Renormalizing spin foam models: quantum cuboids and beyond

Sebastian Steinhaus

sebastian.steinhaus@desy.de

II. Institute f. Theoretical Physics
University of Hamburg

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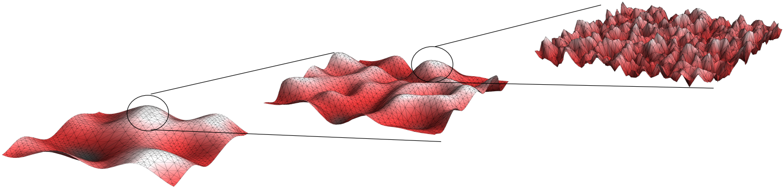
Motivation

- Spin foams are a **microscopic** ansatz for **quantum space-time**
- How can we **'zoom out'** from Planck scale to large scales?



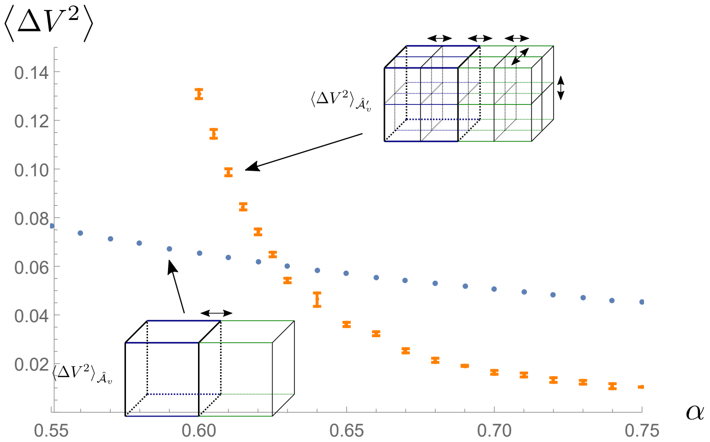
- **Renormalization group** ideal tool: relate theories at different scales
  - Check **consistency** of theory
  - Fate of **diffeomorphism symmetry**
- **Efficiently** extract results from the theory!

Key idea: Relate states across Hilbert spaces to compare transitions!

Numerical techniques are essential!

Proof of principle: renormalizing quantum cuboids

- Compare observable in two 'quantum cuboid' configurations



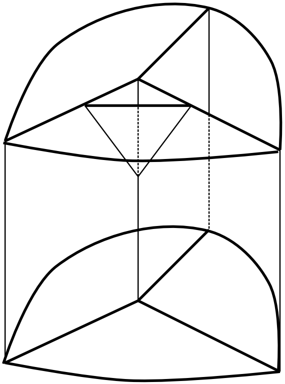
Indications for **phase transition** [Bahr, S.St. PRL '16]:

UV-attractive fixed point with restoration of **diffeomorphism symmetry**?

Outline

1. Spin foams and renormalization
2. Tensor network renormalization
3. Renormalizing restricted spin foam models
4. Outlook and Summary

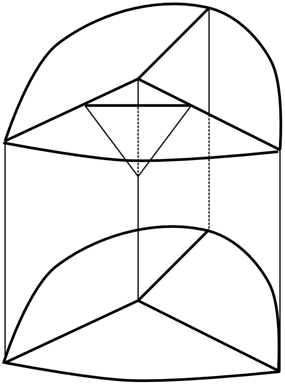
Spin foams in a nutshell [Engle, Pereira, Rovelli, Livine, Freidel, Krasnov, Barrett, Crane,...]



- **Spin foams**: path integral for kinematical states of loop quantum gravity
  - Spin network states on the boundary of the foam
- Spin network $\sim$ **3D geometry**
  - Node: Intertwiner $\iota \sim$ shape of a polyhedron
  - Link: Representation $j \sim$ area of face
- Spin foam **'evolves'** boundary state
  - Irreducible representations j_f on faces f
  - Intertwiners ι_e on edges e

Combinatorics tell how polyhedra are glued together to form a **4D geometry**.

Spin foams in a nutshell II [Engle, Pereira, Rovelli, Livine, Freidel, Krasnov, Barrett, ...]



- Path integral given as a **sum** over all possible **spin foam states** $\{\iota_e, j_f\}$
- Spin foam: assign **amplitude to geometry**
  - $\mathcal{A}_v$: vertex amplitude
  - $\mathcal{A}_e$: edge amplitude
  - $\mathcal{A}_f$: face amplitude
- Amplitude functional $\mathcal{A} : \mathcal{H}_b \to \mathbb{C}$
  - $\mathcal{H}_b$: boundary Hilbert space on a discretisation b
- **Transition amplitude** for $\mathcal{H}_b = \mathcal{H}_i \otimes \mathcal{H}_f^*$
  - $\langle s' | s \rangle_{\mathcal{A}} = \mathcal{A}(s \otimes s')$

$$Z = \sum_{\{j_f\},\{\iota_e\}} \prod_f \mathcal{A}_f(j_f) \prod_e \mathcal{A}_e(\{j_f\}, \iota_e) \prod_v \mathcal{A}_v(\{j_f\}, \{\iota_e\})$$

Spin foams: Results and interesting directions

- **Asymptotic expansion of vertex amplitude** $\mathcal{A}_v \sim e^{iS_R} + e^{-iS_R}$ [Barrett, Dowdall, Fairbairn, Gomes, Hellmann, Conrady, Freidel,...]
  - S_R: **Regge action**, discretisation of general relativity
- **Cosmological constant** in spin foams [Turaev, Viro, Han, Meusburger, Fairbairn, Haggard, Kaminski, Riello,...]
- Recent progress:
  - Proper vertex amplitude [Engle, Zipfel, Vilensky,...]
  - **Lorentzian amplitudes** [Speziale, Dona, Kaminski, Kisielowski, Sahlmann,...] **[see Speziale's and Kisielowski's talks]**
  - Computing **Pachner moves** [Banburski, Chen, Freidel, Hnybida]
- Towards **observations**:
  - Spin foam cosmology [Rovelli, Bianchi, Vidotto, Rennert, Sloan]
  - **Black hole** tunneling from spin foams [Christodoulou, Rovelli, Speziale, Vilensky,...] **[see Rovelli's talk]**

Most of these calculations are done for **few simplices**.
Can we **reliably** extract results from **coarse discretisations**?

Spin foams as a theory

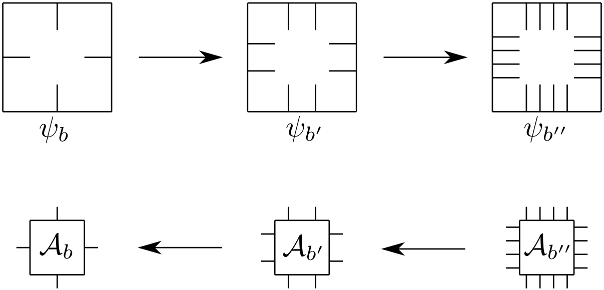
- Do **predictions depend** on **ambiguities**? **[see Thiemann's talk]**
  - Choice of **amplitudes**
  - Choice of **discretisation**
- Can we **approximate** an observable on a coarse **discretisation**?
- Two conceptually different approaches tackling these questions:

- **Group field theory** approach: Summing over all triangulations and topologies [Oriti, Carrozza, Rivasseau, Ben Geloun, Gurau, Bonzom, Lahoche,...] **[see Oriti's talk]**
  - GFT generates spin foam amplitudes as Feynman diagrams
  - **Renormalisable (as a QFT)?** **[see Carrozza's talk]**
  - Spin foam vertices $\sim$ 'atoms of spacetime'

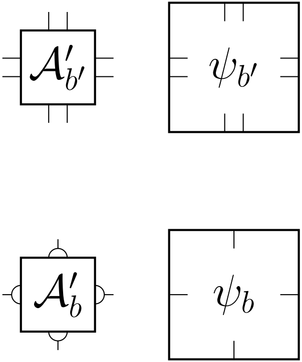
- **Refining approach**: renormalize the spin foam amplitudes by coarse graining [Dittrich, Bahr, S.St., Martin-Benito, Delcamp, Mizera,...]
  - **How is a coarse spin foam related to a fine spin foam?**
  - **Consistent boundary formulation** **[see Dittrich's talk]**
  - 2-complex is a regulator.

Renormalization – how and why? [Dittrich, S.St. '14]

- **Wilsonian renormalization**: order degrees of freedom according to scale
- **Background independence**: use a 'relative' scale
- Represent **same transition** on **different discretisations**
  - Relate states $\phi_b \in \mathcal{H}_b$ and $\psi_{b'} \in \mathcal{H}_{b'}$
  - Embedding maps $\iota_{b'b} : \mathcal{H}_b \to \mathcal{H}_{b'}$
  - Consistency conditions
  - Then we can compare $\mathcal{A}_b(\phi_b)$ and $\mathcal{A}_{b'}(\psi_{b'})$



Spin foam amplitudes and embedding maps



- Consider two spin foam amplitudes, where $b \prec b'$:
  - $\mathcal{A}_b : \mathcal{H}_b \to \mathbb{C}$
  - $\mathcal{A}'_{b'} : \mathcal{H}_{b'} \to \mathbb{C}$
- Relate / **renormalize** spin foam amplitudes:

$$\mathcal{A}'_{b'}(\underbrace{\iota_{b'b}(\psi_b)}_{=\psi_{b'}}) =: \mathcal{A}'_b(\psi_b) \stackrel{?}{=} \mathcal{A}_b(\psi_b)$$

- Consistency of boundary states translated to a relation of amplitudes

- $\iota_{b,b'}$ relates amplitudes $\mathcal{A}_b$ and $\mathcal{A}'_{b'}$, but **need not be dynamical**.
- **Dynamical** embedding maps [Dittrich '12, '14] from **spin foam amplitudes** [Dittrich, S.St. '14]

Embedding maps and cylindrical consistency

- Different embedding maps in loop quantum gravity
- **Ashtekar–Lewandowski vacuum** [Ashtekar, Isham '92, Ashtekar, Lewandowski '95]:
  - Arbitrary refinement of graphs
  - New edges carry spin $j = 0$.
- **BF-vacuum** [Dittrich, Geiller '15, Bahr, Dittrich, Geiller '15]: **[see Geiller's talk]**
  - Refinement of triangulations by Pachner / Alexander moves
  - New closed holonomies are flat.
- **Consistency**: how to **refine boundaries** and how to **add degrees of freedom in vacuum state**.
  - Vacuum refers to representing the same state (w.r.t. an inner product) on a finer boundary.
- Consider as well variational discrete systems [Dittrich, Höhn '12, '13, Höhn '14].

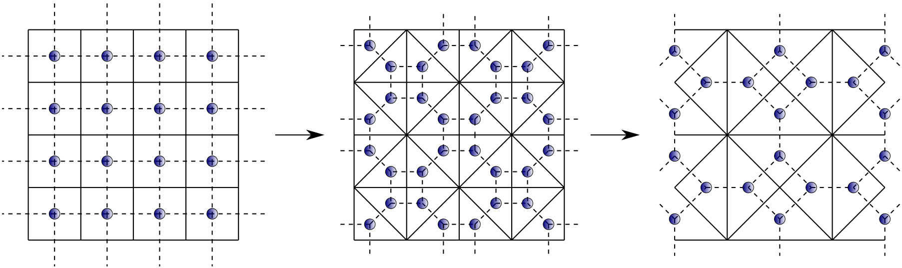
These concepts work well in the kinematical setting.

Can they be extended to the **dynamical / physical** setting?

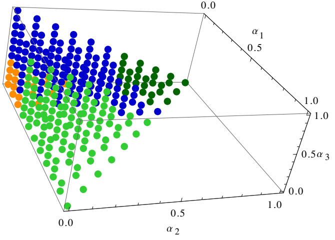
Outline

Tensor network renormalization [Levin, Nave '07, Gu, Wen '09]

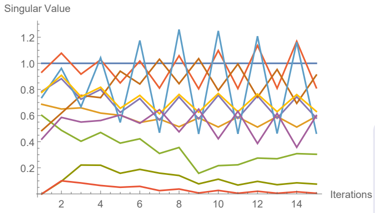
- Tensor network algorithms: **two main steps**
  - **Summing** over 'fine' degrees of freedom
  - **Variable transformation**: from 'fine' to 'coarse' degrees of freedom
- Variable transformation (= **embedding map**) computed from **dynamics**!
  - **Truncation** also determined by dynamics
- **Renormalization group flow** of tensors
- Numerical algorithm



Spin net models



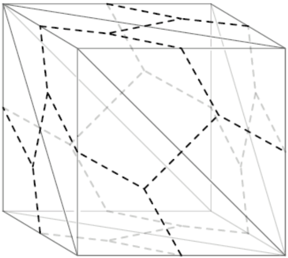
$\alpha = 0.35$

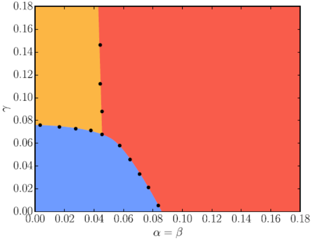


- **2D 'spin' systems**
  - Global symmetry
  - Same degrees of freedom as spin foams
- **Finite groups**:
  - Abelian [Dittrich, Eckert, Martin-Benito '11]
  - Non-Abelian [Dittrich, Martin-Benito, Schnetter '13]
- **Quantum groups**:
  - $SU(2)_k$ spin nets [Dittrich, Martin-Benito, S.St. '14]
  - Toy model matter + 'gravity' [S.St. '15]
  - **BC and EPRL-type models** [Dittrich, Schnetter, Seth, S.St. '16]

- Interesting, **rich phase structure**
- Almost **scale invariance** at phase transitions
- Extensive **optimization** required!

Tensor networks for lattice gauge theories





- **Gauge theory**: redundancy of data
- **Decorated tensor networks**
  - Abelian [Dittrich, Mizera, S.St. '15]
  - Non-Abelian [Delcamp, Dittrich '16]
- Amplitudes assigned to regions 'decorated' by spin network data
  - Explicit symmetry: **fix redundant data**
- **Phase diagram** for 3D $\mathbb{Z}_2$ and S_3 lattice gauge theory.
- Issue: spin network basis **not stable** under coarse graining
  [see Livine's, Riello's and Delcamp's talks]

Basis encoding **curvature** and **torsion** excitations.
More efficient description?

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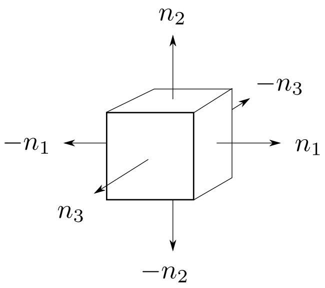
Quantum cuboids: restricted spin foam [Bahr, S.St. '15]

- **Strategy**: study a **subset of partition function**
  - Might contribute **significantly** in specific physical scenarios
- **Quantum cuboids**: 4D EPRL model [Engle, Pereira, Rovelli, Livine '08] on **hypercubic** 2-complex [Kamiński, Kisielowski, Lewandowski '10]
- State sum restricted to specific intertwiner: **cuboid shaped**

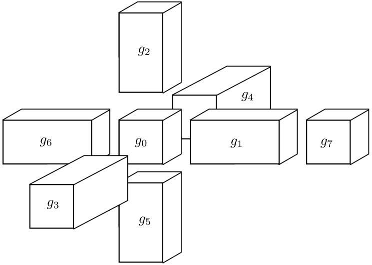


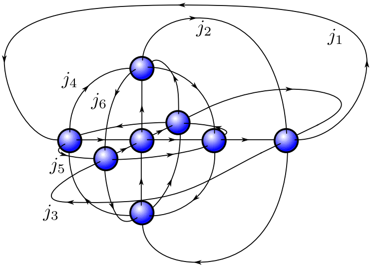
- **Coherent SU(2) intertwiner** [Livine, Speziale '07] for cuboids:

$$|\iota_{j_1,j_2,j_3}\rangle = \int_{\mathrm{SU}(2)} dg\; g \triangleright \bigotimes_{i=1}^{3} |j_i, e_i\rangle \otimes |j_i, -e_i\rangle$$

  - e_i: normal unit vectors in $\mathbb{R}^3$.
  - $e_1 = \exp(-i\pi\,\sigma_2/4) \triangleright e_3$ and $e_2 = \exp(i\pi\,\sigma_1/4) \triangleright e_3$.

Quantum cuboids – amplitudes





- **Vertex amplitude $\mathcal{A}_v$:** contract 8 coherent intertwiners
- $\mathcal{A}_v = \mathcal{A}_v^+ \mathcal{A}_v^-$ for $\gamma < 1$.

$$\mathcal{A}_v^\pm = \int_{\mathrm{SU}(2)^8} dg_a \, e^{S^\pm[g_a]} \,, \text{ with}$$

$$S^\pm[g_a] = \frac{1 \pm \gamma}{2} \sum_l 2j_l \, \ln \langle -\vec{n}_{ab} | \, g_a^{-1} g_b \, | \vec{n}_{ba} \rangle$$

- **Face amplitude $\mathcal{A}_f$:**

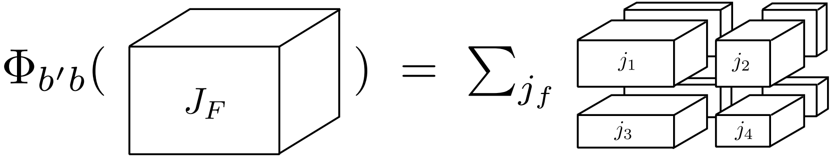
$$\mathcal{A}_f = ((2j_f^+ + 1)(2j_f^- + 1))^\alpha$$

Large-j limit: $S = 0$

Flat geometries – drastic simplification!

Embedding map

- $\hat{\mathcal{A}}_v(j_1, j_2, j_3, j_4, j_5, j_6) \sim \prod_{i=1}^{6} j_i^{2\alpha} \left(\frac{1}{\sqrt{-D}} + \frac{1}{\sqrt{-D^*}} \right)^2$
  - α only parameter
  - D determinant of Hessian matrix
- **Geometric embedding map**:

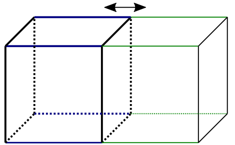


- 'Coarse cuboid' $\approx$ 'Collection of fine cuboids'
- Count number of fine cuboids satisfying $J_F = \sum_f j_f$.

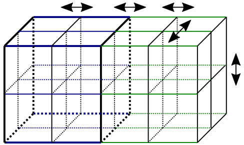
Geometric embedding map, **not dynamical!**

The renormalization prescription [Bahr, S.St. PRL '16]

$\langle \Delta V^2 \rangle_{\hat{\mathcal{A}}_v}$



$\langle \Delta V^2 \rangle_{\hat{\mathcal{A}}'_v}$



- Consider two glued geometric hypercuboids:
  - 'Coarse': 2 hypercuboids
  - 'Fine': 32 hypercuboids + embedding map
- Total volume fixed, **integrate** over remaining variables:
  - 'Coarse': 1 integration
  - 'Fine': 6 integrations
- Numerical **Monte Carlo integration**

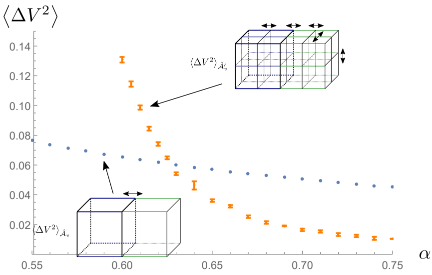
- Compare Variance ΔV^2 of the volume of one coarse cuboid:

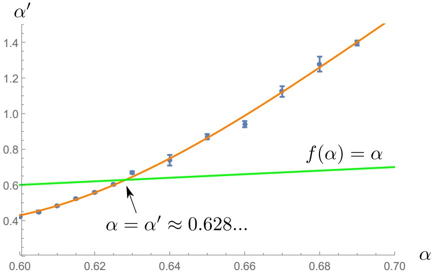
$$\left\langle \Delta V^2 \right\rangle_{\text{Coarse}(\alpha')} \approx \left\langle \Delta V^2 \right\rangle_{\text{Fine}(\alpha)}$$

- Determine **RG flow** for α, i.e. $\alpha \to \alpha'$.

Essentially **project** $\hat{\mathcal{A}}_v^{(\text{fine})}$ back onto $\hat{\mathcal{A}}_v^{(\text{coarse})}$ for different α.

Indications for a UV-attractive fixed point [Bahr, S.St. PRL '16, PRD '17]





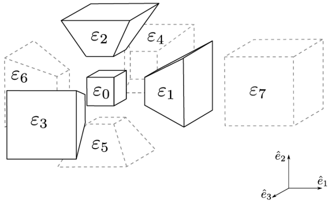
- Renormalization prescription $\alpha'(\alpha)$
  - $\alpha < 0.628... \rightarrow \alpha' < \alpha$
  - $\alpha > 0.628... \rightarrow \alpha' > \alpha$
- **Fixed point** at $\alpha^* \approx 0.628...$
- Close to (almost) **diffeomorphism invariant** amplitude ($\alpha = 0.6067...$).
- Similar for different choices of boundary data
- Error by projection appears to be small

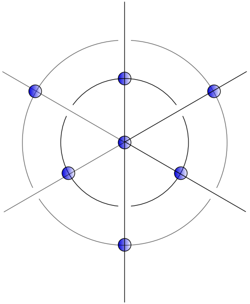
Indications for a **phase transition / UV-attractive fixed point**

Restoration of **diffeomorphism invariance**?

Outline

Renormalizing Quantum Frusta [Bahr, Rabuffo, S.St. w.i.p.]





- Replace cuboids by **frusta** [Bahr, Klöser, Rabuffo, '17]
  - Model expanding / contracting universe
- Include **cosmological constant** [Han '11]
  - Deformation at crossing of planar graph
- Large-j limit:

$$\mathcal{A}_v \sim \frac{\exp(i\,\beta\,S_R)}{-D} + \frac{\exp(-i\,\beta\,S_R)}{-D^*} + \frac{2}{\sqrt{D D^*}} \cos(\gamma\,\beta\,S_R + \beta\,\Lambda\,V)$$

- Additional parameters:
  - $\beta \sim$ Newton's constant
  - Λ: Cosmological constant

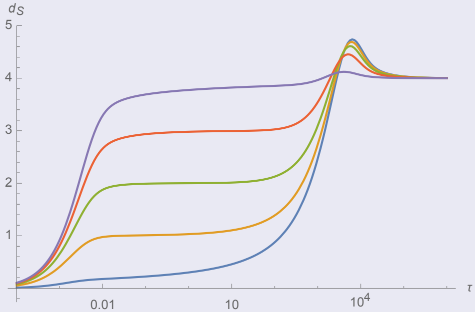
Consistency check – cuboids included!

[see Bahr's and Rabuffo's talks]

Spectral dimension of spin foams

- **Effective dimension: spectral dimension** d_S of spin foams [Modesto, Magliaro, Perini, Caravelli,...]
- **Diffusion process on Quantum Cuboids** [S.St., Thürigen, w.i.p.]
  - Spin foam with 10^{12} vertices + periodic boundary conditions
  - Additional N-periodicity [Sahlmann '10]
  - Laplace operator Δ

Tentative indications for **dimensional reduction / flow** in spin foams (for $\alpha < 0.583$) at **small scales!** **[see Thürigen's talk]**



Summary

- **Refinement approach** to renormalizing spin foams
- **Key idea**: **same transition** represented on **different discretisations**
  - Check consistency of theory
  - Efficiently compute results
- **Tensor network renormalization**: ideal for identifying phases / transitions
  - Rich phase structure in 2D models
  - Phase diagram for 3D lattice gauge theories
- **Restricted spin foam models**: RG flow for 4D quantum cuboids
  - Indications for phase transition and UV-attractive fixed point
  - Hope: approximate well specific physical situation
- Numerics indispensable for this approach: see school 'Making quantum gravity computable' @ Perimeter (`pirsa.org`)

Many **open issues** remain:

Lorentzian signature, full theory, include matter, ...

Thank you for your attention!