Relating LQC with LQG
– Algebraic Aspects –

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- Single out representation of $\mathfrak{A}$ by physical conditions.

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Diffeomorphism invariant state on $\mathfrak{A}, \mathfrak{W}$ LOST, AC, EHT

Unique Diff-invariant state: GNS-rep. $\equiv$ Std. rep. phys. intuition

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	LQG	AWE	ABL	Fleischhack
Unqs	LOST	AC	EHT	EHT
QA	$\mathfrak{A}$	$\mathfrak{W}$	$\mathfrak{A}$	$\mathfrak{A}$
$\mathcal{A}_{\mathrm{red}}$	$\mathcal{A}$	c_1, c_2, c_3	c	c
$\mathcal{P}$	analytic	axes	linear	analytic
$\mathfrak{D}$	Cyl	$C_{\mathrm{AP}}(\mathbb{R}^3)$	$C_{\mathrm{AP}}(\mathbb{R})$	$C_{\mathrm{AP}}(\mathbb{R}) \oplus C_0(\mathbb{R})$
P	Γ	p_1, p_2, p_3	p	p
Diff	$\mathrm{Aut}(P)$	Dil^3_V	Dil^1	Dil^1
		same representation	same representation	

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Ashtekar Variables

$e^I = e^I_\alpha \, \mathrm{d}x^\alpha$ – co-tetrad

$\omega = \omega^{IJ}_\alpha \, \mathrm{d}x^\alpha - \mathrm{SO}(1,3)$

The Holst Action:

$$L = \frac{1}{32\pi G} \int_{(\mathcal{M},[e])} \left(\epsilon_{IJKL} \, e^I \wedge e^J \wedge \Omega^{KL} - \frac{2}{\gamma} \, e^I \wedge e^J \wedge \Omega_{IJ} \right)$$

Spin connection Γ^i_a, and $K^i_a = \omega^{i0}_a$

Legendre transform:

$$A^i_a = \Gamma^i_a + \gamma \, K^i_a \quad \text{(Ashtekar connection)}$$

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Cosmology: $\mathcal{M} \cong \mathbb{R} \times \mathbb{R}^3$ with $\mathbb{R}^3 \rtimes \mathrm{SO}(3) \curvearrowright \{t\} \times \mathbb{R}^3$

- Set $\mathcal{S}$ of invariant (ω, e) parametrized by $((c, w), v)$.
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$$\widetilde{L} = \frac{3V_0[\mathcal{C}]|v|v}{8\pi G\gamma} \frac{\mathrm{d}}{\mathrm{d}x^0}(c-w) - \frac{3NV_0[\mathcal{C}]|v|}{8\pi G\gamma^2}(c^2 - \gamma^2 w^2 - 2cw)$$

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$$\widetilde{L} = p\,\dot{c} - N\sqrt{\frac{3V_0[\mathcal{C}]}{k\gamma^3}}\,|p|^{1/2}c^2 \qquad \text{for} \qquad p = \frac{3V_0[\mathcal{C}]|v|v}{8\pi G\gamma}$$

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The Classical Holonomy-Flux Algebra

Phase space formed by pairs (A, E) with functions:

Cyl	$\Psi\colon A \mapsto h_\gamma^{i,j}(A)$	$\gamma\colon [0,1] \to \mathcal{M}$	embedded analytic
Flux	$X_{S,f}\colon E \mapsto \int_S \widetilde{E}(f)$	$f\colon S \mapsto \mathfrak{su}$	SmVf on Surface S

$$\widetilde{E} = \tfrac{1}{2!}\,\epsilon_{abc}\,E_i^a\,\mathrm{d}x^b \wedge \mathrm{d}x^c \otimes \tau^i \qquad \text{for} \qquad E = E_i^a\,\partial_a \otimes \tau^i$$

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$\mathfrak{A}_{\mathbf{class}}\textbf{:} \quad \mathbf{Cyl} \times \boldsymbol{\Gamma}$

- Γ: $\mathbb{C}$-span of $\quad$ ($X_{S,f}\colon \mathrm{Cyl} \to \mathrm{Cyl}$ derivation)

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- $\mathfrak{D}$: Suitable dense subalgebra of $C_{\mathrm{AP}}(\mathbb{R}) \oplus C_0(\mathbb{R})$

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$\mathfrak{D} \in \{\mathfrak{d}_{\mathrm{AP}}, \mathfrak{d}_{\mathrm{AP}} \oplus \mathfrak{d}_0\}$ for

$$\mathfrak{d}_{\mathrm{AP}} = \{\psi \in C_{\mathrm{AP}}(\mathbb{R}) \cap C^\infty(\mathbb{R}) \mid \psi^{(n)} \in C_{\mathrm{AP}}(\mathbb{R}) \;\; \forall n \in \mathbb{N}\} \subseteq C_{\mathrm{AP}}(\mathbb{R})$$

$$\mathfrak{d}_0 = \{\psi \in C_0(\mathbb{R}) \cap C^\infty(\mathbb{R}) \mid \psi^{(n)} \in C_0(\mathbb{R}) \;\; \forall n \in \mathbb{N}\} \subseteq C_0(\mathbb{R})$$

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Diffeomorphisms

Full theory: $\mathrm{SU}(2)$-bundle P over Σ

- $\mathrm{Aut}(P) \ni \Phi \rightharpoondown \mathfrak{A}_{\mathrm{class}} = \mathrm{Cyl} \times \Gamma$ acts $\{\cdot,\cdot\}$ - preserving via

$$\Psi \mapsto \Psi \circ \Phi^* \qquad X_{S,f} \mapsto X_{\phi(S), f \circ \phi^{-1}}$$

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- $V_0[\mathcal{C}] \mapsto \lambda^3 V_0[\mathcal{C}]$
- $\mathcal{S} \ni (\omega, e) \mapsto ([\Phi_\lambda]_* \omega, [\Phi_\lambda]_* e) \in \mathcal{S}$ $((c, w), v)$ $p = \frac{3V_0[\mathcal{C}]|v|v}{8\pi G \gamma}$

$$(c, v) \mapsto (\lambda^{-1} c, \lambda^{-1} v) \qquad \Longrightarrow \qquad (c, p) \mapsto (\lambda^{-1} c, \lambda p)$$

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- $\mathfrak{A}_{\mathrm{class}} \ni (\psi, zp) \mapsto \left(\psi \circ \Phi_\lambda^*\colon c \mapsto \psi(\lambda^{-1}\cdot c), \lambda \cdot zp\right)$

  Preserve $\{\cdot,\cdot\}$; thus, carry over to the quantum algebra.

Uniqueness of the Invariant State

Quantum holonomy-flux *-algebra $\mathfrak{A} = \mathrm{T}^\bullet(\mathrm{Cyl} \oplus \Gamma)/\mathfrak{J}$ with $\mathfrak{J}$ gen. by

$$\left.\begin{array}{c} a \otimes b - b \otimes a - \mathrm{i} \cdot \{a, b\} \\ \Psi \otimes \Psi' - \Psi\Psi' \end{array}\right\} \quad \mathfrak{A} \text{ spanned by } \widehat{\Psi} \text{ and } \widehat{O} \cdot \widehat{X}_{S,f}$$

$$\Psi_0 - 1$$

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State: $\omega : \mathfrak{A} \to \mathbb{C}$ linear with $\omega(1) = 1$ and $\omega(a^* a) \geqslant 0$ for $a \in \mathfrak{A}$

GNS-construction: $|\omega(a^* b)|^2 \leqslant \omega(a^* a) \cdot \omega(b^* b)$

- $\mathcal{H} := \mathfrak{A}/\mathcal{I}$ with $\langle [a], [b] \rangle := \omega(a^* b)$ and $\mathcal{I} = \{a \in \mathfrak{A} \mid \omega(a^* a) = 0\}$
- Representation: $\varrho : \mathfrak{A} \to \mathfrak{B}(\mathcal{H})$, $\varrho(a) : [b] \mapsto [ab]$

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$\mathfrak{A}$ spanned by $\widehat{\Psi}$ and $\widehat{O} \cdot \widehat{X}_{S,f}$

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Diffeomorphism Invariance: $\omega(\Phi(a)) = \omega(a)$ for each Φ, $a \in \mathfrak{A}$

- $\widehat{X}_{S,f} \in \mathcal{I}$ $\Longrightarrow$ ω uniquely determined by $L \colon \Psi \mapsto \omega(\widehat{\Psi})$

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$$\left.\begin{array}{c} a \otimes b - b \otimes a - \mathrm{i} \cdot \{a, b\} \\ \Psi \otimes \Psi' - \Psi\Psi' \end{array}\right\} \quad \mathfrak{A} \text{ spanned by } \widehat{\Psi} \text{ and } \widehat{O} \cdot \widehat{X}_{S,f}$$

$$\Psi_0 - 1 \qquad\qquad \omega(\widehat{O} \cdot \widehat{X}_{S,f}) = 0$$

State: $\omega : \mathfrak{A} \to \mathbb{C}$ linear with $\omega(1) = 1$ and $\omega(a^* a) \geqslant 0$ for $a \in \mathfrak{A}$

GNS-construction: $\qquad |\omega(a^* b)|^2 \leqslant \omega(a^* a) \cdot \omega(b^* b)$

- $\mathcal{H} := \mathfrak{A}/\mathcal{I}$ with $\langle [a], [b] \rangle := \omega(a^* b)$ and $\mathcal{I} = \{a \in \mathfrak{A} \mid \omega(a^* a) = 0\}$
- Representation: $\varrho : \mathfrak{A} \to \mathfrak{B}(\mathcal{H})$, $\varrho(a) : [b] \mapsto [ab]$

Diffeomorphism Invariance: $\qquad \omega(\Phi(a)) = \omega(a)$ for each Φ, $a \in \mathfrak{A}$

- $\widehat{X}_{S,f} \in \mathcal{I} \quad \Longrightarrow \quad \omega$ uniquely determined by $L : \Psi \mapsto \omega(\widehat{\Psi})$
- L continuous, Φ-invariant $\quad \Longrightarrow \quad L(\Psi) = \int_{\overline{\mathcal{A}}} \mathcal{G}(\Psi) \, \mathrm{d}\mu_{\mathrm{AL}} \qquad$ OL+S

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	Standard	Fleischhack	
$\mathfrak{D}$	$\mathfrak{d}_{\mathrm{AP}}$	$\mathfrak{d}_{\mathrm{AP}} \oplus \mathfrak{d}_0$	
$\mathfrak{A}$	$\mathfrak{A}_{\mathrm{S}}$	$\mathfrak{A}_{\mathrm{F}} = \mathfrak{A}_{\mathrm{S}} \oplus \mathcal{J}(\mathfrak{d}_0)$	$\mathcal{J}(\mathfrak{d}_0) \subseteq \mathcal{I}$
ϱ	ϱ_{S}	$\varrho_{\mathrm{F}} = \varrho_{\mathrm{S}} \oplus 0$	$\mathcal{H}_{\mathrm{S}} \cong \mathcal{H}_{\mathrm{F}} \equiv \mathfrak{d}_{\mathrm{AP}}$

Both $\varrho \colon \mathfrak{A} \to \mathfrak{d}_{\mathrm{AP}} = \mathcal{H}$ with $\langle \psi, \psi' \rangle = \lim_n \frac{1}{2n} \int_{-n}^{n} \overline{\psi}(t) \cdot \psi'(t) \, \mathrm{d}t$

$$\varrho(\widehat{\psi}) \colon \chi \mapsto \pi_{\mathrm{AP}}(\psi) \cdot \chi \qquad \varrho(z\widehat{p}) \colon \chi \mapsto -z\mathrm{i} \cdot \dot{\chi}$$

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The Homogeneous Case

Ashtekar-Campiglia + EHT

Homogeneity + Triad diagonal + Dirac constrained analysis:

Conjugate variables $c^i,\ p_i$ for $i = 1, 2, 3$ with

$$A^i_{\mathbf{a}} = c^i \delta^i_{\mathbf{a}} \qquad E^i_{\mathbf{a}} = \frac{8\pi G\gamma}{V_0[\mathcal{C}]} p_i \delta^i_{\mathbf{a}}.$$

$\mathfrak{A}_{\text{class}} = \mathfrak{D} \times \mathbb{C} \cdot p_1 \times \mathbb{C} \cdot p_2 \times \mathbb{C} \cdot p_3 \qquad \mathfrak{D}$ = generated by $x \mapsto \mathrm{e}^{\mathrm{i}\mu \cdot x^j}$

$$\{(\psi, zp_i), (\psi', z' p_j)\} = -(z\partial_i \psi' - z' \partial_j \psi, 0)$$

Preserved under: $\Phi_{\vec{\lambda}} \colon (x^1, x^2, x^3) \mapsto (\lambda^1 x^1, \lambda^2 x^2, \lambda^3 x^3)$ (act on Cell)

Quantum Algebra:	$\mathfrak{W}_{\text{AC}}$	$\mathfrak{A}_{\text{EHT}}$
$(\psi \in \mathfrak{D})$	$\widehat{\psi}, \exp\left(\mathrm{i} \cdot \mu^i \widehat{p}_i\right)$	$\widehat{\psi}, \widehat{p}_i$

$$\varrho_{\text{AC}}|_{\widehat{\mathfrak{D}}} \quad = \quad \varrho_{\text{EHT}}|_{\widehat{\mathfrak{D}}} \colon \widehat{\psi} \mapsto [\chi \mapsto \psi \cdot \chi]$$

$$\varrho_{\text{AC}}(\exp\left(\mathrm{i} \cdot \mu^j \widehat{p}_j\right))(\psi) = \exp\left(\mathrm{i} \cdot \mu^j \partial_j\right)\psi \qquad \varrho_{\text{EHT}}(\widehat{p}_j)(\psi) = -\mathrm{i} \cdot \partial_j \psi$$

THE END