Hybrid LQC: choice of vacuum state for cosmlogical perturbations in LQC

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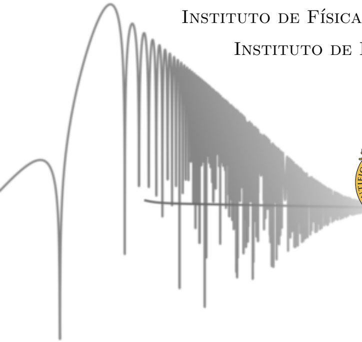

Pontificia Universidad Católica de Chile

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CSIC Consejo Superior de Investigaciones Científicas

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Structure of the talk

1. Cosmological perturbations in LQC and effective equations
2. Vacuum state for cosmological perturbations
   - Adiabatic vacua
   - Non-oscillatory vacuum
   - Other criteria to select a vacuum state
3. What are the corrections coming from LQC?
4. Discussion and conclusions

Cosmological perturbations in LQC

- Most of the approaches to include cosmological perturbations within loop quantum cosmology (LQC) consider a standard quantum field theory (QFT) quantization for the fields describing the perturbations.
- LQC corrections are included in different ways depending on the approach, but ended with fields propagating on an effective, non stationary background.
- It is well known that in general non stationary scenarios there is no criterion to select a unique vacuum for the field quantization.
- Of course, this issue does not only appear within LQC, but also in the standard treatment for inflation within general relativity (GR).
- In GR this problem is (partially) solved focusing on the dynamics during the slow-roll inflationary phase and selecting the Bunch–Davies vacuum.

Hybrid quantization

- We will consider a flat Friedmann–Robertson–Walker spacetime with a minimally coupled scalar field with a quadratic potential $V(\phi) = \frac{1}{2}m^2\phi^2$ and study the equations of motion coming from its *hybrid quantization*:
- Hybrid quantization approach:
  - Combines the polymeric quantization for the homogeneous degrees of freedom with a Fock quantization for the perturbations.
  - Specific "splitting" of degrees of freedom is given by an uniqueness result for the Fock quantization of the fields. Scaling of perturbations.
  - The approach focuses in preserve the symplectic structure of the whole system, following canonical methods for its quantization.
  - This calls for consider perturbations up to quadratic order in the total Hamiltonian.
  - A Schrödinger equation for the perturbations is obtained considering a Bohr–Oppenheimer ansatz, regarding ϕ as a time, and imposing some conditions expected to hold for semiclassical states.

Effective equations of motion

- To study the physical predictions the effective equations of motion are used, neglecting the backreaction and regularized inverse volume corrections.
- Background equations of motion are the same obtained in other approaches:

$$H^2 = \frac{8\pi G}{3}\rho\left(1 - \frac{\rho}{\rho_{\max}}\right); \qquad \rho_{\max} = \frac{3}{8\pi G\gamma^2\Delta} \approx 0.41\rho_{\mathrm{Pl}};$$

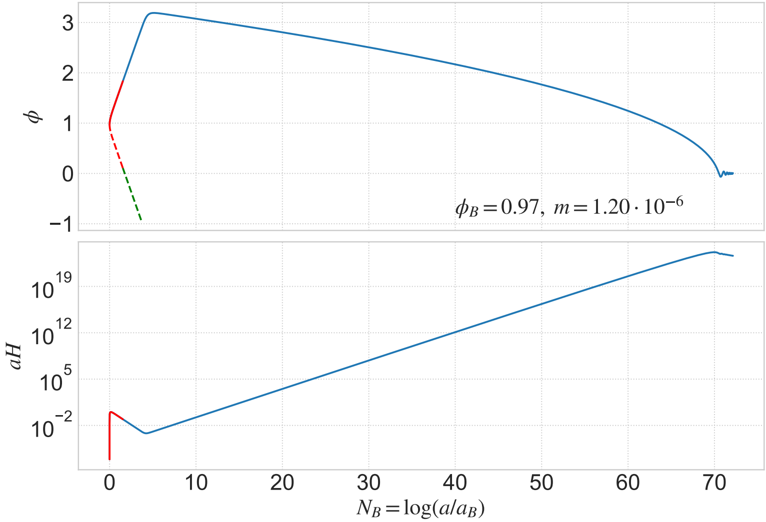
$$\dot{H} = -4\pi G\left(\rho + \mathcal{P}\right)\left(1 - \frac{2\rho}{\rho_{\max}}\right); \qquad \Delta = 4\sqrt{3}\pi\gamma\ell_{\mathrm{Pl}}^2.$$

- Equation for the Mukhanov–Sasaki variable are:

$$v_k'' + (k^2 + s^{(s)}(\eta))v_k = 0$$

  - $s^{(s)}(\eta)$ recovers GR time dependent mass for $\rho \lesssim 10^{-2}\rho_{\max}$.
  - Around the bounce it takes positive values (for kinetic dominated bounce), and therefore it differs from the time dependent mass for other approaches.

Background evolution



Quantization of the cosmological perturbations

- Fields describing cosmological perturbations are quantized using standard QFT techniques:
  - Scalar perturbations: $\mathcal{R} = \frac{\mathcal{V}}{z}$; $\mathcal{V} \equiv$ Mukhanov–Sasaki variable; $z = \frac{a\dot{\phi}}{H}$
  - Tensor perturbations: $h^{+,\times} = \frac{\mathcal{U}}{a}$;
- Fields to be quantized using QFT, $\mathcal{V}$ and $\mathcal{U}$, are selected imposing the background symmetries plus unitary evolution.
- A choice of vacuum is tantamount to the choice a complete set of complex "positive frequency" solutions to the equations of motion:

$$\left\{v_k \mid v_k v_k'^* - v_k' v_k^* = i\right\};$$

then,

$$\hat{\mathcal{V}} = \frac{1}{(2\pi)^{3/2}} \int \mathrm{d}\vec{k} \left(\hat{a}_k\, v_k(\eta)\, e^{i\vec{k}\cdot\vec{x}} + \hat{a}_k^\dagger\, v_k^*(\eta)\, e^{-i\vec{k}\cdot\vec{x}}\right); \quad k = |\vec{k}|$$

- Primordial power spectra: $\mathcal{P}_{\mathcal{R}}(k) = \frac{k^3}{2\pi^2} \frac{|v_k|^2}{z^2}\Big|_{\eta_{\mathrm{end}}}$

Choice of vacuum/Initial conditions

- One can select such set of solutions, $\{v_k\}$, by means of initial conditions $\{(v_{k,0}, v'_{k,0})\}$ at a certain chosen time η_0.
- General initial conditions:

$$v_{k,0} = \frac{1}{\sqrt{2D_k}}, \quad v'_{k,0} = \sqrt{\frac{D_k}{2}}\left(C_k - i\right), \quad D_k \in \mathbb{R}^+, \ \ C_k \in \mathbb{R}.$$

Adiabatic states

- A way to obtain initial conditions with suitable ultraviolet behavior is by means of *adiabatic states*, which consider the ansatz:

$$v_k(\eta) = \frac{1}{\sqrt{2W_k}} e^{-i\int^{\eta} W_k(\bar{\eta})\mathrm{d}\bar{\eta}},$$

then: $$W_k^2 = k^2 + s(\eta) - \frac{1}{2}\frac{W_k''}{W_k} + \frac{3}{4}\left(\frac{W_k'}{W_k}\right)^2$$

- An adiabatic state of order $\mathfrak{n}$ is obtained giving an approximated solution $W_k^{(\mathfrak{n})}$ such that $\left(W_k^{(\mathfrak{n})} - W_k\right) \underset{k\to\infty}{\sim} \mathcal{O}\left(k^{-1-\mathfrak{n}}\right)$.

- Associated initial conditions: $D_k = W_k|_{\eta_0}, \quad C_k = -\frac{W_k'}{2W_k^2}\Big|_{\eta_0}$

Considered adiabatic states

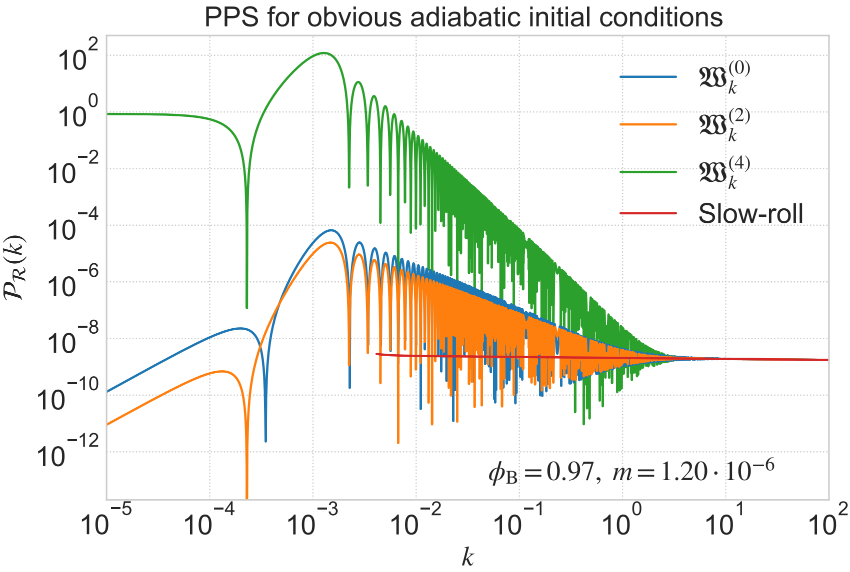
$$W_k^2 = k^2 + s(\eta) - \frac{1}{2}\frac{W_k''}{W_k} + \frac{3}{4}\left(\frac{W_k'}{W_k}\right)^2$$

- For every adiabatic order there is still an infinite ambiguity in selecting a vacuum.
- Here we consider two procedures to select instances of adiabatic vacua for each order, $\mathfrak{n}$:

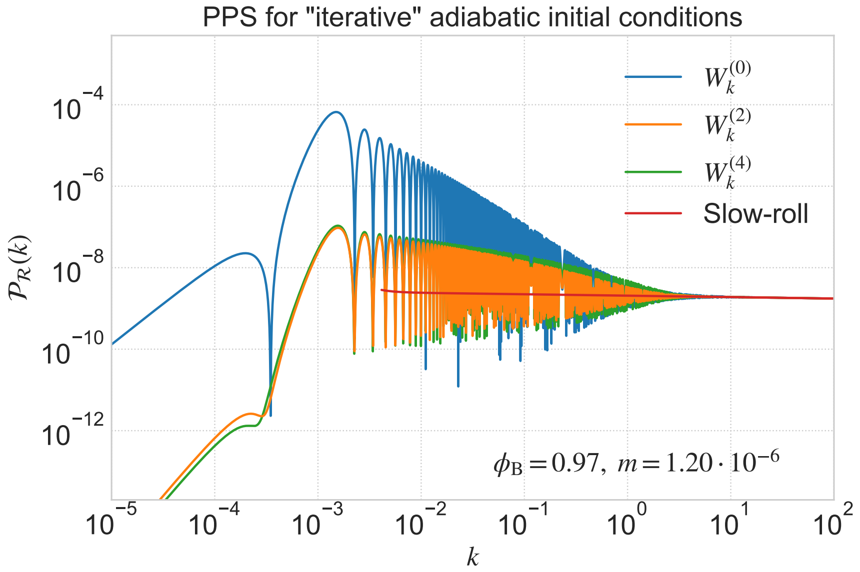
1. $\mathfrak{W}_k^{(\mathfrak{n})} = \sigma_{-1}k + \sigma_0 + \sum_{i=1}^{\mathfrak{n}} \frac{\sigma_i}{k^i},$ (so-called *obvious vacuum* of order $\mathfrak{n}$)
2. Using an iterative process where $W_k^{(\mathfrak{n}+2)}$ is obtained from $W_k^{(\mathfrak{n})}$, starting the process with $W_k^{(0)} = k$.

- Examples: $\mathfrak{W}_k^{(2)} = 1 + \frac{s}{2k};$ $W_k^{(2)} = \sqrt{k^2 + s}.$

Primordial power spectra for adiabatic vacua



Primordial power spectra for adiabatic vacua



Caveats of adiabatic states

Adiabatic states have a series a caveats:

- Only fix ultraviolet (small scale) behavior.
- The vacuum selected by a procedure depends on the initial time selected.
- The considered instances of adiabatic vacua may lead to ill defined initial conditions for some scales.
- The procedures shown do not select the Bunch-Davies vacuum in de Sitter spacetimes at any adiabatic order, unless one selects $\eta_0 = -\infty$.

Non-oscillatory vacuum

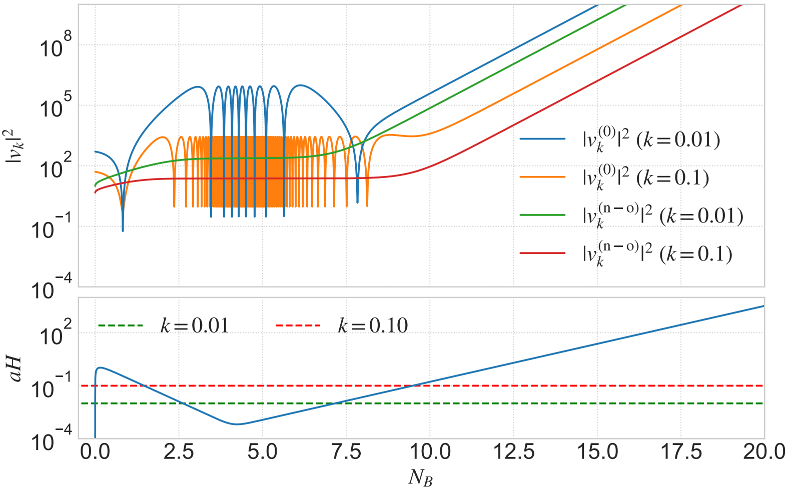
We have proposed a *constructive* criterion to select a suitable vacuum state:

- Mode by mode, it selects the solution that minimizes the time oscillations of the fluctuations in a given temporal interval,

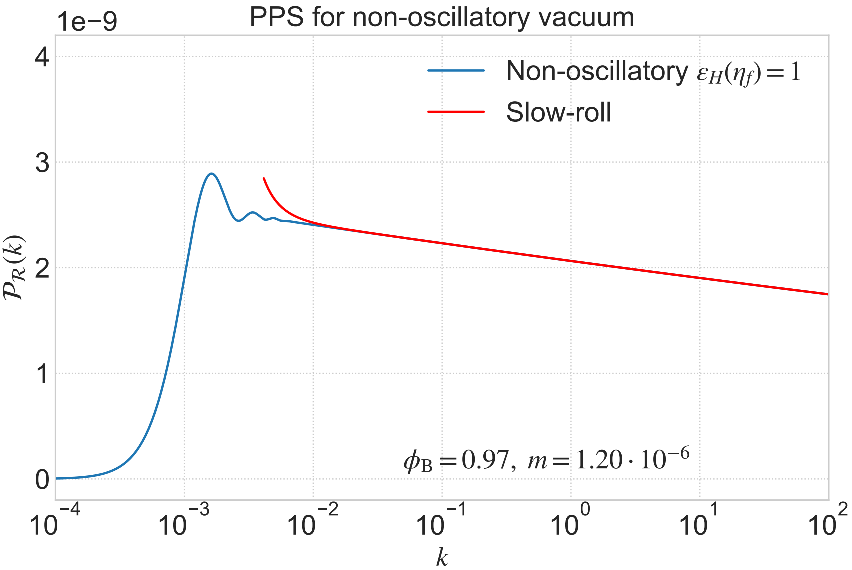
$$\int_{\eta_i}^{\eta_f} \left|\partial_\eta |v_k(\eta)|^2\right| \mathrm{d}\eta$$

- It recovers the privileged vacuum for static spacetimes and the Bunch-Davies vacuum in de Sitter spacetimes when $\eta_i = -\infty$.
- Provides meaningful initial conditions.
- Vacuum selected depends on time interval selected.
- We select the initial time as the time at the bounce and the final time as the beginning of the inflationary phase.

Evolution of the fluctuations



Primordial power spectra for non-oscillatory vacuum



Non-oscillatory vacuum and adiabatic states

- Any positive frequency solution $v_k^{(n)}$ can be obtained from a referential one $v_k^{(r)}$ by means of a Bogoliubov transformation:

$$v_k^{(n)} = \alpha_k v_k^{(r)} + \beta_k v_k^{(r)*}, \qquad |\alpha_k|^2 - |\beta_k|^2 = 1$$

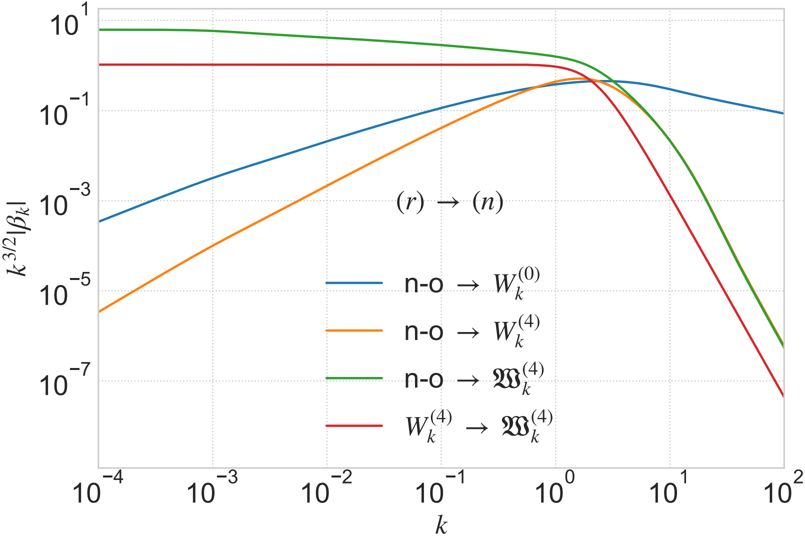
where:

$$\alpha_k = -i\left[(v_{k,0}'^{(r)})^* v_{k,0}^{(n)} - v_{k,0}^{(r)*} v_{k,0}'^{(n)}\right], \qquad \beta_k = i\left[v_{k,0}'^{(r)} v_{k,0}^{(n)} - v_{k,0}^{(r)} v_{k,0}'^{(n)}\right].$$

- When considering two solutions for adiabatic states, the asymptotic behavior of $|\beta_k|$ is determined by the one with smaller adiabatic order:

$$|\beta_k| \underset{k\to\infty}{\sim} \mathcal{O}\left(k^{-2-\mathfrak{m}}\right), \quad \text{with } \mathfrak{m} \text{ the smaller order.}$$

Non-oscillatory vacuum and adiabatic states



Other criteria to select a vacuum state

Instantaneous vacuum by Agullo, Nelson and Ashtekar

- This criterion considers the *adiabatically renormalized* energy-momentum tensor selecting the vacuum state that vanish its expectation value mode by mode at the given time.
- Seems to provide better large scale behavior that the obvious 4th order adiabatic vacuum.
- Infinite freedom in the adiabatic renormalization of the energy momentum tensor.

Other criteria to select a vacuum state

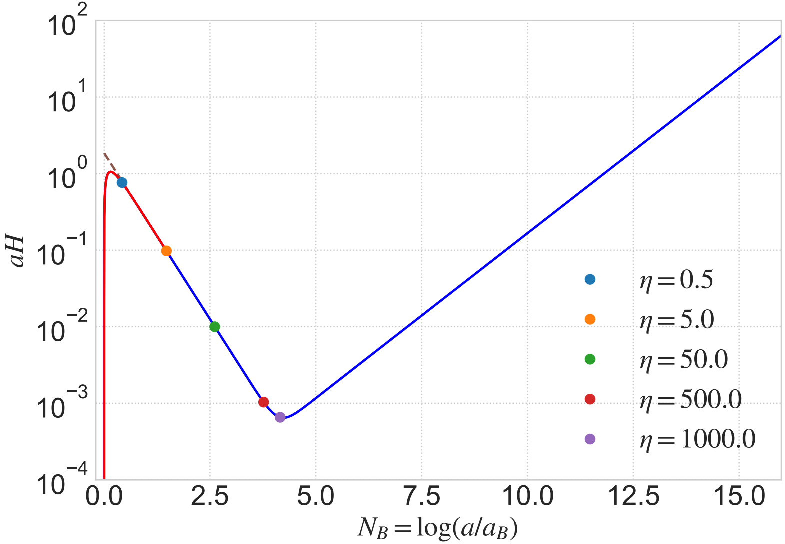
Recent proposal by Ashtekar and Gupt

- Here it is proposed an interplay between the behavior of the fluctuations at the period in which LQC correction are important, $\rho \lesssim 10^{-4}\rho_B$, and at the end of the inflationary phase.
- Mode by mode, the vacuum state selected is the one that minimizes $|\mathcal{R}_k|^2$ at the end of inflation, among the ones contained in a ball of states for which their quantum Weyl curvature is below an specific bound.
- This proposal leads to very appealing primordial power spectra, showing an averaged suppression, that provides a good fitting with observations.
- Solutions selected oscillate during the kinetic dominated phase (with a controlled amplitude) and freeze out when exiting the Hubble horizon in a "destructive interference".

What are truly the corrections coming from LQC?

- For general adiabatic vacuum we saw that we obtain a region of scales with large oscillations and an averaged enhancement of power, when comparing with the slow-roll spectra.
- It is often believed that those large modifications are coming directly from LQC.
- Nonetheless, they rather come from giving initial conditions before the inflationary phase, and such corrections also appear when considering GR.

What are truly the corrections coming from LQC?



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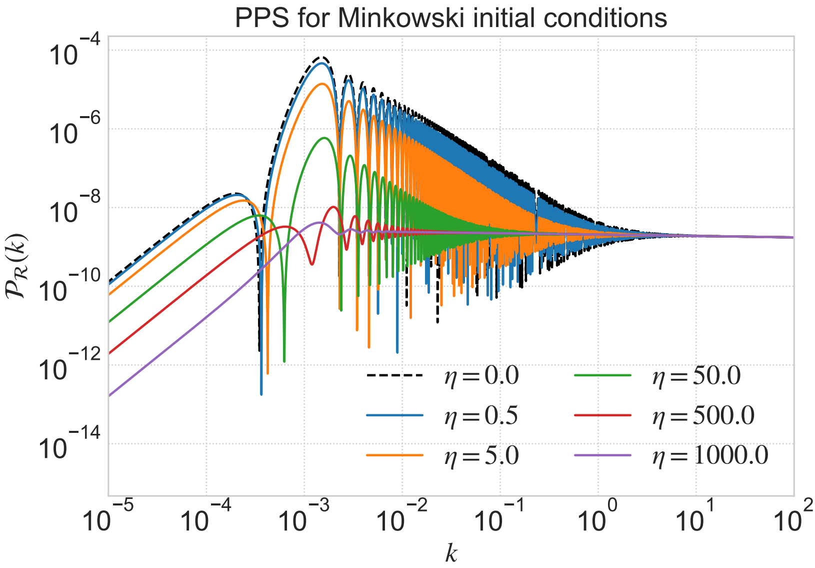

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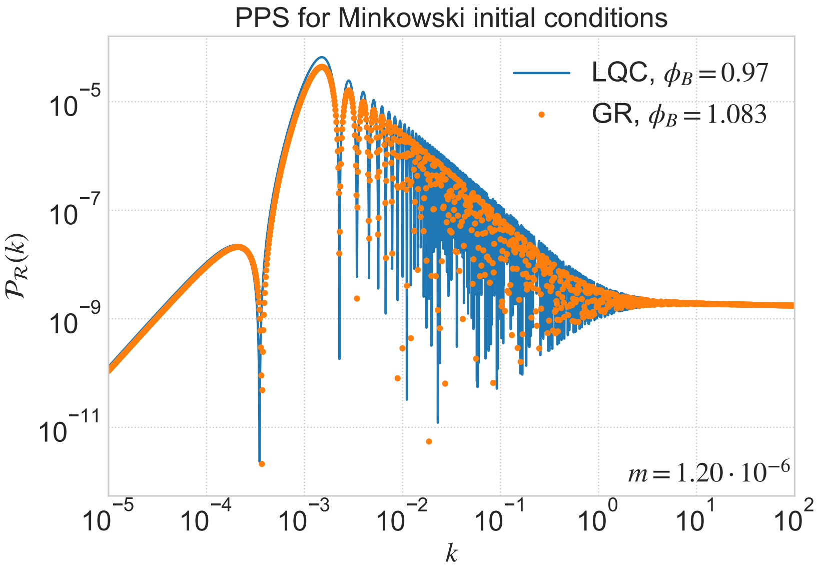


Discussion

- Predictions for the primordial power spectra in LQC are highly dependent in the choice of vacuum for the cosmological perturbations.
- In LQC, as in GR, we have no criteria to select a unique vacuum.
- Adiabatic states are useful to constraint the ultraviolet behavior, but there is still infinite freedom.
- From the two procedures studied to select instances of adiabatic initial conditions, the iterative process seems to be more stable and provide better large scale behavior.
- The non-oscillatory vacuum takes into account the dynamics of the perturbations in a temporal range. It leads to primordial power spectra without large oscillations and a power suppression for large scales.
- Large oscillations in the primordial power spectra are due to impose initial conditions before the inflationary phase, rather than effects coming from LQC. Power suppression for large scales also takes places in GR.

Discussion

Thank you for your attention!

Dependence on LQC parameters

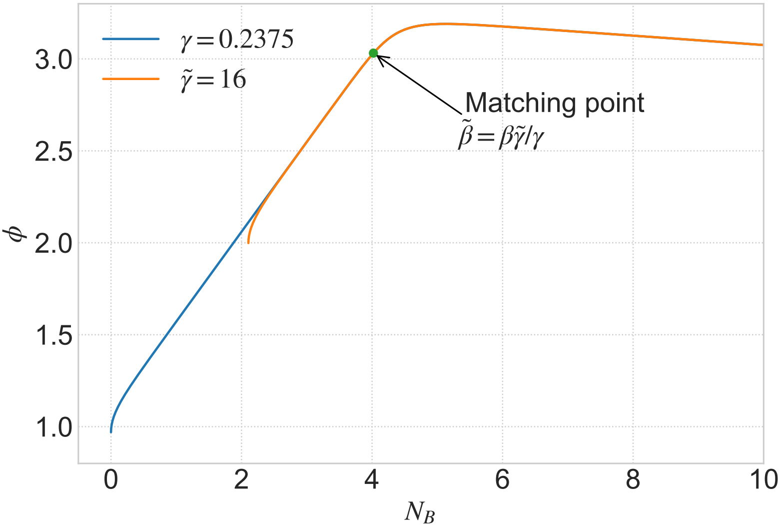
- The energy density at the bounce depend on two extra parameters included in LQC.

$$\rho_{\max} = \frac{3}{8\pi G \Delta \gamma^2}$$

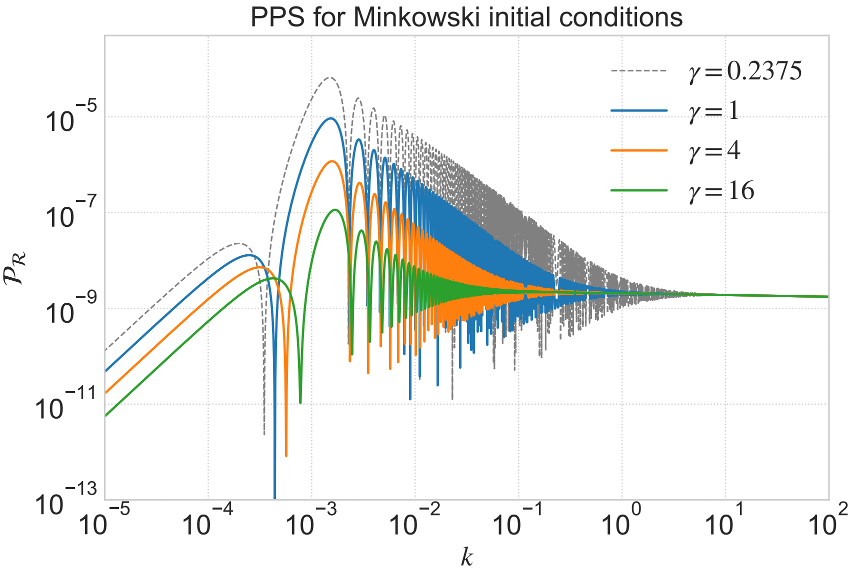
1. Immirzi parameter: usually fixed as $\gamma = 0.2375$ form black hole entropy computations (but there is not consensus on this value).
2. Δ parameter: usually fixed as twice the minimum eigenvalue of the area operator in LQG (but this value is linked heuristically to consider specific graphs and spin representations)

- Moreover, it has been shown that when considering more spread quantum states the energy density at the bounce is lower.
- In order to study the dependence of physical predictions on the energy density at the bounce, we have allowed different values for the Barbero-Immirzi parameter, that lead to the same trajectories far away from the bounce.

Matching of trajectories



Dependence of the power spectra on γ



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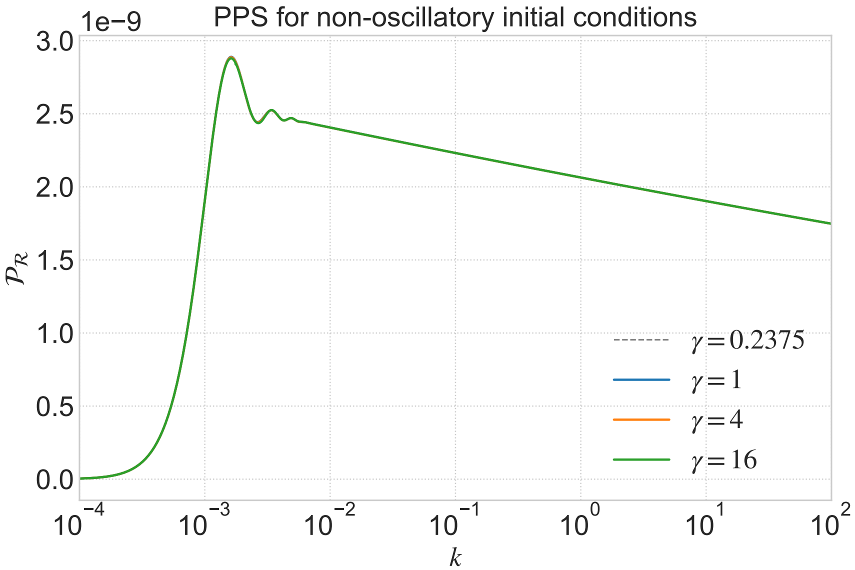

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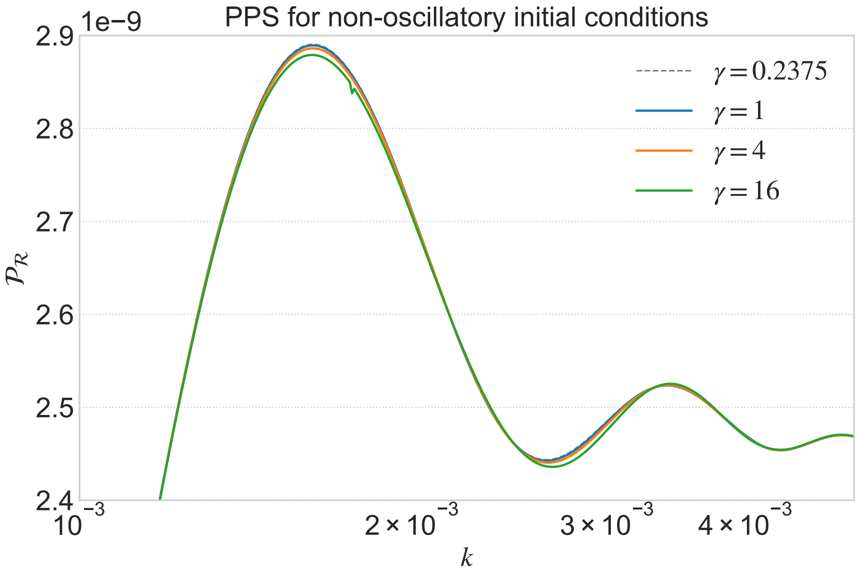
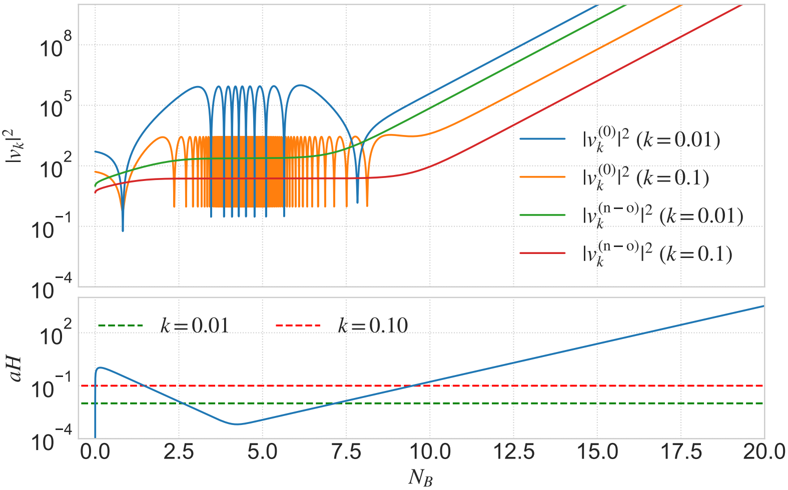


Evolution of positive frequency solutions



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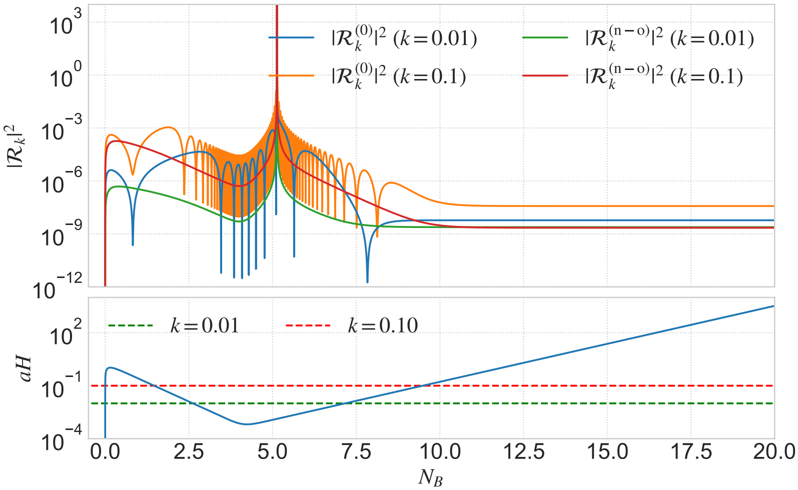
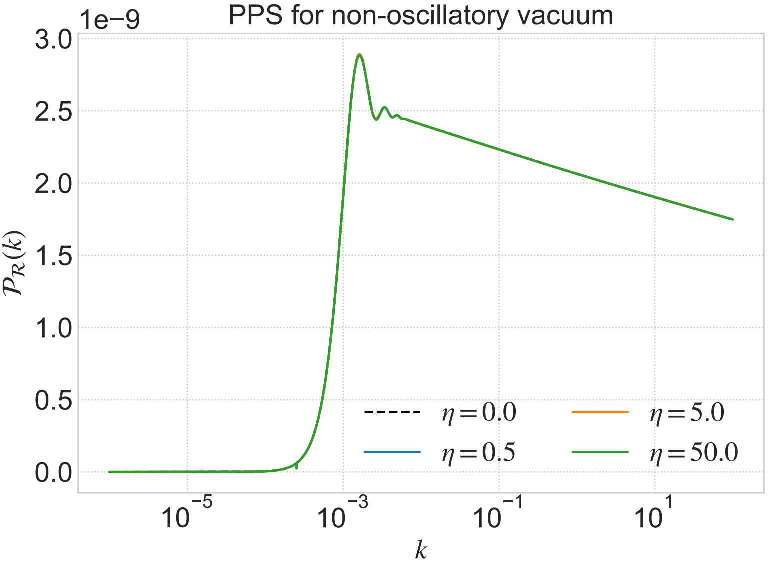


Non-oscillatory vacuum with η_i after the bounce



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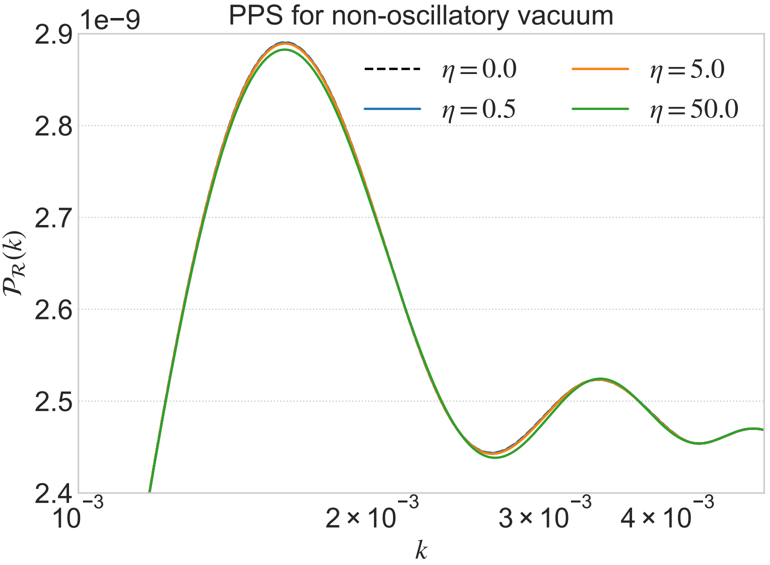
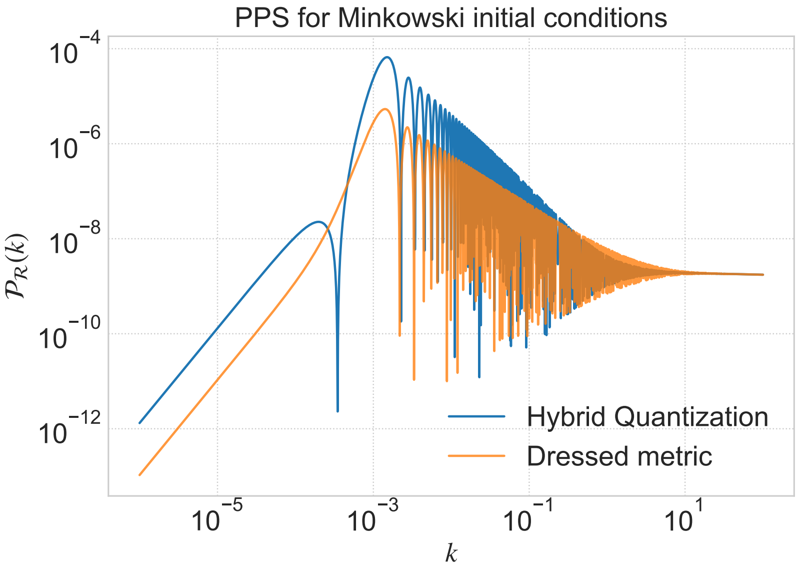


Hybrid quantization vs dress metric approach



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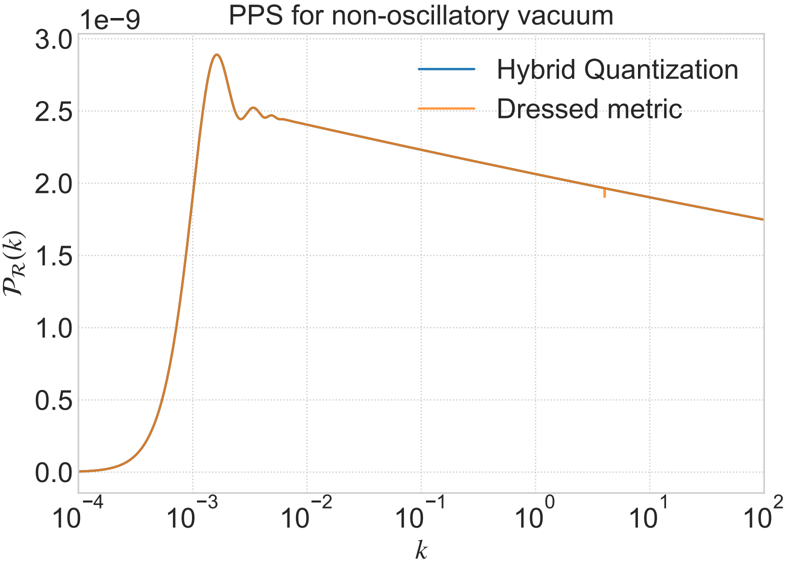

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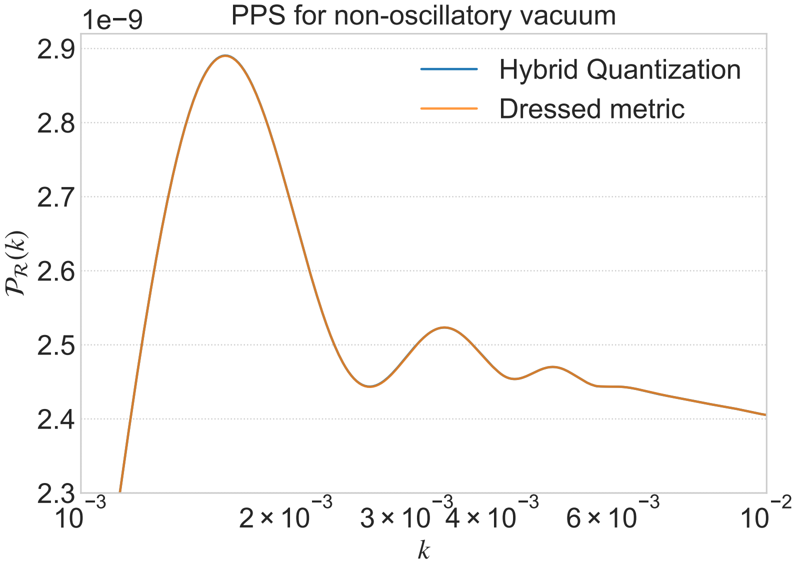

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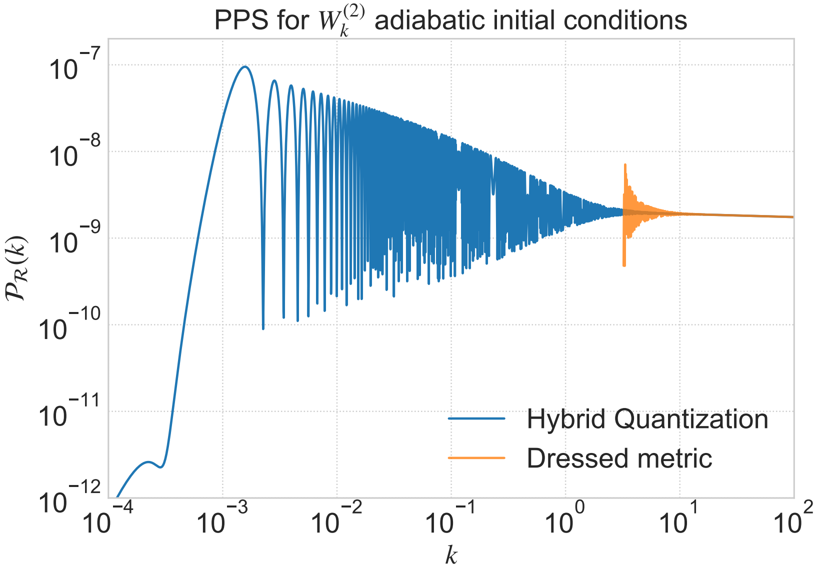

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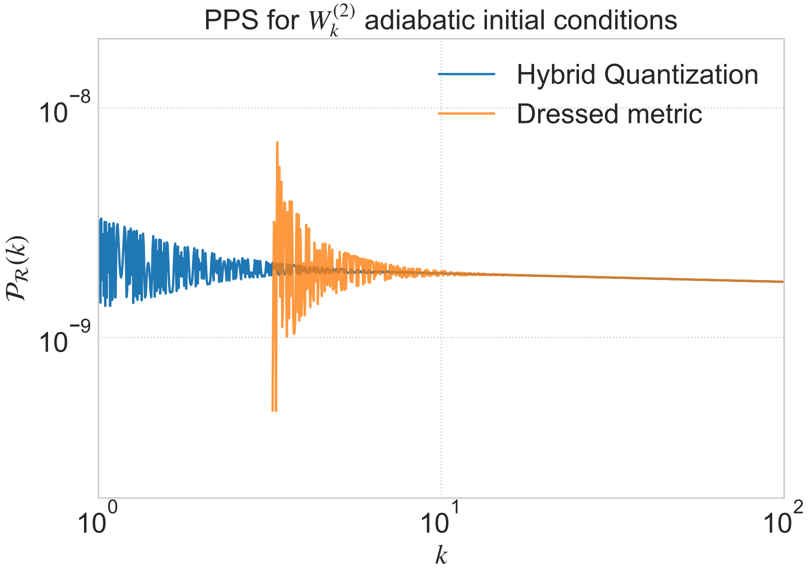

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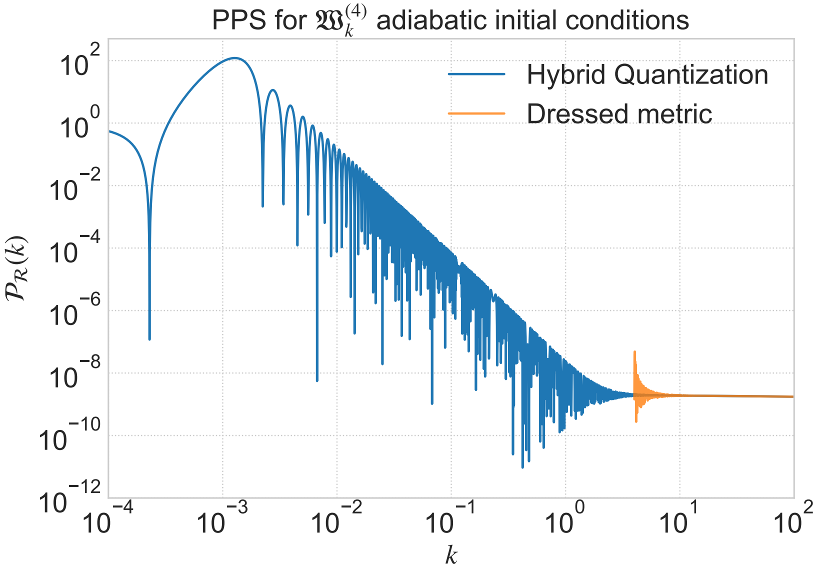

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