Squeezed spin networks and entanglement

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Introduction

Q1. How to recognize semiclassical states in quantum gravity?

Mean values $\langle \hat{q}_{ab}(n) \rangle = \bar{q}_{ab}(n)$

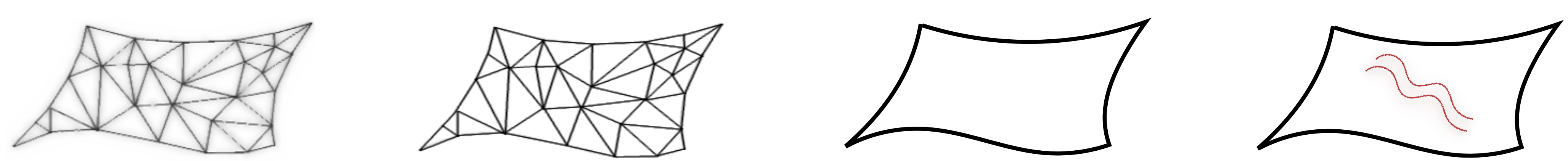
Peakedness $\langle (\Delta \hat{q}_{ab}(n))^2 \rangle \ll \epsilon$

} *large scales* $\bar{q}_{ab}(x)$

Correlations $\langle \hat{q}_{ab}(m) \hat{q}_{ab}(n) \rangle - \bar{q}_{ab}(m) \bar{q}_{ab}(n) = \langle \hat{h}_{ab}(m) \hat{h}_{ab}(n) \rangle$

Fluctuations: $\hat{h}_{ab}(m) = \hat{q}_{ab}(m) - \bar{q}_{ab}(m)$

} $\hat{h}_{ab}(x)$



spin netwok state	*discrete geometry (Regge, twisted)*	*classical spacetime*	*Linearized gravity on classical spacetime*
$\lvert \Gamma, \mathrm{i}_n, j_\ell \rangle$	$\bar{q}_{ab}(n)$	$\bar{q}_{ab}(x)$	$\bar{q}_{ab}(x) + \hat{h}_{ab}(x)$

Fluctuations of the quantum geometry should match the fluctuations of linearized gravity on the mean geometry at large scales.

Introduction

Q1. How to recognize semiclassical states in quantum gravity?

- Fluctuations display long-range correlations: $\langle \hat{h}(m)\hat{h}(n)\rangle \propto 1/d(m,n)^2$

  Behavior of two-point function for general states in QFT on curved spaces is universal in small scales $\ell \ll R_{macro}$ (Hadamard condition). Asymptotic form for sufficiently close points should be valid for $L_{Pl} \ll \ell$ in quantum gravity.

- Entanglement entropy satisfies an area law. $S_{EE}(R) \propto A(\partial R)$

Bianchi-Myers Conjecture: Area law a characteristic feature of semiclassical states in QG.

"Entanglement as the architecture of spacetime" *[Bianchi, Myers '14]*

- ✦ Disentangling QG dofs breaks connectivity of space *[van Raamsdonk '10]*.
- ✦ Geometry from entanglement in LQG *[Chirco et al '17]*.

Introduction

Q2. How to construct semiclassical states in LQG?

Bosonic representation: LQG in terms of constrained harmonic oscillators *[Girelli, Livine '05]*.

Coherent states	=	Simplest class of states with arbitrary 1-point functions
Squeezed states	=	Simplest class of states with arbitrary 2-point functions

- Squeezed states in bosonic representation ⟶ Squeezed spin networks

- Take coherent and squeezed spin networks as candidate semiclassical states. Search for sector with long-range correlations and area law for the entanglement entropy.

Plan of the talk

1. Bosonic representation of LQG

2. Squeezed spin network states

3. Long range correlations in cubulations

Part One

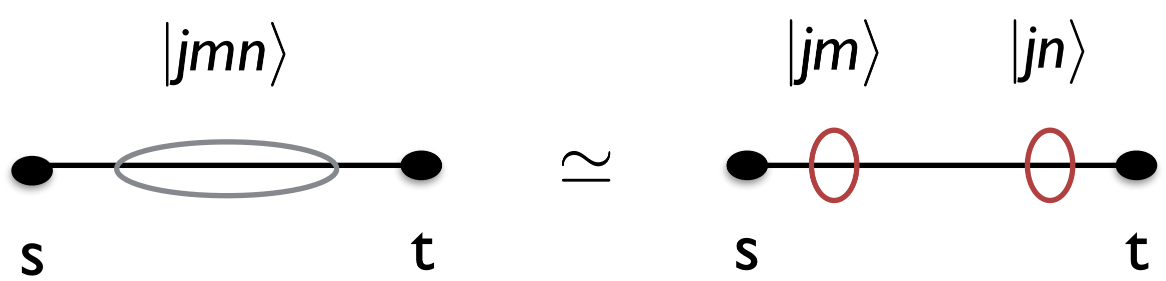
The bosonic representation

Bosonic representation [Girelli, Livine CQG 22 (2005)]

Hilbert space of LQG on a graph Γ : $\mathcal{H}_\Gamma = L^2[SU(2)^L/SU(2)^N] \quad \ni \quad \psi(U_1, \ldots, U_L)$

At each link, basis of Wigner matrices: $|jmn\rangle = D^j_{mn}(U_\ell)$ *gauge invariant*

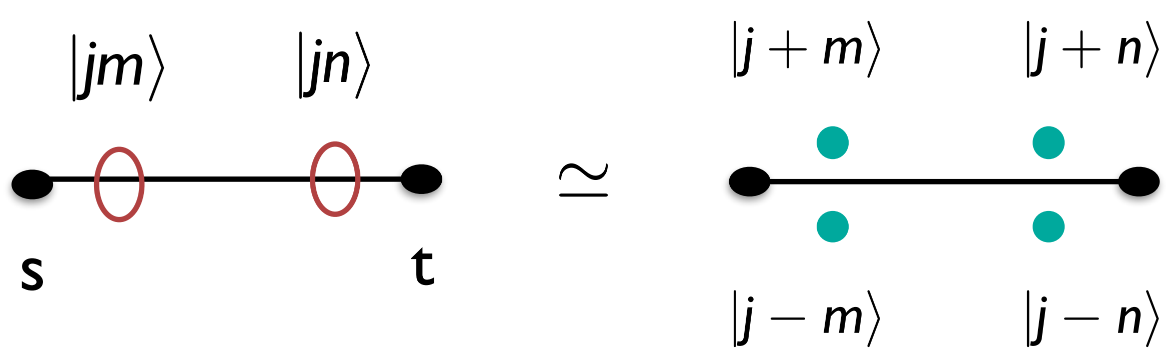
First transformation



$L^2[SU(2)^L] \subset \mathcal{H}^{2L}_{spin}$

Area matching: $C_\ell \equiv l_s - l_t \approx 0$

Second transformation (Schwinger model)

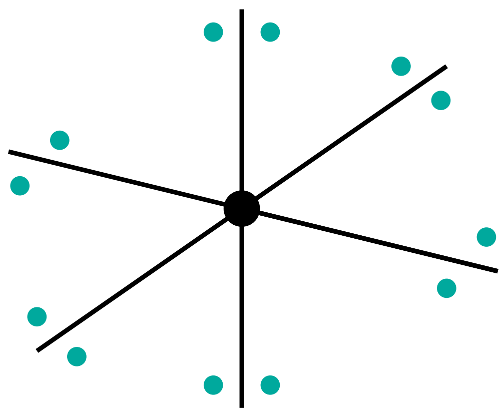


$\mathcal{H}^{2L}_{spin} \simeq \mathcal{H}^{4L}_{osc} \equiv \mathcal{H}_{\mathcal{S}}$

$$|jm\rangle = \frac{(a^{0\dagger})^{j+m}\,(a^{1\dagger})^{j-m}}{\sqrt{(j+m)!(j-m)!}}$$

LQG states on Γ mapped into states of a bosonic lattice with 4L oscillators.

Bosonic representation



Bosonic variables: $a^A_{n\mu},\, a^{A\dagger}_{n\mu}\ ,\ A = 0, 1$

LQG Hilbert space: $\mathcal{H}_\Gamma = P_G\, P_A\, \mathcal{H}^{4L}_{osc}$

Gauss constraint (→ P_G)   *Area matching constraint* (→ P_A)

Holonomy-flux algebra

$$\vec{J}_i = \frac{1}{2}\,\vec{\sigma}_{AB}\, a_i^{A\dagger} a_i^B \quad , \quad I_i = \frac{1}{2}\delta_{AB}\, a_i^{A\dagger} a_i^B \qquad i = (n\mu)$$

$$(h_\ell)^A{}_B \equiv (2I_t + 1)^{-\frac{1}{2}} \left(\epsilon^{AC}\, a^\dagger_{t\,C}\, a^\dagger_{s\,B} - \epsilon_{BC}\, a_t^A\, a_s^C\right) (2I_s + 1)^{-\frac{1}{2}}$$

Wilson loops

$$W_\alpha = \mathrm{tr}\left(h_{|\alpha|} h_{|\alpha|-1} \cdots h_1\right)$$

$$W_\Phi = \prod_k W_{\alpha_k} \quad , \quad \text{for multiloops } \Phi = \{\alpha_1, \alpha_2, \dots\}$$

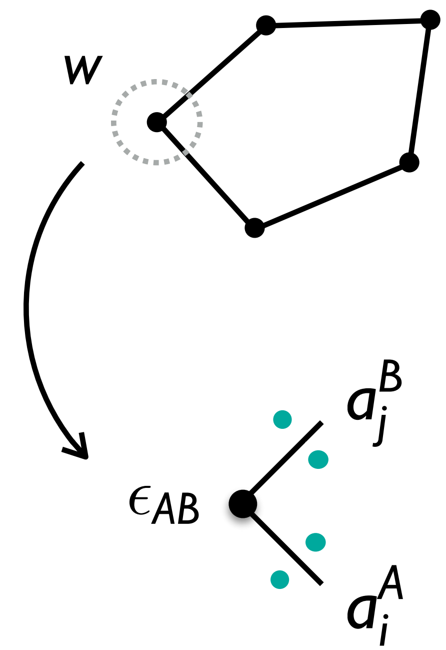
[Livine, Tambornino JMP 53 (2012) and PRD 87 (2013)]

[Bianchi, Guglielmon, Hackl, NY, PRD 94 (2016) and 1605.05356]

Loop expansion [Bianchi, Guglielmon, Hackl, NY, PRD 94 (2016)]

New structure: ***Notion of normal order in the bosonic representation.***

⟶ Normal-ordered Wilson loop $:W_\Phi: |0\rangle = F^\dagger_\Phi |0\rangle = |\Phi\rangle$



Wedge operator $F_w = F_{ij} = \epsilon_{AB} a_i^A a_j^B$

Loop operator $F_\alpha = \prod_r F_{w_r}$

Multiloop operator $F_\Phi = \prod_k F_{\alpha_k}$

Wedge operators $F_{ij}, F^\dagger_{ij}$ the same as in U(N) formalism.

[Freidel, Livine JMP 51 (2010) and JMP 52 (2011); Dupuis, Livine CQG 28 (2011)]

Loop expansion

$$P_\Gamma = \sum_\Phi \frac{1}{\prod_\ell (2j_\ell)! \ \prod_n (J_n + 1)!} F^\dagger_\Phi |0\rangle\langle 0| F_\Phi$$

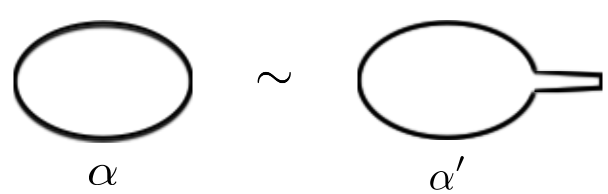
Loop expansion of the projector

Provides resolution of the identity in a basis of loop states in $\mathcal{H}_\Gamma$

$$|\Psi\rangle = \sum_{\Phi} c_\Phi |\Phi\rangle \ , \qquad c_\Phi = \frac{\langle 0|F_\Phi|\Psi\rangle}{\prod_\ell (2j_\ell)! \prod_n (J_n + 1)!}$$

Overcompleteness of loop basis can be managed with new loop states.

- Retracing identities automatically solved. For loops with tails, loop state vanishes.

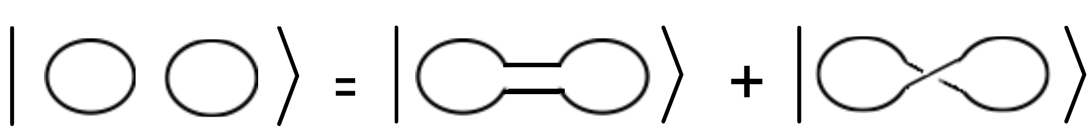


retracing identity

$$F_{\alpha'} = 0 \implies |\alpha'\rangle = 0$$

Only one loop in each equivalence class contributes to the loop expansion.

- Local Mandelstam identities encoded in Plücker identities for F’s at common node.



$$\Longrightarrow \qquad F_{ij}F_{kl} = F_{ik}F_{jl} + F_{il}F_{kj}$$

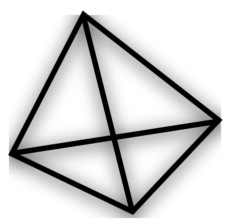
At each node: Plücker identities

Entanglement entropy

✦ Locality in Γ: quantum polyhedra as atoms of space (*twisted geometries*).

Intertwiner space = quantization of classical phase of polyhedron [Bianchi, Doná, Speziale PRD83 (2011)]

States on fixed graph as quantum twisted geometries [Freidel, Speziale PRD82 (2010), Rovelli PRD82 (2010)]



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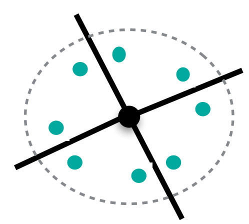


Quantum polyhedron | *Intertwiner space* $\mathcal{H}_n$

Local algebra of observables at node n generated by operators of $2|n|$ oscillators.

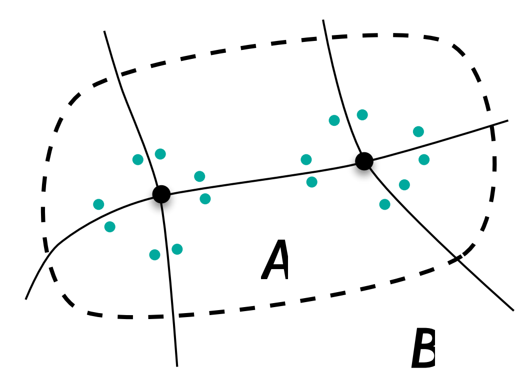
$$\mathcal{H}_{\mathcal{S}} = \bigotimes_n \mathcal{H}_{osc}^{2|n|}$$

✦ Decomposition of Γ: two complementary sets of nodes $\mathcal{V}_A$ and $\mathcal{V}_B$

$$\mathcal{V} = \mathcal{V}_A \cup \mathcal{V}_B \quad \Longrightarrow \quad \mathcal{H}_{\mathcal{S}} = \mathcal{H}_A \otimes \mathcal{H}_B$$

$$\rho_A = \mathrm{Tr}_B \left(|\Psi\rangle\langle\Psi| \right)$$

Entanglement entropy: $\quad S_A = -\mathrm{Tr}_A[\rho_A \log \rho_A]$



Part Two

Squeezed spin network states

Squeezed spin network states

❖ Construction of squeezed spin networks:

1. Start with reference vacuum *(AL)*

$$|0\rangle = |AL\rangle$$

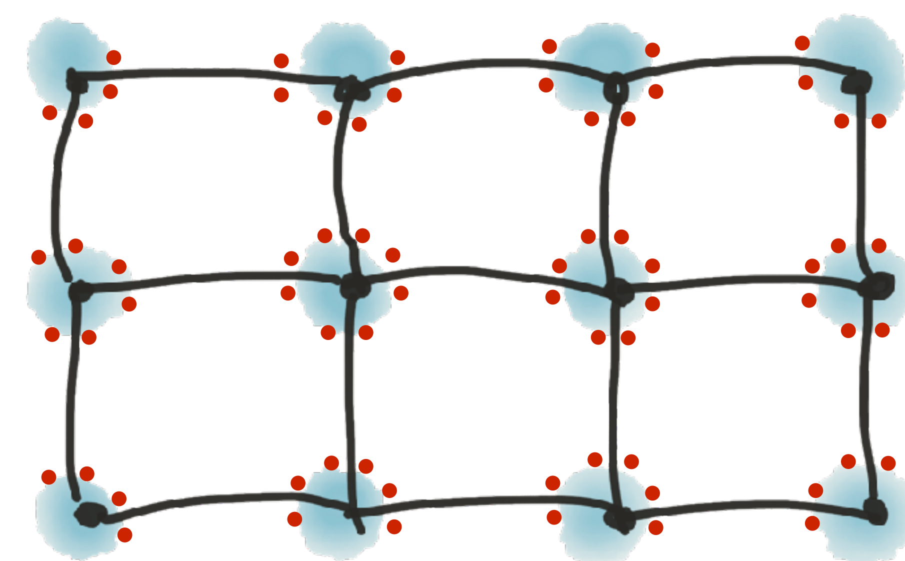
2. Squeeze

Symplectic transformation $M : V \to V$

Squeezed vacuum $U(M)|0\rangle$

3. Project to LQG space

$$P_\Gamma U(M)|0\rangle$$



At each oscillator, local bosonic fields φ_i, π_i

Symplectic structure: $\Omega_{ab} = \{\xi_a, \xi_b\}, \ \xi = (\varphi, \pi)$

Space of linear observables spanned by local fields is a symplectic vector space V.

Symplectic group and squeezing

Bosonic space $\mathcal{H}_S$ carries a representation of the symplectic group $Sp(8L, \mathbb{R})$.

Generators	$E_{ij}^{AB} = \frac{1}{2}\left(a_i^{A\dagger} a_j^B + a_j^B a_i^{A\dagger}\right)$	***Preserve vacuum***
(Symplectic algebra)	$F_{ij}^{AB} = a_i^A a_j^B \quad , \quad F_{ij}^{AB\dagger} = a_i^{A\dagger} a_j^{B\dagger}$	***Squeezing***

Under a symplectic transformation, reference vacuum transforms into a squeezed state. It is sufficient to consider $M_\gamma \in Squeeze(J_0)$.

Bosonic squeezed state: $|\gamma\rangle \equiv U(M_\gamma)|0\rangle \propto \exp\left(\gamma_{AB}^{ij} a_i^{a\dagger} a_j^{B\dagger}\right)|0\rangle$

- ✦ Parametrized by two-point functions.
- ✦ Entanglement entropy as a simple function of γ [Bianchi, Hackl, NY PRD92 (2015)].
- ✦ Ground states of quadratic Hamiltonians. Area law states if Hamiltonian is local.

Projection to LQG space

Squeezed spin network state

$$|\Gamma, \gamma\rangle = P_\Gamma \underbrace{\exp[\gamma^{ij}_{AB} a^{A\dagger}_i a^{B\dagger}_j]|0\rangle}_{\text{bosonic squeezed state}}$$

<u>Loop expansion</u>

$$c_\Phi = \frac{1}{\prod_\ell (2j_\ell)! \; \prod_n (J_n + 1)!} \int \frac{d^{4L}z \, d^{4L}\bar{z}}{\pi^{4L}} Z_\Phi \; e^{-z^A_i \bar{z}^i_A + \frac{1}{2}\gamma^{AB}_{ij} \bar{z}^i_A \bar{z}^j_B}$$

Loop amplitudes can be computed for local squeezing matrix and its perturbations.

<u>Small squeezing</u>

strong coupling expansion

$$|\Gamma, \gamma\rangle = |0\rangle + \sum_{\square} c_{\square} F^\dagger_{\square} |0\rangle + \sum_{\square\!\square} c_{\square\!\square} F^\dagger_{\square\!\square}|0\rangle$$
$$+ \sum_{\square} c_{\square} F^\dagger_{\square}|0\rangle + \sum_{\square} c_{\square} F^\dagger_{\square}|0\rangle + \sum_{\square\,\square} c_{\square\,\square} F^\dagger_{\square} F^\dagger_{\square}|0\rangle + \ldots$$

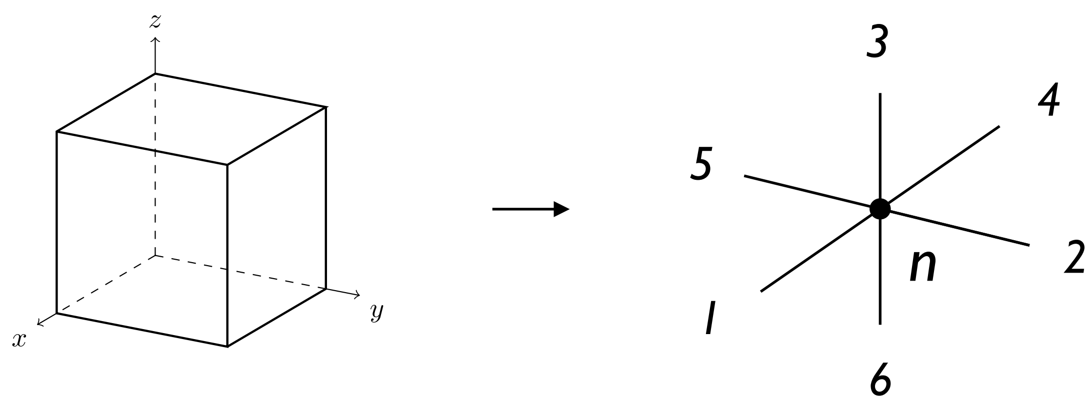
More on squeezed spin network states: See <u>Hackl's talk, 16:25</u>

<u>Part Three</u>

Long range correlations

Semiclassical states for cubic lattice

- Geometry of a cubic lattice described with spinor variables:



<u>3d vectors from spinors:</u>

$$\vec{v}(z) = \frac{1}{2}\vec{\sigma}_{AB}\,\bar{z}^A z^B\,, \quad z \in \mathbb{C}^2$$

$$z_{n\mu} = z_\mu\,, \quad \vec{v}(z_\mu) \text{ are the } \pm\hat{x}^i \text{ directions}$$

- Semiclassical states:

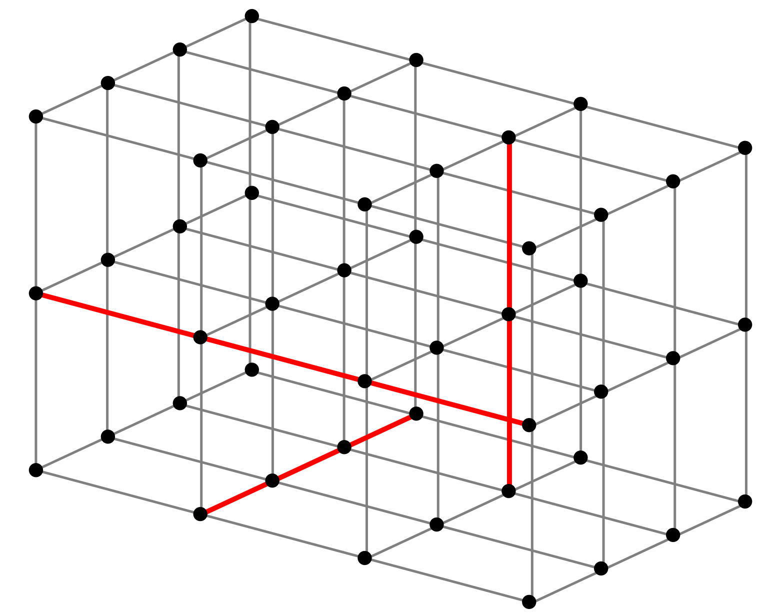
1. Coherent states $\quad |z\rangle = P_\Gamma \exp(\lambda\, z_A^{n\mu} a_{n\mu}^{A\dagger})|0\rangle$

2. Squeezed states $\quad |\gamma\rangle = P_\Gamma \exp\left(\lambda\,[\gamma(z)]_{AB}^{(m\mu)(n\nu)} a_{m\mu}^{A\dagger} a_{n\nu}^{B\dagger}\right)|0\rangle$

Spin-spin correlations: $\quad C_{\ell\ell'} = \langle I_\ell I_{\ell'}\rangle - \langle I_\ell\rangle\langle I_{\ell'}\rangle\,, \quad I_\ell = \frac{1}{2}\delta_{AB}\, a_{s(\ell)}^{A\dagger} a_{s(\ell)}^B$

Correlations for coherent states (large spins, $j_\ell \gg 1$)

1. Probability distribution factorizes over one-dimensional sublattices.



$$P_\lambda \propto \prod_{links} e^{-\frac{4}{\lambda^2}\left(j_\ell - \frac{\lambda^2}{2}\right)^2} \prod_{nodes} \prod_{a} e^{-\frac{1}{2\lambda^2}(j_{a+3} - j_a)^2}$$

2. Correlations within each sublattice:

$$C(R) \equiv C_{j_i, j_{i+R}} = \frac{j_0}{5} e^{-2.3R} \quad \Longrightarrow \quad \text{Correlation length:} \quad \xi = 0.43$$

(in lattice units)

Similar result for heat kernel states $|\{H_\ell\}, t\rangle$, $H_\ell \in SL(2, \mathbb{C})$ [Thiemann CQG18 (2001)] [Bianchi et al PRD82 (2010)]

$$C(R) \simeq \frac{j_0}{(1 + 4tj_0)} [2(1 + 4tj_0)]^{-R} \quad \Longrightarrow \quad \xi < \frac{1}{\log 2} \simeq 1.44$$

Correlations in the limit of small spins

Coherent states satisfy the following properties:

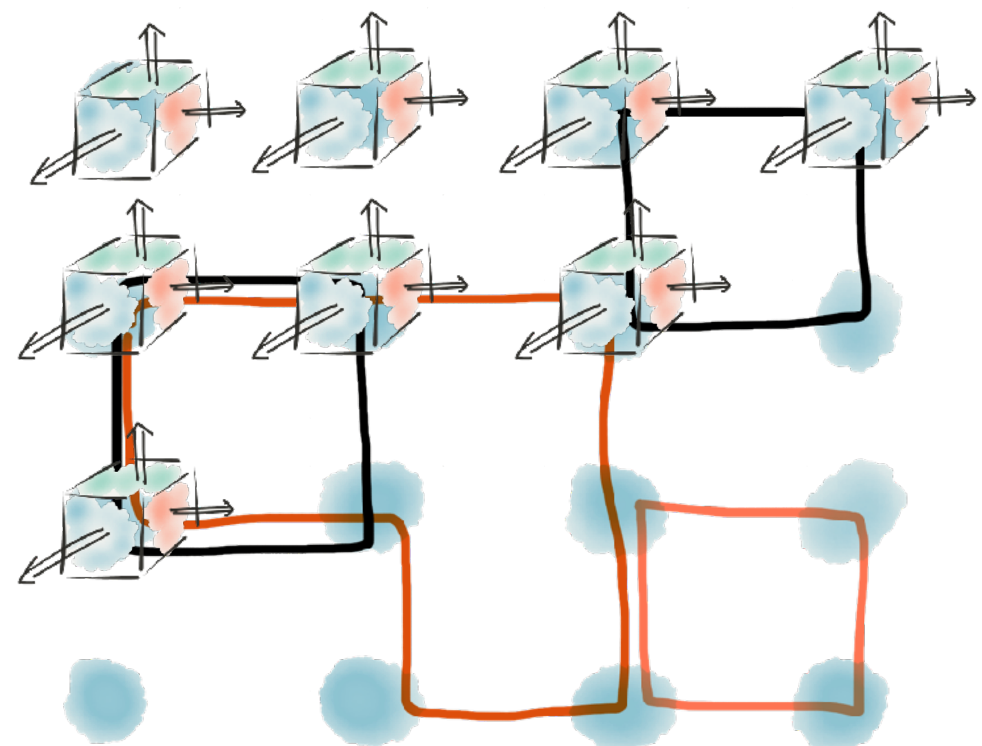
(i) Factorizable state $\qquad c_{\Phi_1 \sqcup \Phi_2} = c_{\Phi_1} c_{\Phi_2}$

(i) Fast decay of amplitudes $\qquad c_{\Phi} \leq A \lambda^{\beta|\Phi|} \qquad$ (with $\beta = 2$)

Loop expansion dominated by contributions from small loops.

$$C(R) \propto \lambda^{2\beta R}$$

Short ranged correlations in the limit of small spins.



[Bianchi, Guglielmon, Hackl, NY]

Correlations for squeezed states

Squeezed state peaked at cubic geometry

Choose: $$\gamma^{AB}_{(m\mu),(n\nu)} = \lambda\, \epsilon^{AB}\, \epsilon_{CD} z^C_\mu z^D_\nu \,(\delta_{mn} + \varepsilon\, f_{mn})$$

fixes scale factor (→ λ)

areas peaked along Euclidean directions (→ $z^C_\mu z^D_\nu$)

$\varepsilon \ll 1$ *, encodes correlations* (→ εf_{mn})

Average values of local observables fixed by λ and z's. For small spins:

$$\langle j \rangle = \frac{2}{3^8}\lambda^8\,, \qquad \langle W \rangle = \frac{\lambda^4}{2\cdot 3^4}\cos[2(\xi_i + \xi_j)]$$

For $\varepsilon = 0$, the state is factorizable. Correlations are short ranged.

Correlations for squeezed states

For nonzero ε, we find for the spin-spin correlations:

$$C_{\ell\ell'} = \frac{2^6}{3^{16}} \lambda^{16} \varepsilon^2 f^2_{s(\ell)s(\ell')}$$

The function f can be chosen to scale with the inverse of the distance, yielding an inverse square law for the correlations:

$$C_{\ell\ell'} \propto 1/(d_{s(\ell)s(\ell')})^2$$

✦ *The distance d does not refer to a background geometry. It is encoded in the state, being determined by the diagonal part of the squeezing matrix.*

✦ *Area law for the entanglement entropy.* [Bianchi, Hackl, NY]

Quantum shape matching

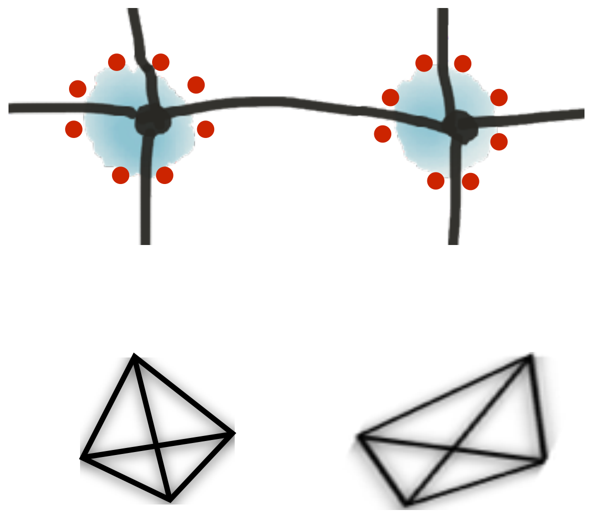
Twisted geometry: Fluctuations in shapes of nearby tetrahedra uncorrelated

Uniformly saturate the mutual information for neighboring nodes:

$$I_{AB} = \frac{1}{2}(S_A + S_B - S_{AB})$$

$$\frac{(\langle \mathcal{O}_A \mathcal{O}_B \rangle - \langle \mathcal{O}_A \rangle \langle \mathcal{O}_B \rangle)^2}{2 \|\mathcal{O}_A\|^2 \|\mathcal{O}_B\|^2} \leq I(A, B)$$

Correlations in fluctuations of the geometry are enhanced for adjoining tetrahedra.



Twisted geometry

Resulting state is peaked at shape-matched configurations.

For dipole graph, EPR-like state with perfectly correlated geometry fluctuations.

[Baytas, Bianchi, NY]

Conclusion

- Bosonic representation

  Resolution of the identity in a new basis of loop states

  Entanglement entropy defined in the extended bosonic space

- Construction of squeezed spin network states

- Long range correlations on cubulations

  Squeezed spin network states can be used to encode correlations that reproduce vacuum fluctuations of massless fields in the continuum

  Area law for the entanglement entropy

- Quantum shape matching