New Boundary Degrees of Freedom

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based on work in collaboration with Laurent Freidel and Alejandro Perez

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ma per seguir virtute e canoscenza
SISSA
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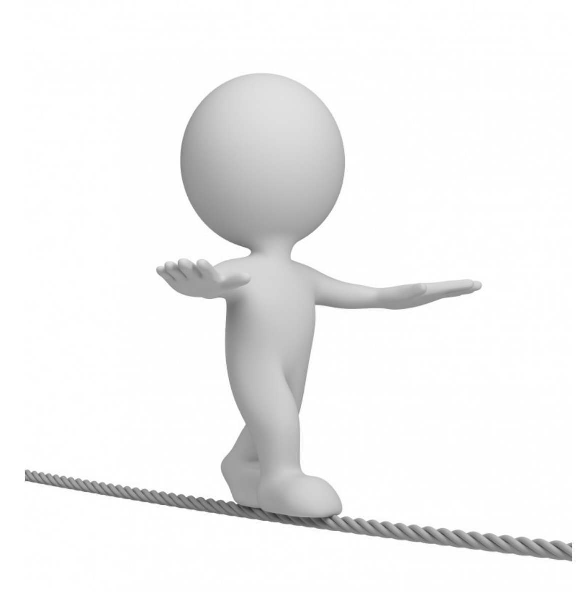
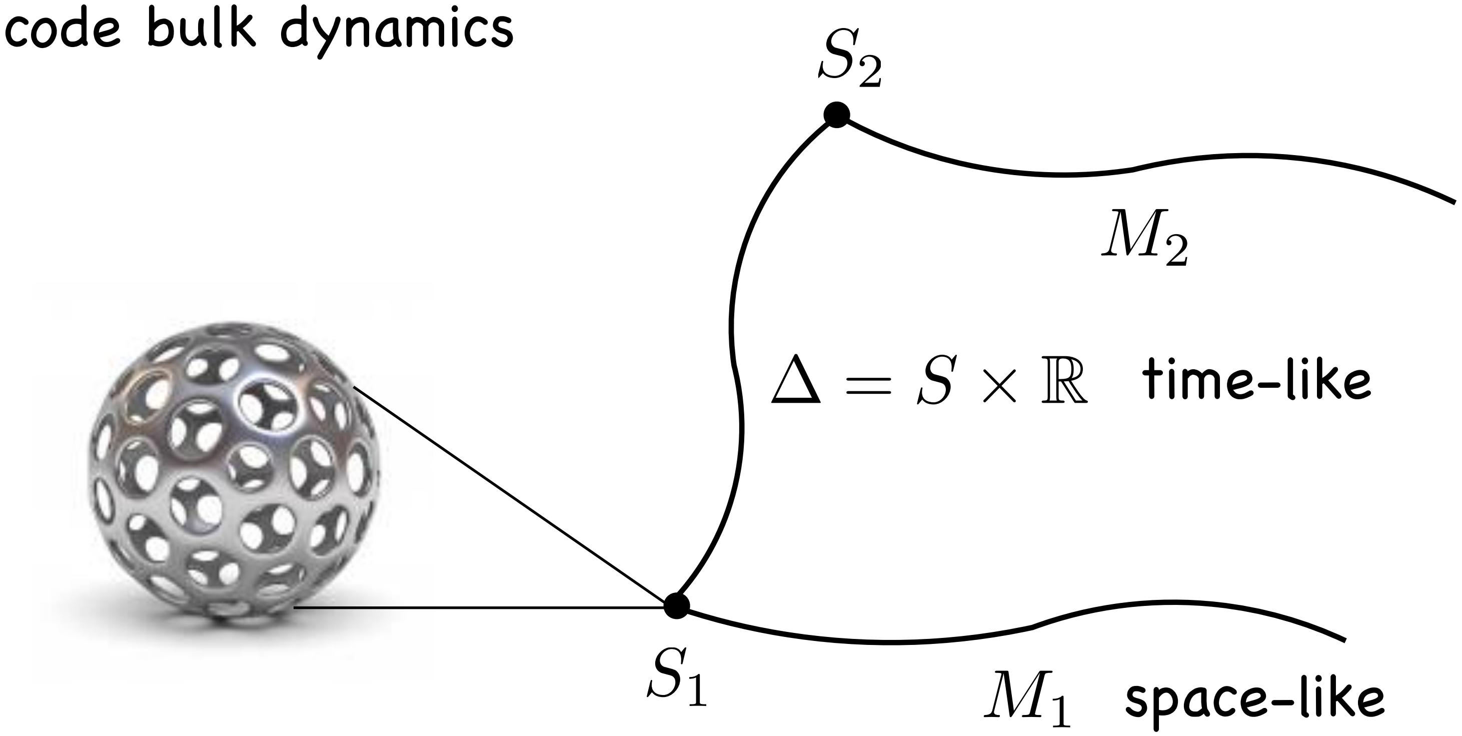
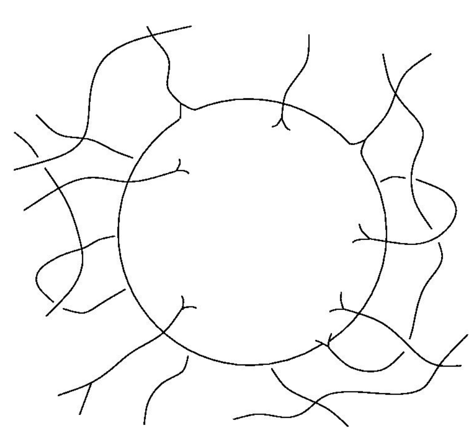
We want to explore life on the edge:



- A lot of physics going on at the boundary
- A place where to define observables (charges)
- A place where to encode bulk dynamics





Fundamental geometry discreteness input

A couple of old ideas:

- ✦ Boundaries break gauge symmetries and new degrees of freedom appear when trying to restore them

[Benguria, Cordero, Teitelboim '77]; [Teitelboim '95]; [Carlip '99]...

recently revisited [Donnelly, Freidel '16]; [see Freidel's talk]

- ✦ CFT degrees of freedom naturally dwell around punctures

[Witten '89]; [Moore, Seiberg '89]...

explored in the context of LQG in [Smolin '95]; [Freidel, Krasnov, Livine '10]; [Ghosh, DP '14]

- One thing we all care about in LQG: The Hamiltonian constraint

$$e_i \wedge F^i(A) + \cdots = 0$$

- One thing we lack in LQG: A triad operator $\hat{e}_i$

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☞ We need to extend the LQG kinematical Hilbert space...

○ How many birds, N, can we kill with one stone?... N ≥ 2

✦ In presence of a boundary, we need to add a boundary term to the Holst action in order to make it differentiable

✦ This leads to an extended phase-space with boundary DOF parametrized by the triad

✦ In presence of background bulk curvature excitations, we can define a triad operator whose algebra includes a central charge

✦ We can use CFT techniques to construct a Fock representation yielding a normalizable vacuum

The new degrees of freedom that arise from the presence of a boundary are physical:

- They represent the set of all possible boundary conditions that need to be included in order to reconstruct the expectation value of all gravity observables

- They are needed in the reconstruction of the total Hilbert space in terms of the Hilbert space for the subsystems (edge states/soft modes): They encode entanglement between subsystems

- They correspond to partial observables ([Rovelli '01]; [Dittrich '04]), which could represent detectors on a boundary or physical boundary conditions

- They also represent the degrees of freedom that one needs in order to couple the subsystem to another system in a gauge invariant manner

Ingredients

- A generalized Gibbons-Hawking-York boundary term

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- The new boundary degrees of freedom organize themselves under the representation of the Virasoro symmetry group. Local conformal invariance realized through the thickening of the spin network links into spin-tubes (regularization required to define $\hat{e}_i$)

originally postulated in [Smolin '95], [Major, Smolin '95]

and recently resurfaced in the context of spin foam amplitudes computation [Haggard, Han, Kaminski, Riello '15]

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- Background geometry assumption: the tangential curvature of the connection vanishes everywhere on the boundary except at the location of a given set of punctures. Motivated also by the new, dual vacuum of loop gravity

[Gambini, Pullin '97]; [Bianchi '09]; [Freidel, Geiller, Ziprick '13]; [Dittrich, Geiller '14]; [see Geiller's talk]

Phase space analysis

➢ We want to consider the canonical structure of general relativity in the first order formalism on a 3d slice that possesses a 2d boundary punctured by (dual) spin network links.

We start from a formulation of gravity on a manifold $M \times \mathbb{R}$ with a boundary two sphere S^2

3d space-like hypersurface

$$S = \frac{1}{2\gamma\kappa}\left[\int_{M\times\mathbb{R}} E^{IJ} \wedge F_{IJ}(\omega) + \int_{S^2\times\mathbb{R}} e_I \wedge \mathrm{d}_A e^I\right]$$

$\kappa = 8\pi G\,,\quad \gamma :$ Barbero-Immirzi parameter, $\quad e^I :$ Frame field, $\quad \omega^{IJ} :$ 4d spin connection

$$A^{IJ} := (\omega^{IJ} + \gamma * \omega^{IJ})$$

Boundary 'connection'

$$E^{IJ} = [\underbrace{(e^I \wedge e^J)}_{} + \gamma * (e^I \wedge e^J)]|_{bulk}$$

Flux defined by the simplicity constraint

gives the Holst term which vanishes on-shell

The boundary action term: $$S_{bound} = \frac{1}{\gamma}\int_{S\times\mathbb{R}} e_I \wedge \mathrm{d}_A e^I = \int_{S\times\mathbb{R}} -\ast\,\omega^{IJ}\wedge(e_I\wedge e_J) + \frac{1}{\gamma} e_I \wedge d_\omega e^I$$

$$\delta\omega_{IJ}: \quad E^{IJ}|_{Bulk} = (e^I\wedge e^J) + \gamma \ast (e^I \wedge e^J)|_{Boundary}$$

Simplicity constraint on the boundary

- By choosing a Lorentz gauge where one of the tetrad is fixed to be the normal to the boundary, it is easy to see that the first component is simply given by the integral of the well known Gibbons-Hawking-York boundary density:

$$\ast\omega^{IJ}\wedge(e_I\wedge e_J) \quad\to\quad 2\sqrt{h}K\,,$$ where $h =$ determinant of the induced metric on the boundary
$K =$ trace of the boundary extrinsic curvature

- The second one is a new addition to the standard boundary term of the metric formulation, which vanishes on shell due to the torsion free condition (Cartan eq.):

$\gamma^{-1} e_I \wedge d_\omega e^I$ is a natural complement to the Holst term in the bulk action $\gamma^{-1}F_{IJ}(\omega)\wedge e^I \wedge e^J$

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☞ The boundary action is not gauge invariant, but its variation is!

✳ For different actions and boundary conditions proposals at outer or inner boundaries in the I order formalism see e.g. [Ashtekar, Engle, Sloan '08]; [Bianchi, Wieland '12]; [Bodendorfer, Neiman '13]; [Corichi, Reyes, Vukasinac '16]; [Wieland's talk]

As the entire Hamiltonian treatment that will follow makes use of the Ashtekar-Barbero connection formulation, we need to appeal to the availability of the extra structure that allows for the introduction of such variables:

normal to M — Ashtekar-Barbero connection — coframe fields tangent to M

✦ Time gauge: $dn^I = 0\,, \quad n = e^I n_I = e^0 \qquad A^i = \Gamma^i + \gamma K^i\,, \qquad \Sigma_i = \frac{1}{2}\epsilon_{ijk}\, e^j \wedge e^k$

Symplectic 2-form: $$\Omega = \Omega_M + \Omega_{S^2} = \frac{1}{\kappa\gamma}\int_M (\delta A^i \wedge \delta\Sigma_i) + \frac{1}{2\kappa\gamma}\int_{S^2} (\delta e_i \wedge \delta e^i)$$

The extended phase space

Poison brackets: $\{A^i_a(x), \Sigma^j_{bc}(y)\} = \kappa\gamma\delta^{ij}\epsilon_{abc}\delta^3(x-y) \qquad \{e^i_a(x), e^j_b(y)\} = \kappa\gamma\,\delta^{ij}\epsilon_{ab}\delta^2(x-y)$

bulk phase space — boundary phase space

The bulk fields are given by an SU(2) valued flux-2-form Σ^i and an SU(2) valued connection A^i

satisfying the standard Gauss, diffeomorphism and scalar bulk constraints

- Bulk fields (Σ_i, A^i) can be seen as background fields that commute with the boundary field e^i

- Following from the imposition of the time gauge, we demand the boundary condition : $\delta e_0|_\Delta = 0$

However, we do not demand the triad e^i to be fixed: We let the boundary geometry fluctuate at will, and this turns out to be the source of the boundary degrees of freedom.

The preservation of the gauge and diffeomorphism symmetry in the presence of the boundary imposes the validity of additional boundary constraints

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Boundary constraints:

- Boundary Gauss law $\Sigma_i|_{bulk} = \dfrac{1}{2}[e,e]_i|_{boundary}$

Boundary simplicity constraint: Matching of bulk and boundary area elements

The simplicity constraint is here the condition enabling the preservation of SU(2) symmetry in the presence of a boundary

Initially, e commutes with all the bulk fields A and Σ, and Σ commutes with itself:

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- Boundary diffeomorphism constraint $\mathrm{d}_A e^i = 0$

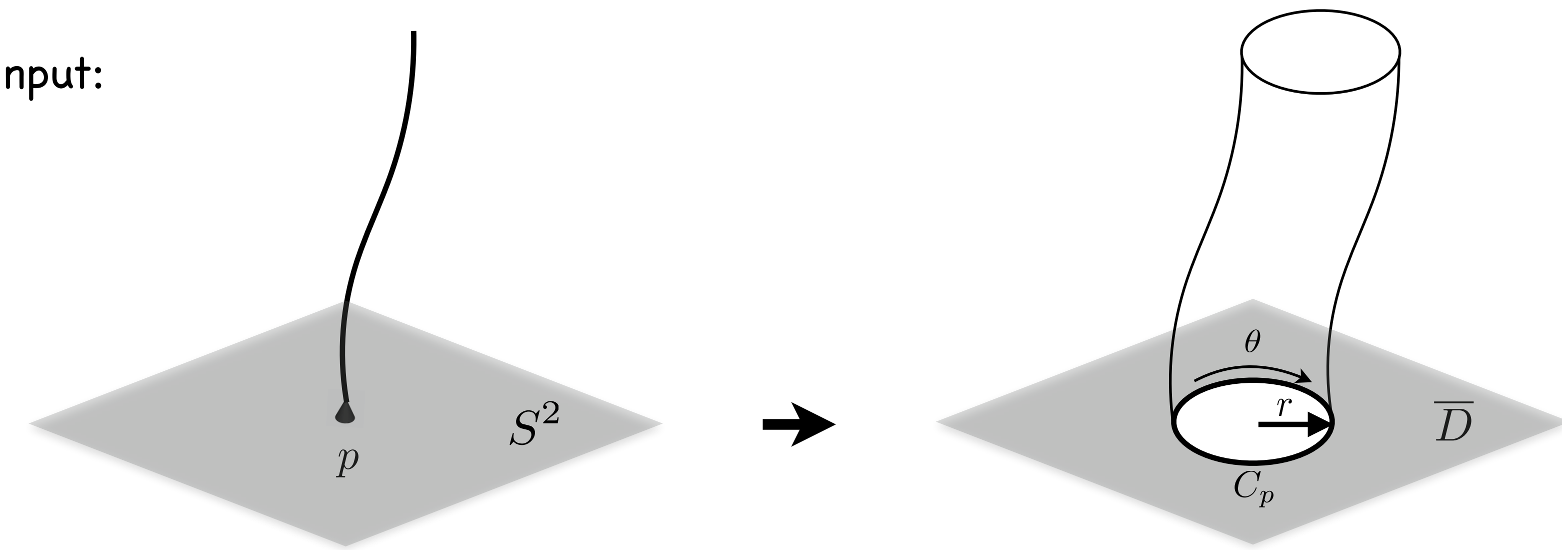
Cartan first structure equation + condition on the embedding

The set of all admissible boundary frames can be thought of as labelling the set of possible boundary geometries which satisfy the boundary equation of motion given by the staticity constraint (which replaces the boundary condition $\delta h = 0$)

Background geometry

➢ Fundamental discreteness input:

$$S^2 = \overline{S} \cup_p D_p$$



$(z, \bar{z})$ complex directions tangential to S^2

Because the components $\Sigma^i_{z\bar{z}}$ and $(A^i_z, A^i_{\bar{z}})$ commute, we can a priori fix them to any value on the boundary:

The simplicity constraint determines the value of the boundary flux $\Sigma^i_{z\bar{z}}$ in terms of the boundary frames $(e^i_z, e^i_{\bar{z}})$:

Given a disk D embedded in S^2

$$\Sigma^i_D \equiv \int_D \Sigma^i = \frac{1}{2} \int_D [e, e]^i$$

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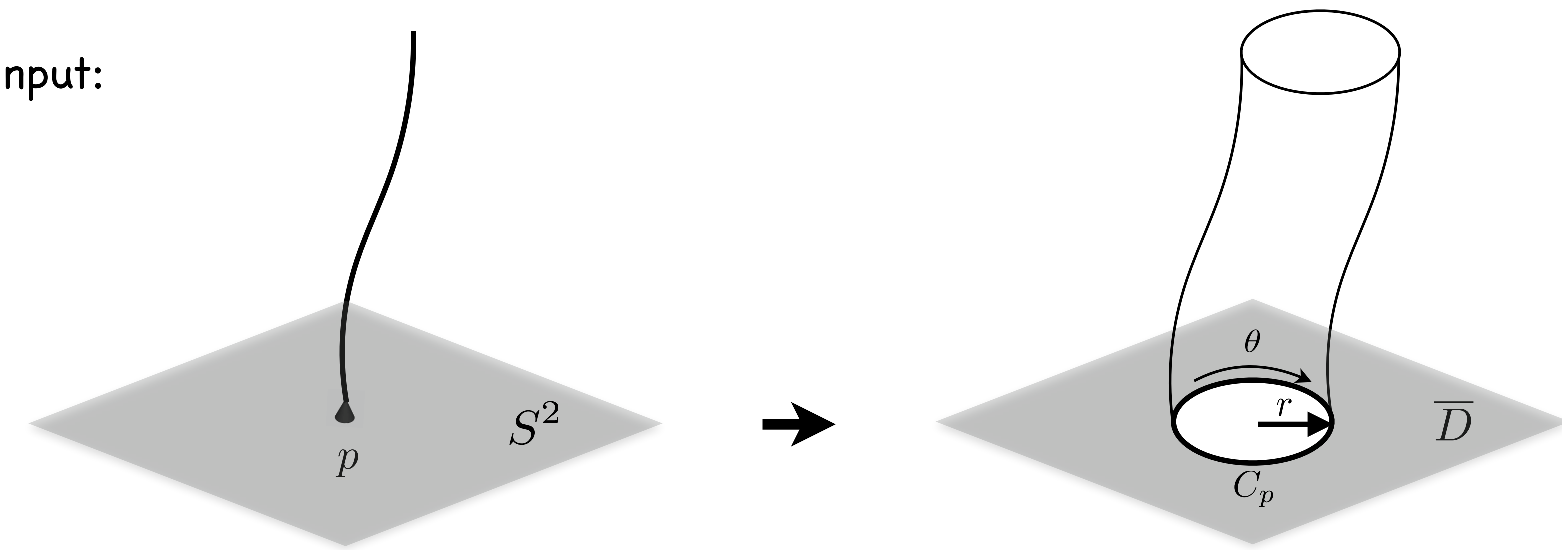

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- We choose the tangential curvature of A to vanish everywhere on the sphere except at the location of a given set of N punctures defined by the endpoint of (dual) spin-network links:

$$F^i(A)(x) = 2\pi \sum_p K^i_p \delta^{(2)}(x, x_p)$$

the SU(2) Lie algebra elements K^i_p parametrize the background curvature of the boundary

$$\left(g_{D_p} \equiv P \exp \oint_{C_p} A = \exp 2\pi K_p \right)$$

Covariant Hamiltonian formulation

Hamiltonian generators associated to the boundary constraints:

The Poisson bracket is related to the symplectic structure via $\{F, G\} = \Omega(\delta_F, \delta_G)$

where δ_F is the Hamiltonian variation generated by F, $\Omega(\delta_F, \delta) = \delta F$

The two generators associated with the Gauss and diffeo constraints are obtained from the symplectic structure through

$$\Omega(\delta_\alpha, \delta) = \delta G_D(\alpha), \qquad \Omega(\delta_\varphi, \delta) = \delta S_D(\varphi)$$

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➥ $$G_D(\alpha) \equiv \frac{1}{\kappa\gamma}\left(\frac{1}{2}\int_D \alpha^i [e, e]_i - \int_M \mathrm{d}_A \alpha^i \wedge \Sigma_i\right)$$

boundary extension of the Gauss constraint

By integrating by parts the bulk term we see
that it imposes the Gauss Law $\mathrm{d}_A \Sigma^i = 0$ and
the boundary simplicity constraint $\Sigma_i = 1/2[e, e]_i$

also generator of internal rotations

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$$S_D(\varphi) \equiv \frac{1}{\kappa\gamma}\left(\int_D \mathrm{d}_A\varphi^i e_i + \int_M F_i(A)\wedge[e,\varphi]^i\right)$$

boundary extension of tangent diffeos generator

where $\varphi^i = \varphi^a e^i_a$ and

$\varphi = \varphi^a\partial_a$ is a vector tangent to M

also generator of the transformations

$$\delta_\varphi e^i = \mathrm{d}_A\varphi^i = L_\varphi e^i$$

Gauge vs. Symmetry

In presence of gauge symmetry, on the constrained surface $\overline{\Gamma}$ consisting of points that satisfy the constraints in the theory, the pullback $\overline{\Omega}$ is degenerate (for first class constraints): $\overline{\Omega}(\delta_G, \delta) = 0\,, \forall \delta$

$\delta_G =$ gauge (degenerate) directions and the associated Hamiltonian generators are generators of gauge symmetries

However, in presence of boundaries we can have $\overline{\Omega}(\delta_S, \delta) = \delta H_S \neq 0$

$\delta_S =$ True symmetries of the theory and the associated Hamiltonian generators represent conserved Hamiltonian charges of physical (partial) observables

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★ In our case, $S_D(\varphi) \equiv \frac{1}{\kappa\gamma}\left(\int_D \mathrm{d}_A \varphi^i e_i + \int_M F_i(A) \wedge [e, \varphi]^i\right)$

is the diffeo Hamiltonian generators associated to Hamiltonian vector field δ_φ and

$$\overline{\Omega}(\delta_\varphi, \delta) = \delta Q(\varphi)\,, \quad \text{where} \quad Q(\varphi) = \frac{1}{\kappa\gamma}\oint_{\partial D} \varphi^i e_i$$

$\Longrightarrow$ $\boxed{\text{Diffeos associated to vector fields which do not vanish on the circle represent true symmetries}}$

(as for asymptotically flat spacetimes, where some non-trivial diffeos at infinity are associated to the generators of the Poincaré group)

Algebra of boundary constraints

Algebra: from $\quad \Omega(\delta_\alpha,\delta_\beta)=\delta_\beta G_D(\alpha)\,,\quad \Omega(\delta_\varphi,\delta_\phi)=\delta\phi S_D(\varphi)\,,\quad \Omega(\delta_\alpha,\delta_\varphi)=\delta_\varphi G_D(\alpha)$

where $\quad \delta_\alpha e^i=[\alpha,e]^i\,,\quad \delta_\alpha\Sigma^i=[\alpha,\Sigma]^i\,,\quad \delta_\alpha A^i=-d_A\alpha^i$

$$\delta_\varphi\Sigma^i=L_\varphi\Sigma^i=\varphi\lrcorner d_A\Sigma^i+d_A(\varphi\lrcorner\Sigma)^i\,,\quad \delta_\varphi A^i=L_\varphi A^i=\varphi\lrcorner F^i(A)\,,\quad \delta_\varphi e^i=L_\varphi e^i=d_A(\varphi\lrcorner e^i)$$

➥

$$\begin{aligned}
\{G_D(\alpha),G_D(\beta)\}&=G_D([\alpha,\beta])\,,\\
\{G_D(\alpha),S_D(\varphi)\}&=\int_{\partial D}([\varphi,\alpha]_i e^i)+S_D([\alpha,\varphi])\,,\\
\{S_D(\varphi),S_D(\varphi')\}&\hat{=}\int_{\partial D}(\varphi^i \mathrm{d}_A\varphi'_i)-\int_D F^i[\varphi,\varphi']_i
\end{aligned}$$

$G_D=S_D=0$

the boundary diffeomorphism algebra is second class with the appearance of central extension terms supported on the boundary

$\Rightarrow$ At the punctures some of the previously gauge degrees of freedom become now physical

Kac-Moody charges

➢ Our goal now is to study the quantisation of this boundary system in the presence of the background fields

Boundary charges: $Q_D(\varphi) \equiv \frac{1}{\sqrt{2\pi\kappa\gamma}} \int_D \mathrm{d}_A \varphi^i \wedge e_i$ where $\varphi = \varphi^i \tau_i$ SU(2)-valued field

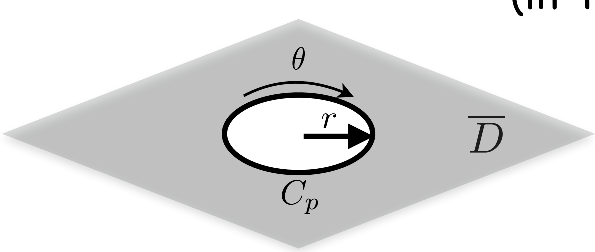
$\rightarrow \quad Q_{\overline{D}}(\varphi) \hat{=} -\sum_p Q_p(\varphi)$ and $Q_p(\varphi) \hat{=} \frac{1}{\sqrt{2\pi\kappa\gamma}} \oint_{C_p} \varphi^i e_i$

on-shell of $\mathrm{d}_A e_i = 0$

By means of the PB $\{e^i_a(x), e^j_b(y)\} = \kappa\gamma\, \delta^{ij} \epsilon_{ab} \delta^2(x,y)$ and solving the condition $F^i(A) = 2\pi K^i \delta(x)$

in the neighborhood of the puncture and fixing the gauge freedom: $A = K_p \mathrm{d}\theta$

(in this gauge the gauge field is constant and the fields are periodic)



$$\Rightarrow \quad \{Q_p(\varphi), Q_{p'}(\psi)\} = \frac{\delta_{pp'}}{2\pi} \oint_{C_p} (\varphi^i \mathrm{d}\psi_i - K^i_p[\varphi,\psi]_i \mathrm{d}\theta)$$

By defining the modes $Q^j_n \equiv Q(\tau^j e^{i\theta n})$, where $[\tau^i, \tau^j] = \epsilon^{ijk}\tau_k$ anti-hermitian basis

➥ $\{Q^i_n, Q^j_m\} = -i\left(n\delta^{ij} + K^{ij}\right)\delta_{n+m}$, where $K^{ij} \equiv -i\epsilon^{ijk}K_k$

✦ In the case where the curvature vanishes we simply get $\{Q^i_n, Q^j_m\} = -in\delta^{ij}\delta_{n+m}$

☞ $U(1)^3$ Kac-Moody algebra with central extension equal to 1

✦ In the presence of curvature, we obtained a three dimensional abelian Kac-Moody algebra twisted by K :

Let us work in a complex basis: $\tau^a = (\tau^3, \tau^+, \tau^-)$, $\tau^\pm = (\tau^1 \mp i\tau^2)/\sqrt{2}$, $[\tau^3, \tau^\pm] = \pm i\tau^\pm$, $[\tau^+, \tau^-] = i\tau^3$

where $K = k\tau_3$ and $K^{a\bar{b}}$ is diagonal, with $(a = 3, +, -)$, $\bar{a} = (3, -, +)$

then the twisted Kac-Moody algebra can then be written compactly as $(\{\cdot,\cdot\} \to -i[\cdot,\cdot])$:

$$\boxed{[Q^a_n, Q^b_m] = \delta^{a\bar{b}}(n + k^a)\delta_{n+m}}$$

where $k^a := (0, +k, -k)$, $\sum_a k_a = 0$

A Kac-Moody algebraic structure follows directly from the gravitational symplectic structure when distributional configurations (punctures) are considered

The theory associated with k and with $k+1$ are equivalent.
This equivalence corresponds to the fact that at the quantum level the connection is compactified, a fact that here is derived completely naturally in the continuum.

Virasoro generators

➢ Generator of boundary diffeomorphisms along a vector field $v^a \partial_a$ tangent to S^2:

$$\Omega_D(L_v, \delta) = \delta T_D \quad \text{, where} \quad L_v e^i := v \lrcorner \mathrm{d}_A e^i + \mathrm{d}_A(v \lrcorner e^i)$$

$$\rightarrow \quad T_D = \frac{1}{2\kappa\gamma} \int_D L_v e^i \wedge e_i \hat{=} \frac{1}{2\kappa\gamma} \oint_{\partial D_p} (v \lrcorner e^i) e_i$$

Hamiltonian conserved charges associated to diffeos tangent to the circle

We can introduce the modes $L_n^{(p)} := T_{D_p}(\exp(i\theta n)\partial_\theta)$, explicitly

$$L_n = \frac{1}{2\pi} \oint e^{i\theta n} T_{\theta\theta} \, \mathrm{d}\theta \quad \text{, where} \quad T_{\theta\theta} = \frac{\pi e^i_\theta e_{\theta i}}{\kappa\gamma}$$

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The SET modes can be obtained from the Kac-Moody modes Q^a_n through the Sugawara construction:

At the quantum level $L_n = \frac{1}{2}\sum_a\sum_{m\in\mathbb{Z}} : Q^a_m Q^{\bar{a}}_{n-m} :$, where $: Q^a_n Q^b_m : = \begin{cases} Q^b_m Q^a_n & \text{if } n+k^a>0 \\ Q^a_n Q^b_m & \text{if } n+k^a \leq 0 \end{cases}$

$$\Rightarrow \quad [L_n, L_m] = (n-m)L_{n+m} + \frac{c}{12}n(n^2-1)\delta_{n+m,0}$$

On the completely algebraic level, we obtain a Virasoro algebra with $c=3$

$$[L_n, Q^a_m] = -(m+k^a)Q^a_{n+m}$$

the currents are primary fields of weight 1 twisted by k

Loop gravity fluxes and Intertwiner

Can we recover the SU(2) local symmetry algebra generated by Σ^i for transformations that affects only the boundary modes while leaving the background fields invariant?

We want to consider SU(2) rotation labelled by α that leaves the connection fixed ($d_A \alpha = 0$).
In the case when the curvature takes integer values, there is a set of rotations: $\alpha^a = a^a e^{-ik^a \theta}$

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$$M^a = \oint_D \Sigma^a = \boxed{-\epsilon^a{}_{bc} \sum_{n\neq 0} : \frac{\tilde{Q}^b_n \tilde{Q}^c_{-n}}{2n} :}\,, \quad \tilde{Q}^a_n = Q^a_{n-k^a}$$ and $$[M^3, M^\pm] = \pm i M^\pm\,, \quad [M^+, M^-] = i M^3$$

Infinite dimensional analog of the Schwinger representation: a new representation of the su(2) Lie algebra generators in terms of the U(1)^3 twisted Kac-Moody ones

Loop gravity fluxes and Intertwiner

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Infinite dimensional analog of the Schwinger representation: a new representation of the su(2) Lie algebra generators in terms of the U(1)^3 twisted Kac-Moody ones

$$[L_n, M^a] = 0 \quad \Rightarrow$$ Each puncture carries a representation of $Vir \times \mathrm{SU}(2)$

New boundary symmetry group whose associated charges represent new boundary observables

- In general $\sum_p L_n^p = -L_n^{\overline{D}}\,, \qquad \sum_p M^p = -M^{\overline{D}}$ $\rightarrow$ violation of the closure constraint

The flux doesn't vanish outside the punctures. It is not the original Ashtekar-Lewandowski vacuum. On the other hand,

$$F^i(A)(x) = 2\pi \sum_p K_p^i \delta^{(2)}(x, x_p) \quad \longrightarrow \quad \text{Curvature vanishes outside the punctures}$$

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However, this vacuum is now a normalizable Fock vacuum carrying a representation of the Virasoro algebra: Key role played by the presence of central charges

Under coarse graining, curvature is generated and this implies that the closure constraint is violated [Livine '14]

New vacuum that allows for curvature in the bulk: `Virasoro' intertwiner

Other LQG vacua which also admit a Fock representation are those used to construct GFT condensates [Gielen, Oriti, Sindoni '13]; [Oriti, DP, Ryan, Sindoni '15]; [see Gielen's and Oriti's talks]

By studying the solution space outside the punctures, the frame fields can be expressed in terms of three scalar fields factorized into the sum of holomorphic and anti-holomorphic modes, which allow us to re-express the frame fields in terms of conserved currents. From the symplectic form written in terms of these currents we recover the $U(1)^3$ Kac-Moody algebra

The nature of the symmetry generators $(L_n^{\overline{D}}, Q_n^{\overline{D}a}, M^{\overline{D}})$ associated with the complementary region can be described in terms of a 3D auxiliary string : The loop gravity string [Freidel, Perez, DP '16]

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➥ In our construction:

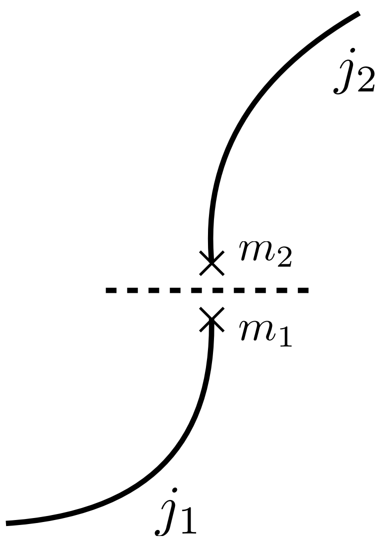
The 2-dimensional metric on the boundary g_{AB} = The string energy-momentum tensor T_{AB}

(However, for a full reconstruction of the energy-momentum tensor in terms of currents, we need to study more in the detail the e_r^i component)

The loop gravity flux Σ_D^a = The string angular momentum along ∂D

Horizon entropy

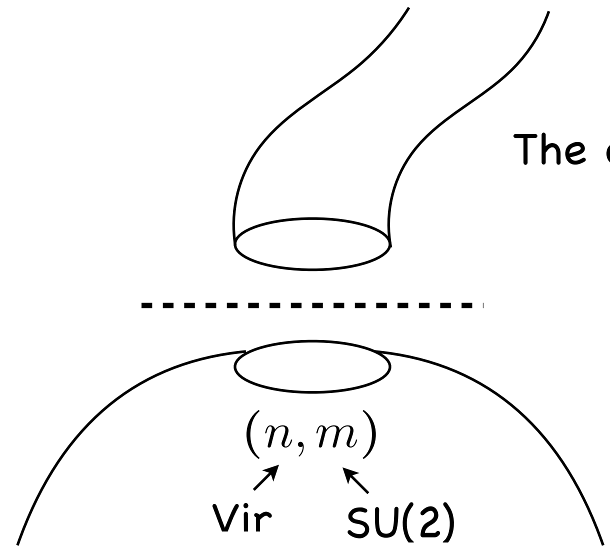
SU(2) boundary symmetry



$$\rightarrow \quad S_E = \log(d_j)$$

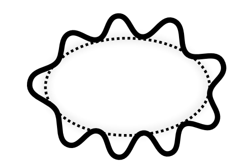
[Donnelly '08]

Vir x SU(2) boundary symmetry



The edge modes encode entanglement

$$\rightarrow \quad S_E = S(\Delta_0(j))$$



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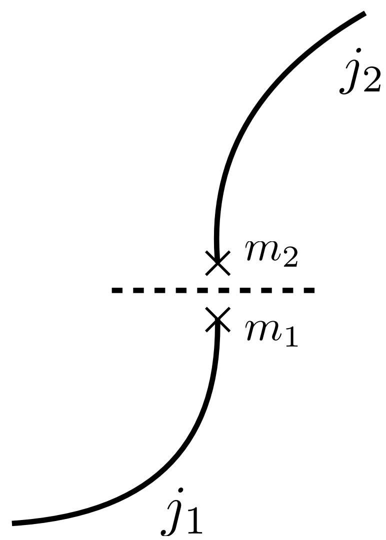

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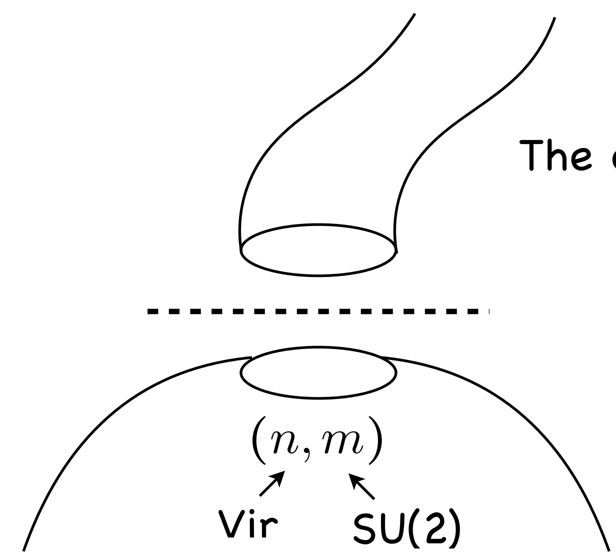

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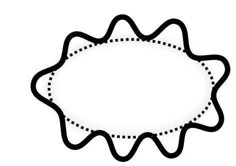
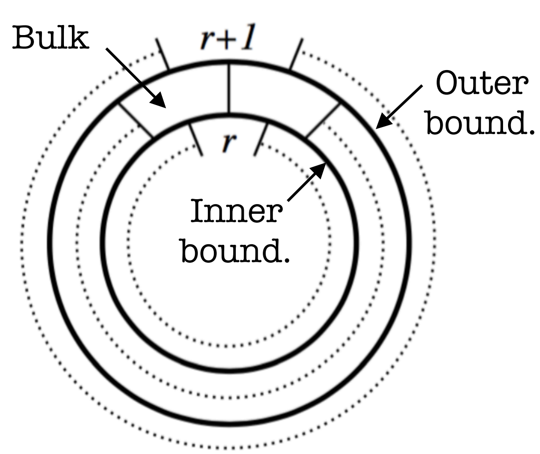
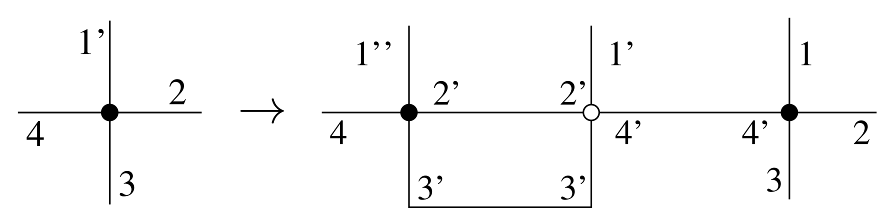


- New boundary DOF in order to account for BH entropy have also been introduced in the GFT condensate approach to the continuum limit [Oriti, DP, Sindoni '16]; [Oriti's talk]



Start with a seed state and then act with refinement operators in order to generate a sum over triangulations



(Sum over graphs) x (single vertex Hilbert space degeneracy) $\Rightarrow$ Bekenstein-Hawking entropy formula

Summary

Introduction of an action boundary term implementing the simplicity constraint as a boundary EOM leads to the existence of soft modes / edge states of gravity at the boundary (attached to the punctures of a locally flat geometry), yielding an extension of the LQG kinematical Hilbert space.

- These new surface charges represent new physical DOF necessary to restore gauge symmetry in presence of a boundary;

- They carry a representation of a twisted $U(1)^3$ Kac-Moody symmetry, encoding a Virasoro algebra and an SU(2) symmetry;

- They can be used to represent at the quantum level, and in terms of a Fock vacuum, not only the flux operator but the triad itself (string-like excitations living in a 3D internal target space);

- They encode the entanglement between subsystems which could account for BH entropy.

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  [Feidel, Krasnov, Livine '10]; [Faulkner, Guica, Hartman, Myers, Mark Van Raamsdonk '13]; [Dittrich, Mizera, Steinhaus '14]; [Strominger '17]

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- We have shown how gravitational observables can be used to define a CFT: Possible application of AdS/CFT ideas/techniques (Ryu-Takayanagi formula in LQG/GFT context)?

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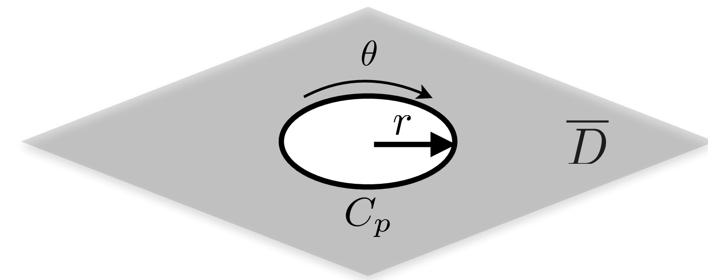
- We want to blow things up: Can new CFT observables be defined in quantum gravity and used to describe matter (emerging from broken diffeos)? Possible connection between our vacuum and different GFT phases? [see Carrozza's talk]

VIRASORO SPIN NETWORK

The string target space

Away from the punctures, on $\overline{D}$: $F(A) = 0 \quad \rightarrow \quad A = g^{-1}\mathrm{d}g$



$$g(z_p + re^{2i\pi}) = e^{2\pi K_p} g(z_p + r)$$

the group element around the circle is quasi-periodic

$\longrightarrow$

$$e^i = (g^{-1}\hat{e}^i g)\,, \quad \varphi^i = (g^{-1}\hat{\varphi}^i g)$$

In the hatted frame the connection vanishes and

$$\hat{e}(z_p + re^{2i\pi}) = e^{2\pi K_p}\hat{e}(z_p + r)$$

We assume that this gauge is chosen and we can therefore neglect the connection A in the following.

The Hamiltonian action of the Kac-Moody charges on e^i generates the translational gauge transformation

$$\hat{e}^i \rightarrow \hat{e}^i + \mathrm{d}\hat{\varphi}^i$$

If we concentrate on those transformations with $\hat{\varphi}^i = 0$ on the boundaries (i.e. the tangent bulk diffeomorphism that are trivial at punctures but otherwise move them around), then we can define a natural gauge fixing by choosing a background metric η_{ab} and imposing the condition

$G^i \equiv \eta^{ab}\partial_a \hat{e}^i_b = 0$ This is a good gauge fixing : solutions to $0 = \delta_\varphi G = \{G, S_{\bar{D}}(\hat{\varphi})\} = \Delta\hat{\varphi}^i$

satisfying the boundary condition $\hat{\varphi}^i = 0$ on C_p is the trivial solution $\hat{\varphi}^i = 0$ everywhere on $\overline{D}$

★ Remaining degrees of freedom live only on the boundary C_p , no residual (diffeo) gauge left on $\overline{D}$

General solution of the staticky constraint $\mathrm{d}\hat{e}^i = 0:$ $\quad \hat{e}^i = \sqrt{\frac{\kappa\gamma}{2\pi}}\,\mathrm{d}X^i$

After plugging this solution into the gauge condition $G^i = \mathrm{d} * \hat{e}^i = 0$ we obtain: $\quad \Delta X^i = 0$

By means of the complex structure induced by the introduction of η_{ab}
we can parametrize the solution of the staticity constraint and the gauge fixing for the remaining degrees of freedom in terms of holomorphic and anti-holomorphic modes:

$$X^i = X^i_+(z) + X^i_-(\bar{z})\,, \quad \text{where} \quad \partial_{\bar{z}} X^i_+ = 0\,,\ \partial_z X^i_- = 0$$

➥ The frame fields are proportional to the conserved currents

$$\hat{e}^i_z = \sqrt{\frac{\kappa\gamma}{2\pi}}\, J^i\,, \quad \hat{e}^i_{\bar{z}} = \sqrt{\frac{\kappa\gamma}{2\pi}}\, \bar{J}^i\,, \qquad \text{where} \qquad J^i := \partial_z X^i, \qquad \bar{J}^i := \partial_{\bar{z}} X^i$$

- These two copies are not independent as they are linked together by the reality condition $(\hat{e}^i_z)^* = \hat{e}^i_{\bar{z}}$

In an internal frame where $K_p = k_p \tau^3$ and in the complex basis $\tau^a = (\tau^3, \tau^\pm)$ which diagonalises the adjoint action

the currents satisfy the quasi-periodicity condition $J^a(z_p e^{2i\pi}) = e^{-2i\pi k_p^a} J^a(z_p)\,, \quad \bar{J}^a(\bar{z}_p e^{2i\pi}) = e^{2i\pi k_p^a} \bar{J}^a(\bar{z}_p)$

We can now pull back the symplectic structure to the solutions of $G^i = 0$ and $\mathrm{d}\hat{e}^i = 0$

parametrized by the scalar fields X^i :

$$\Omega_{\bar{D}} = -\sum_p \Omega_p \qquad \text{where} \qquad \Omega_p = \frac{1}{2\kappa\gamma} \int_{D_p} \delta e_a \wedge \delta e^a = \frac{1}{4\pi} \int_{C_p} \delta X_a \, \mathrm{d}\delta X^a$$

We are now ready to rederive the Kac-Moody algebra in terms of the current algebra.

Recall that the holomorphicity of the currents and quasi-periodicity condition imply that the currents admit the expansion

$$z J^a(z) = \sum_{n \in \mathbb{Z}} J_n^a z^{-n-k^a}\,, \quad \bar{z} \bar{J}^a(\bar{z}) = \sum_{n \in \mathbb{Z}} \bar{J}_n^a \bar{z}^{-n+k^a}$$

In order to make sense of such expansions we have to restrict to curvatures that satisfy the condition that

$k^a \in \mathbb{Z}/N\,, N \in \mathbb{N}$: this is a pre-quantization condition on the distributional curvature at puncture

The reality condition gives the identification $(J_n^a)^\dagger = \bar{J}_n^{\bar{a}}$

Mode expansion: the k integer case

$$X^a(z,\bar z) = x^a(z,\bar z) - \sum_{n+k^a\neq 0} \frac{J^a_n z^{-n-k^a}}{(n+k^a)} - \sum_{n-k^a\neq 0} \frac{\bar J^a_n \bar z^{-n+k^a}}{(n-k^a)}$$

zero mode position and momentum

where $\quad x^a(z,\bar z) = \tilde x^a + \theta P^a\,, \quad \tilde x^a := x^a + (J^a_{-k^a} + \bar J^a_{k^a})\ln r\,, \quad P^a := i(J^a_{-k^a} - \bar J^a_{k^a})$

Let us also introduce the mode expansion

$$e^a_\theta = \sqrt{\frac{\kappa\gamma}{2\pi}}\left(\sum_n e^{-i\theta(n+k^a)} Q^a_n\right)\,, \qquad Q^a_n = i(r^{-n-k^a} J^a_n - r^{n+k^a}\bar J^a_{-n})$$

Direct replacement of the expansion in the symplectic form shows that

$$\Omega = \sum_a \left(\frac{1}{2}\delta\chi^a \wedge \delta P^{\bar a} + i \sum_{n+k^a\neq 0} \frac{\delta Q^a_n \wedge \delta Q^{\bar a}_{-n}}{2(n+k^a)}\right)$$

$$\Rightarrow \qquad \{Q^a_n, Q^b_m\} = -i\delta^{a\bar b}(n+k^a)\delta_{n+m}, \qquad \{\chi^a, P^b\} = 2\delta^{a\bar b}$$

U(1)^3 Kac-Moody algebra plus a zero mode algebra

CFT Hilbert space

An arbitrary $\underbrace{\text{holomorphic primary field } \phi(z)}_{z \to f(z)\,,\quad \phi(z) \to \left(\frac{\partial f}{\partial z}\right)^h \phi(f(z))}$ of weight h has a mode expansion $\phi(z) = \sum_{N \in \mathbb{Z}} \phi_N z^{-N-h}$

where the modes are give by $\phi_N = \oint \frac{dz}{2\pi i} z^{N+h-1} \phi(z)$

Let us consider the state $|h\rangle = \phi(0)|0\rangle$ created by an holomorphic field $\phi(z)\,, z \to 0\,,$ of weight h

From the operator product expansion between the stress-energy tensor $T(z)$ and a primary field $\phi(z)$

$$[L_N, \phi(0)] = 0\,,\ N > 0 \quad \Rightarrow \quad L_0|h\rangle = h|h\rangle\,, \quad L_N|h\rangle = 0\,, \quad \text{for all } N > 0$$

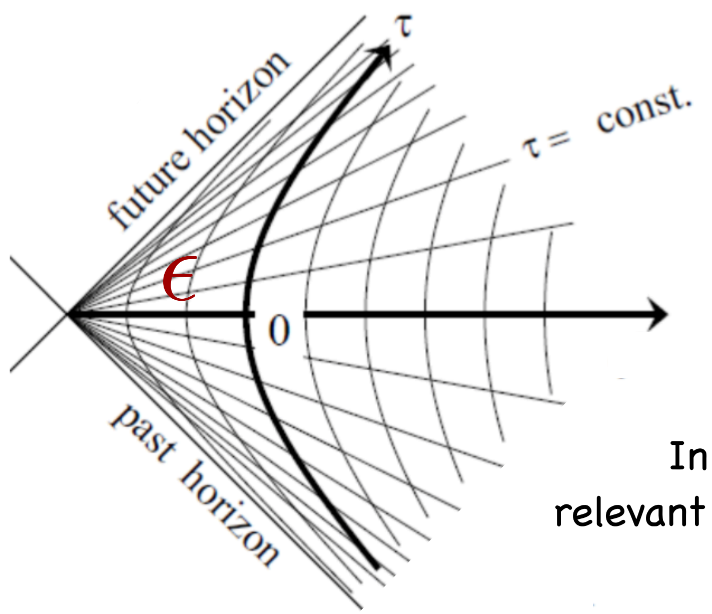
$|h\rangle$ is called a **highest weight state**. All other states in the highest weight representation can be constructed by repeated application of $L_{-N}\,, N > 0$ on $|h\rangle$ by virtue of the Virasoro algebra $[L_0, L_{-N}] = N L_{-N}$

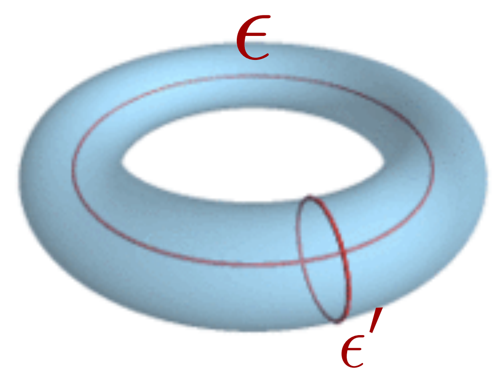
Each application L_{-N} increases the conformal dimension of the state by N, i.e. $L_0(L_{-N}|h\rangle) = (h+N)(L_{-N}|h\rangle)$

The excited states obtained in this way are called **descendants**

Each irreducible unitary highest weight representation of the Virasoro algebra is characterised by the pair (c, h)

Implications for BH entropy

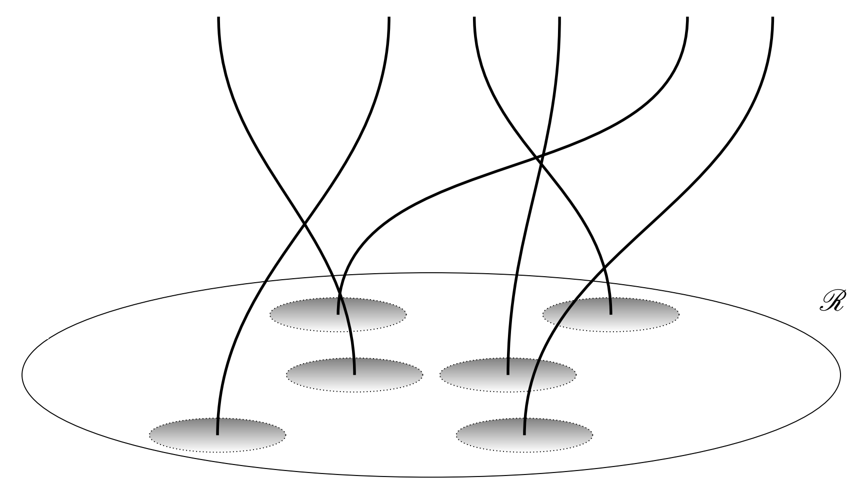


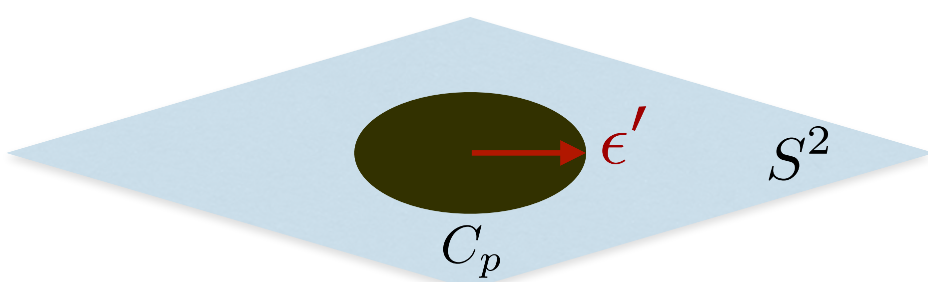


In the Euclidean,
relevant for thermodynamics

Torus modular parameter: $\tau = i\dfrac{\epsilon}{\epsilon'}$

Coarse grained region





local notion of energy for a stationary (thermal equilibrium) observer [Frodden, Ghosh, Perez '13]

CFT Hamiltonian

$$H^{CFT}_{\mathscr{R}} \equiv \sum_{p\in\mathscr{R}} H^{CFT}_p = \frac{L_0^{\mathscr{R}} - \frac{c_{\mathscr{R}}}{24}}{\epsilon'} = \frac{a_{\mathscr{R}}}{8\pi\epsilon G_N}$$

The eigenstates of $H^{CFT}_{\mathscr{R}}$ are labelled by strings of integers $|n_1, n_2, \cdots, n_{p(\mathscr{R})}\rangle$ associated with the level of descendants at each of the $p(\mathscr{R})$ punctures in region $\mathscr{R}$

$\longrightarrow$ Representation of a Virasoro algebra with central charge $c_{\mathscr{R}} = 3p(\mathscr{R})$

conformal dimension

$L_0^{\mathscr{R}}$ eigenvalue: $\Delta = \sum_{p\in\mathscr{R}} (h_p + n_p)$ $\quad\longrightarrow\quad$ $\Delta - \frac{c_{\mathscr{R}}}{24} = \frac{a_{\mathscr{R}}}{8\pi|\tau|G_N}$

descendant level

Asymptotic density of states
$(\Delta >> 1)$
$\longrightarrow$ Cardy formula: $\rho(\Delta) \approx \exp 2\pi\sqrt{\frac{c_{\mathscr{R}}(\Delta - \frac{c_{\mathscr{R}}}{24})}{6}}$

➢ The number of states compatible with a macroscopic area $a_{\mathscr{R}}$ will be dominated by configurations with the maximum amount of punctures (all spin-1/2 configuration dominates the counting) $\longrightarrow$ $c_{\mathscr{R}} = \frac{3}{4\pi\gamma G_N}$

➢ Macroscopic BH are possibile, i.e. the convergence of the partition function is assured, for maximal possible (Hagedorn) temperature $\longrightarrow$ $|\tau| = |\tau_h| = \frac{1}{\gamma}$

Possible relation with
the simplicity constraint
(see [Bianchi '14])

$$\Longrightarrow \quad \boxed{\rho(\Delta) \approx \exp \frac{a_{\mathscr{R}}}{4G_N}}$$