rovelli, Smolin, Lewandowski... '90s; Dittrich, Geiller, Bahr '15].

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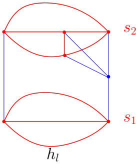
  $\rightarrow$ amplitudes $\mathcal{A}_{s,C}$ associated to a 2-complex C with boundary spin-network state s.



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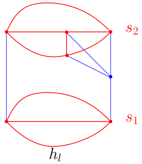

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- Two *a priori* distinct strategies to extract $\mathcal{A}_s$, and therefore **complete the definition of spin foam models**:

  1. **Lattice interpretation**: refining and coarse-graining C (and s)
     $\Rightarrow \mathcal{A}_s \equiv \lim_{C \to \infty} \mathcal{A}_{s,C}$ [Dittrich, Bahr, Steinhaus, Martin-Benito, Delcamp,... '10s]

  2. **QFT interpretation**: quantum space-time histories, to be summed over
     $\Rightarrow \mathcal{A}_s \equiv \sum_{C | \partial C = s} w_C \mathcal{A}_{s,C}$ [De Pietri, Rovelli, Freidel, Oriti... '00s, '10s]

Summing over spin foams

QFT interpretation:

Superposition principle → spin foams are viewed as Feynman amplitudes, to be summed over: [Boulatov '92;... De Pietri, Rovelli, Freidel, Oriti, Ryan, Livine... '00s, '10s]

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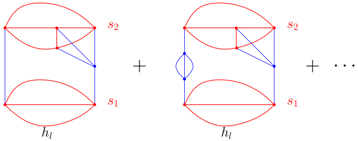

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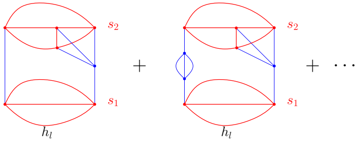

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This may be implemented by a **Group Field Theory**: [Talk by Oriti]

- a (Euclidean) **quantum field theory** for a (complex) field φ;
- defined on d-copies of a **local symmetry group** of gravity: $\varphi = \varphi(g_1, \dots, g_d)$;
- with **combinatorially non-local** interactions.

⇒ This embedding of spin foam amplitudes into a GFT uniquely fixes the combinatorial weights w_C.

Renormalization of GFT/Spin Foams: motivations

GFT/Spin Foam amplitudes are **generically divergent** $\rightarrow$ we need to **regularize** and then **renormalize** the theory.

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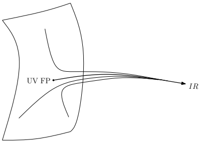
Physical motivations:

1. **flow** of the quantum dynamics under coarse-graining?



2. **quantization / discretization ambiguities**: what are the **universal features** of the known models? [EPRL, DL, BO, ...]

3. macro-physics from microscopic dynamics: how do we extract the **low-energy limit of LQG**? are there several **quantum phases**? compatibility with **general relativity**?

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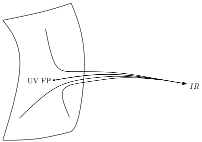

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> GFT renormalization programme:
> step-by-step generalization of standard renormalization techniques, until we are able to tackle 4d quantum gravity proposals.

[Benedetti, Ben Geloun, Bonzom, SC, Dine Samary, Gurau, Lahoche, Oriti, Rivasseau, Vignes-Tourneret...]

General structure of a GFT and long-term objectives

Typical structure of a GFT: (complex) field $\varphi(g_1, \ldots, g_d)$, $g_\ell \in G$, with partition function

$$Z = \int [\mathcal{D}\varphi]_\Lambda \exp\left(-\overline{\varphi} \cdot \mathcal{K} \cdot \varphi + \sum_{\{\mathcal{V}\}} t_\mathcal{V} \, \mathcal{V} \cdot (\overline{\varphi}\varphi)^{p_\mathcal{V}} \right) = \sum_{\mathcal{G}} \prod_i (t_{\mathcal{V}_i})^{n_{\mathcal{V}_i}(\mathcal{G})} \mathcal{A}_{\mathcal{G},\Lambda}$$

Main objectives of the GFT research programme:

1. Model building: define the **theory space**.
   e.g. spin foam amplitudes $\mathcal{A}_\mathcal{G}$ + *combinatorial considerations (tensor models)* $\rightarrow d$, G, $\mathcal{K}$, $\{\mathcal{V}\}$ *and* $[\mathcal{D}\varphi]_\Lambda$.

2. Perturbative definition: prove that the spin foam expansion is **consistent** in some range of Λ.
   e.g. perturbative renormalization.

3. Systematically explore the theory space: **effective continuum regime reproducing GR** in some limit?
   e.g. functional RG, constructive methods, condensate states...

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Simple enough but non-trivial set-up: [Ben Geloun, Rivasseau '11; SC, Oriti, Rivasseau '12; ...]

- G = **compact Lie group**: $\mathrm{SU}(2)$ for definiteness.
- Standard bosonic kinetic term: $\mathcal{K} = m^2 - \sum_{\ell=1}^{d} \Delta_\ell$.
- $[\mathcal{D}\varphi]_\Lambda$ = integration over **gauge invariant fields**, with **cut-off Λ on large spins**.
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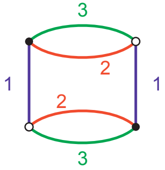
$\Rightarrow$ sum over $\mathrm{SU}(2)$ lattice gauge theory amplitudes.

Trace invariants

Trace invariants of fields $\varphi(g_1, g_2, \ldots, g_d)$ labelled by *d*-**colored graphs** / **bubbles** b:

$b =$ 

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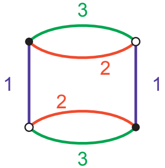
$b =$ 

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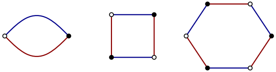
$(d = 2)$  $\cdots$

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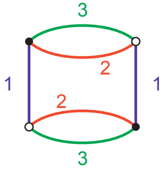
$b =$ 

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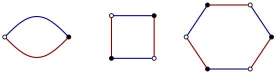
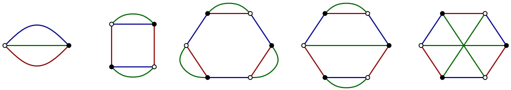
$(d = 2)$  $\dots$

$(d = 3)$  $\dots$

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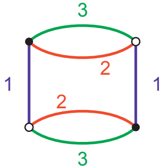
$b =$ 

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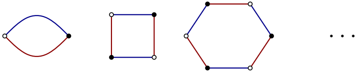
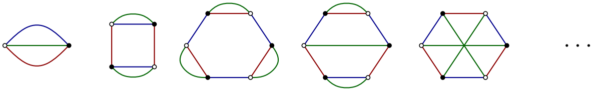
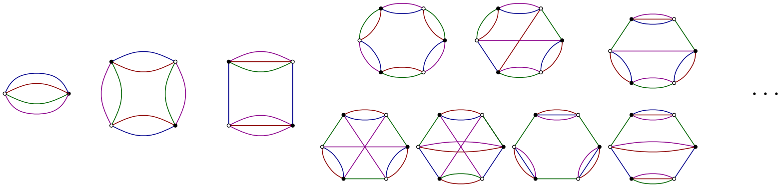
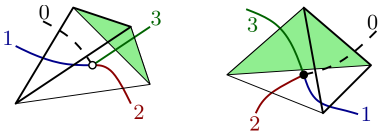
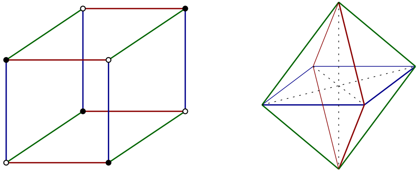
$(d = 2)$ 

$(d = 3)$ 

$(d = 4)$ 

d-colored graphs can canonically be associated to *d*-dimensional triangulations:





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Colored cell decompositions of pseudo-manifolds

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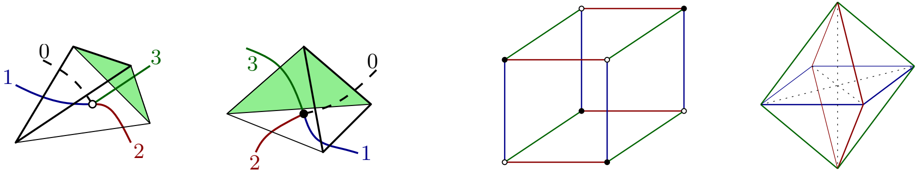

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Theorem: [Pezzana '74] Any PL manifold can be represented by a colored graph. In general, a $(d+1)$-colored graph represents a triangulated **pseudo-manifold** of dimension d.

⇒ **Crystallisation theory** [Cagliardi, Ferri et al. '80s]; introduced in GFTs / tensor models in '09 [Gurau '09...].

Nice book and reviews: "Random Tensors", Gurau (2016) and [Gurau, Ryan '11; Ryan '16]

Gauge invariance condition and Feynman amplitudes

- In order to connect to spin foams, the GFT path integral must be restricted to **gauge invariant fields**:

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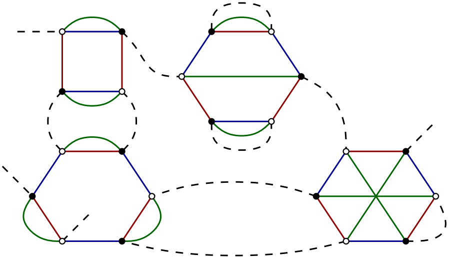
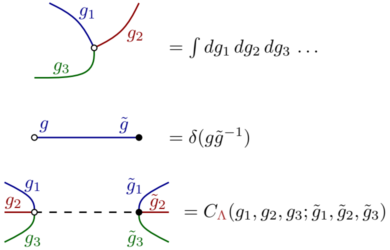
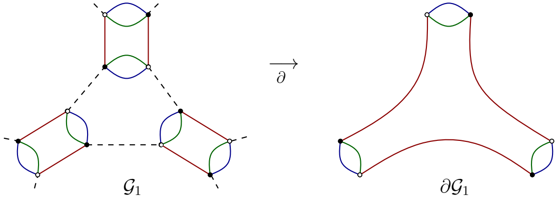
- The Feynman diagrams are $(d+1)$**-colored graphs**:





Bulk-boundary combinatorial correspondence

To each Feynman diagram $\mathcal{G}$ can be associated a canonical **boundary graph** $\partial\mathcal{G}$:



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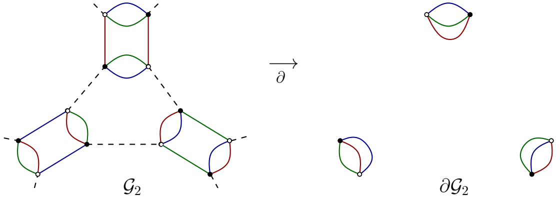

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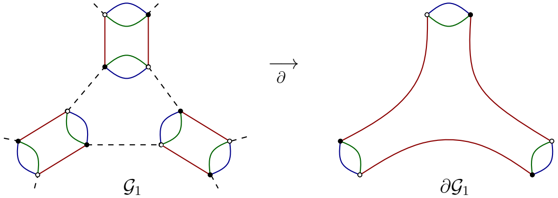

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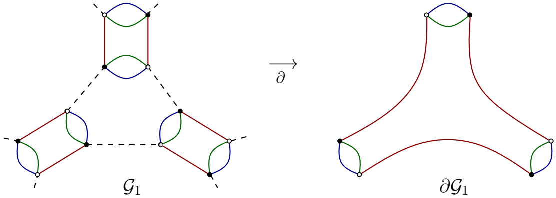

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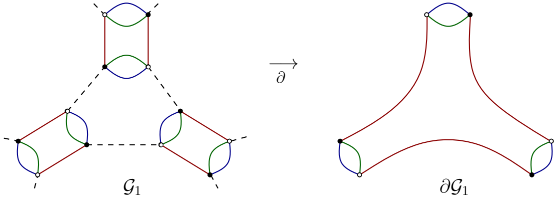

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This is **not automatically possible**, but it has been proven to work in the present theory space. This relies on a **delicate interplay** between the divergent structure of the theory and the topology of the divergent Feynman diagrams:

- the divergences are supported on **flat bulk holonomies**;
- the leading order Feynman diagrams ('melonic graphs') and their boundaries have **trivial topologies**.

[SC, Oriti, Rivasseau '13...]

Perturbative renormalization

$$\mathcal{Z}_\Lambda = \int_{\varphi \in \mathrm{Inv}} [\mathcal{D}\varphi \mathcal{D}\overline{\varphi}]_\Lambda \ \exp\left(-S_\Lambda(\varphi, \overline{\varphi})\right) = \sum_{\mathcal{G}} \prod_b (-t_b^\Lambda)^{n_b(\mathcal{G})} \mathcal{A}_{\mathcal{G},\Lambda}$$

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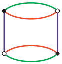
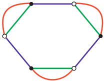
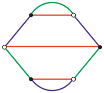
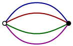
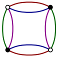
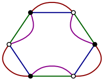
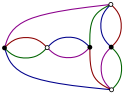
- Perturbatively renormalizable actions for $d = 3$ and $d = 4$ $\mathrm{SU}(2)$ GFTs:

$S_\Lambda(\varphi, \overline{\varphi}) \underset{(d=3)}{=} -\overline{\varphi} \cdot \Delta \cdot \varphi + t_2^\Lambda$  $+ \, t_4^\Lambda$  $+ \, t_{6,1}^\Lambda$  $+ \, t_{6,2}^\Lambda$ 

$\underset{(d=4)}{=} -\overline{\varphi} \cdot \Delta \cdot \varphi + t_2^\Lambda$  $+ \, t_4^\Lambda$  $+ \, t_{6,1}^\Lambda$  $+ \, t_{6,2}^\Lambda$ 

Perturbative renormalization

$$\mathcal{Z}_\Lambda = \int_{\varphi \in \mathrm{Inv}} [\mathcal{D}\varphi \mathcal{D}\overline{\varphi}]_\Lambda \ \exp\left(-S_\Lambda(\varphi, \overline{\varphi})\right) = \sum_{\mathcal{G}} \prod_b (-t_b^\Lambda)^{n_b(\mathcal{G})} \, \mathcal{A}_{\mathcal{G},\Lambda}$$

2. Perturbative definition: prove that the spin foam expansion is **consistent** in some range of Λ.
e.g. perturbative renormalization.

- Rigorous multiscale methods → **renormalizablity theorems (BPHZ) at all orders in perturbation theory**. [Ben Geloun, Rivasseau, SC, Oriti, Vignes-Tourneret, Ousmane Samary...]

- Perturbatively renormalizable actions for $d = 3$ and $d = 4$ $\mathrm{SU}(2)$ GFTs:

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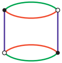
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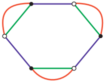
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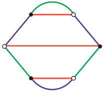

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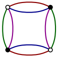
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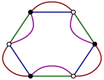
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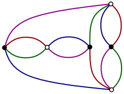

- Perturbative **beta functions** can be extracted → prescription for how to consistently adjust t_b^Λ as one changes Λ. [SC '14...]

Non-perturbative RG flow

3. Systematically explore the theory space: **effective continuum regime reproducing GR** in some limit?
   e.g. functional RG, constructive methods, condensate states...

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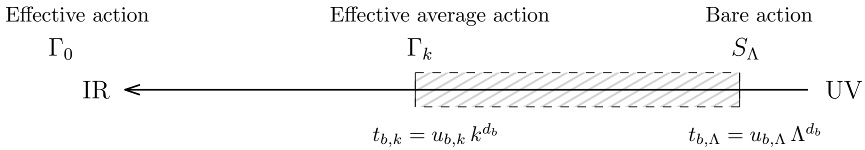
Effective average action [Wetterich '93, Morris '93...] [Benedetti, Ben Geloun, Oriti '14; Benedetti, Lahoche '15...]



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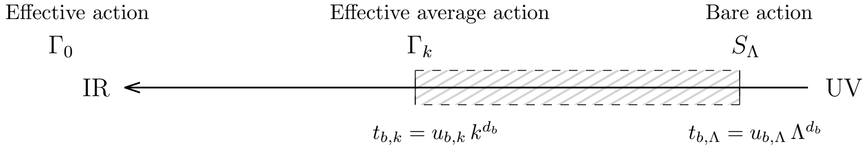
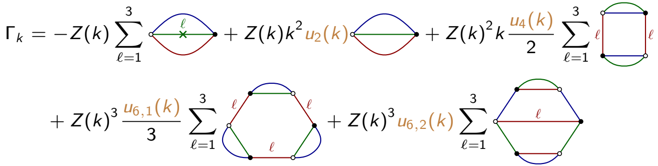
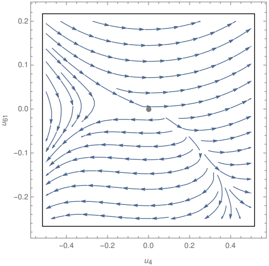
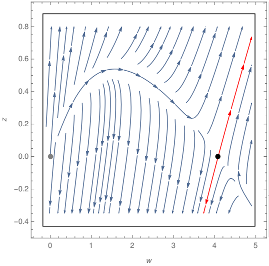


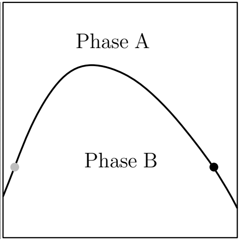
Work within a truncation e.g. φ^6 truncation in $d = 3$:



Wetterich (exact) equation $\rightarrow$ truncated non-perturbative flows e.g. φ^6:







$\Rightarrow$ **Non-perturbative UV fixed point** with **1 IR-relevant direction**.

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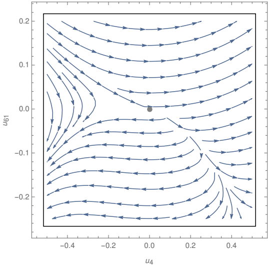

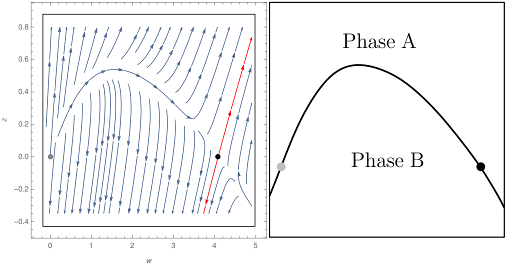

⇒ **Non-perturbative UV fixed point** with **1 IR-relevant direction**.

This statement is **robust under refinement** of the truncations:

$d = 3$

n	4	6	8	10	12
θ_1	2.5	2.7	2.8	2.9	3.0
θ_2	−0.37	−0.31	−0.28	−0.28	−0.31
θ_3	–	−1.7	−1.6	−1.6	−1.7
θ_4	–	–	−4.0	−4.1	−4.3
θ_5	–	–	–	−6.6	−6.7
θ_6	–	–	–	–	−9.5
η	−0.70	−0.82	−0.94	−1.1	−1.2

$d = 4$

n	4	6	8	10	12
θ_1	2.1	2.2	2.3	2.4	2.5
θ_2	−0.42	−0.33	−0.25	−0.16	−0.11
θ_3	–	−1.5	−1.3	−1.3	−1.3
θ_4	–	–	−3.4	−3.2	−3.4
θ_5	–	–	–	−5.5	−5.6
θ_6	–	–	–	–	−8.0
η	−0.64	−0.71	−0.77	−0.84	−0.95

Summary of results

- GFT = QFT **completion of Spin Foam models**.
- **Colored trace invariants** → **infinite-dimensional** but manageable GFT theory space (stability under the RG flow).
- Appreciable number of **renormalizable GFTs** known e.g. in the theory space of $\mathrm{SU}(2)$ **gauge invariant GFTs**.
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Take-home message: Spin foam renormalization schemes can be defined; they rely on a delicate interplay between the **combinatorics**, the **topology** and the **geometric data** of the foams. Extra care will be required in QG models.

GFT at Loops:
Chirco, de Cesare, Finocchiaro, Freidel, Gielen, Kegeles, Kotecha, Livine, Oriti, Perez-Sanchez, Pithis, Raasakka, Rovelli, Thürigen, Wilson-Ewing, Zhang...

Not at Loops, but contributed significantly to GFT/TFT renormalization:
Baratin, Benedetti, Ben Geloun, Bonzom, Eichhorn, Gurau, Krajewski, Koslowski, Lahoche, Ousmane Samary, Rivasseau, Ryan, Smerlak, Tanasa, Toriumi, Vignes-Tourneret...

Steps towards quantum gravity applications

Long-term questions:

- Is there (at least) one phase in which (a sector of) continuous **general relativity** is recovered in the classical limit of some 4d Lorentzian model?

- Can these questions be addressed within a **well-controlled approximation scheme**, allowing to subsequently compute **quantum corrections** to classical general relativity?

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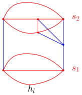
Some technical and conceptual stumbling blocks:

- **Lorentzian signature** $\Rightarrow$ non-compact groups? [Ben Geloun, Martini, Oriti '15 '16]
- Structure of the divergences in the presence of **simplicity constraints**? [Riello '13; Bonzom, Dittrich '13; Chen '16]
- expansion around **non-trivial vacua**, and connection to **GFT condensates**? [Plenaries by Geiller and Gielen]
- **Propagators** in 4d gravity? Appropriate notion of **scale**? Role of the **Barbero-Immirzi** parameter? [Charles, Livine '15]
- **Observables** and **symmetries**? [Baratin, Girelli, Oriti '11; Baratin, Freidel, Gurau '14; Kegeles, Oriti '16; Dittrich '10s]

Strategy I: build up quantum geometries from elementary building blocks.

- **Loop quantum gravity** and **spin foams**:

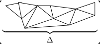
$G_{\mu\nu} = 8\pi G T_{\mu\nu} \xrightarrow[\text{quantization}]{}$

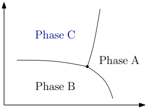


Planck-scale quantum geometry

- **GFT formulation**:

$\sum_{\Delta} w \left(\underbrace{\quad}_{\Delta} \right) \xrightarrow[\text{renormalization}]{}$





Effective phase diagram

Epilogue

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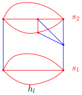

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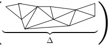
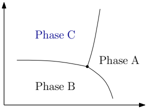
$\sum_{\Delta} w \Big($  $\Big)$ $\xrightarrow[\text{renormalization}]{}$

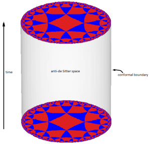


Effective phase diagram

Strategy II: Assume some weak background structure, e.g. at a boundary.

- **String theory** and **AdS/CFT correspondence**:

(https://commons.wikimedia.org/)



Bulk geometry from boundary CFT dynamics

Epilogue

- SYK model: N **Majorana fermions** ψ_i with random q-valent interactions: [Sachdev, Ye '93; Sachdev '10; Kitaev '15...]

$$H_{\rm int} = \sum_{1 \leq i_1 < \ldots < i_q \leq N} j_{i_1 \ldots i_q} \, \psi_{i_1} \psi_{i_2} \cdots \psi_{i_q}$$

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- Same type of Schwinger-Dyson equations as in tensor models and GFTs ('melonic' equations) $\Rightarrow$ **SYK conformal regime from tensor models**.

  [Witten '16; Gurau '16; Klebanov, Tarnopolsky '16...]

Technical convergence of strings and loops in simplified models $\rightarrow$ unexpected dialogue opportunity! [See talk by Verlinde, and also Pranzetti, Chirco...]

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