On the dynamics in LQG deparametrized models: Perturbative approach

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Plan of the talk

I. Deparametrization & LQG

II. Physical Hamiltonian operators in LQG models

III. Approximating the dynamics: a perturbative method

IV. Summary & outlook

Deparametrization

$$S = \int d^4x \; L_G + L_R + L_M$$

Deparametrization ⟶ Matter fields as coordinates

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E.G: time ref. mod.

$$L_R = -\frac{1}{2}\sqrt{|g|}\,[\rho \; g^{\mu\nu}(\nabla_\mu T)(\nabla_\nu T) + \Lambda(\rho)]$$

Massless K.G. scalar field*	Non-rotational dust**
$C_a = 0$	$C_a = 0$
$h := \sqrt{-2\sqrt{\det(q)}C}$	$h := -C$

*[C. Rovelli, L. Smolin 93'], [M. Domagala, K. Giesel, W. Kaminski, J. Lewandowski 10']
**[J. Brown, K. Kuchar 94'], [V. Husain, T. Pawlowski 12'], [K. Giesel, T. Thiemann 12']

Some history…

- Reference fluid: Einstein, Hilbert, Landau, ….
  (different terminologies)

- Phenomenological fluid: DeWitt, Lund, Wald, Unruh, Smolin …
  (minisuperspace/quantization)

- Physical reference fluid: Kuchar, Isham, Hartle, Rovelli, Smolin, Torre, …

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Deparametrization ⟶ Quantization ⟶ LQG

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Deparametrization & LQG

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Motivation

- Test-ground for LQG quantization methods
- Insights on the continuum limit of LQG
- Insights on the dynamics of Matter fields in presence of QG

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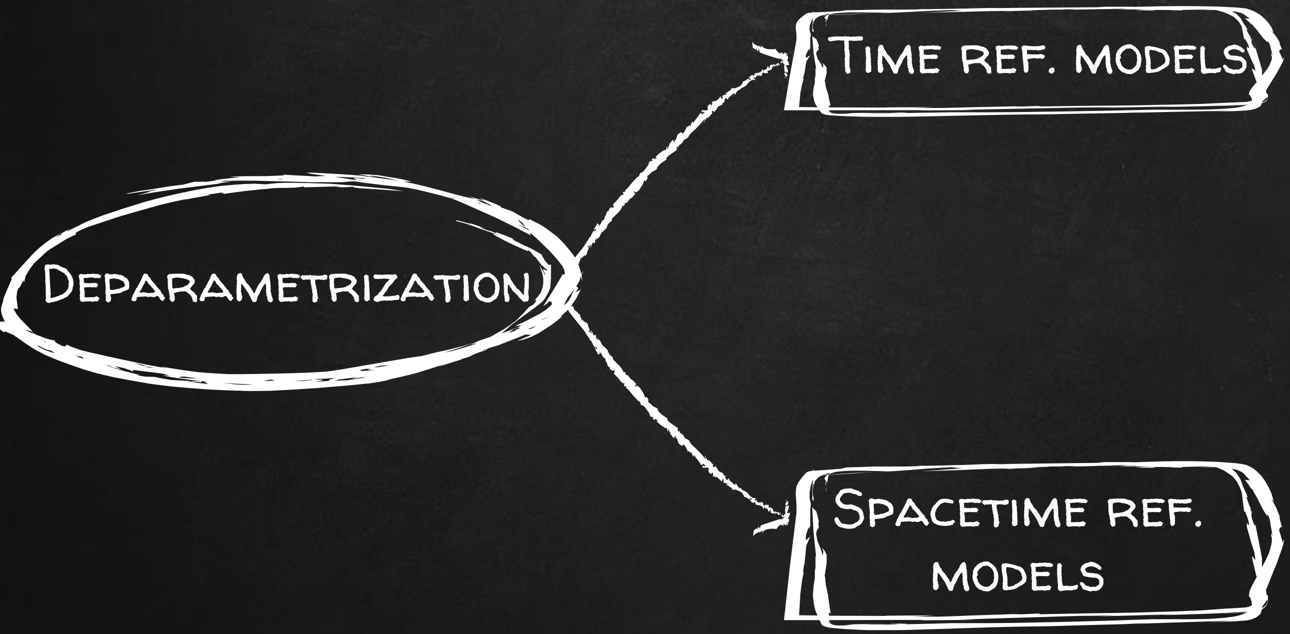
First steps...

- Complete quantum gravity models:
  * Physical Hilbert space
  * Admissible Hamiltonian operator
- Computable dynamics (time evolution)

LQG deparametrized models



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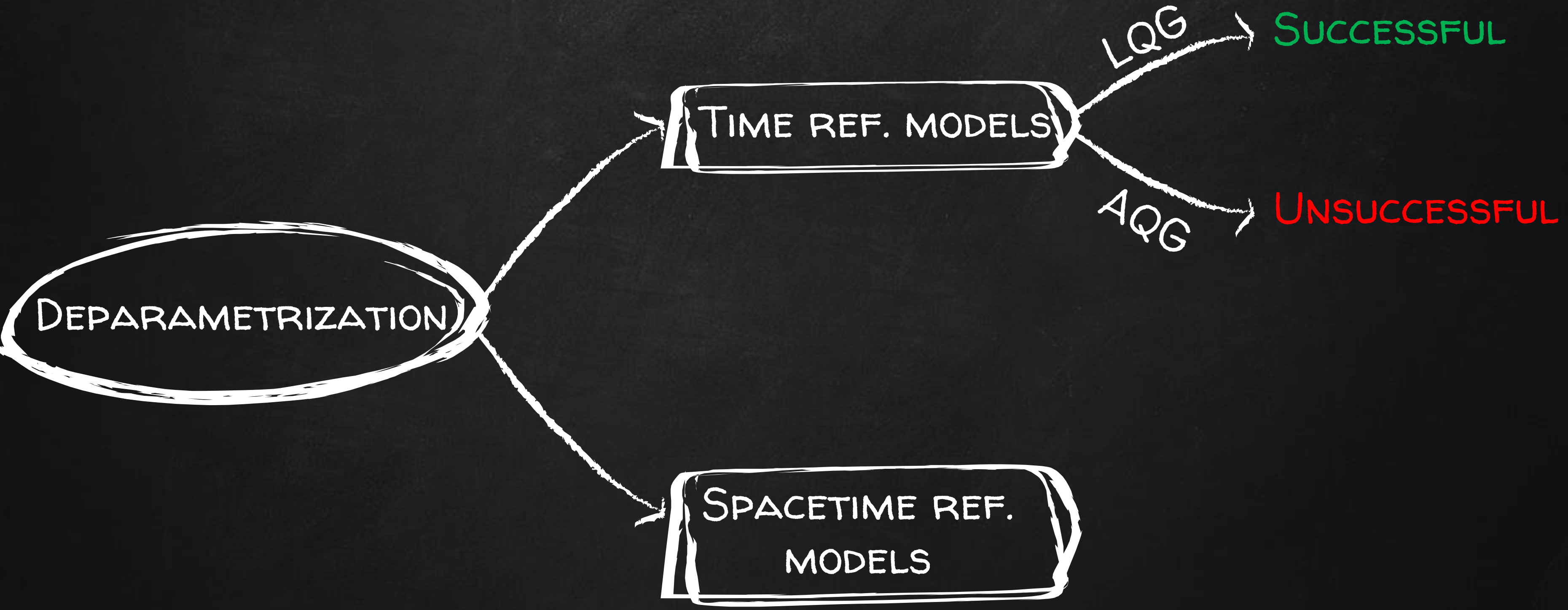

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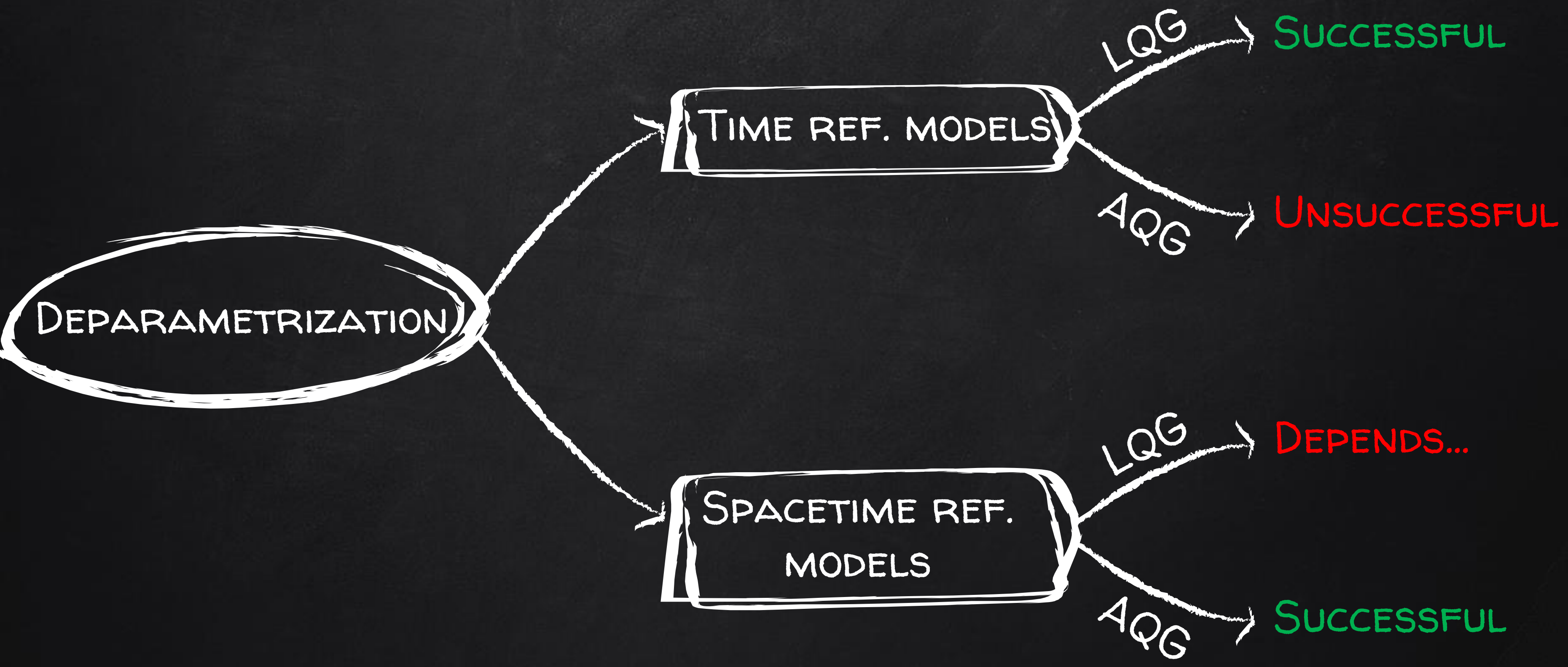

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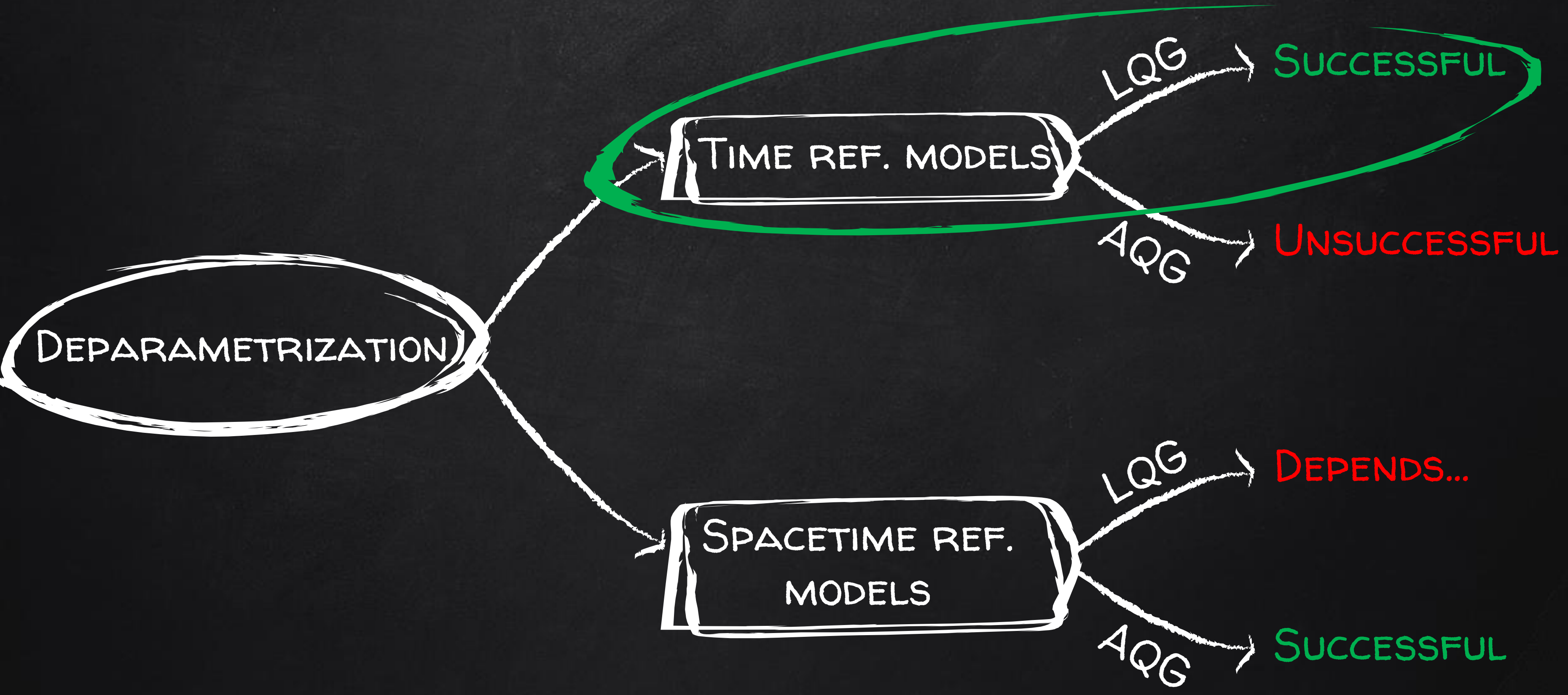

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LQG deparametrized models

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→ Proper implementation of the functional C

* Self-adjoint Hamiltonian operator
* Spatial diffeomorphism invariant
* Computable dynamics

Physical Hamiltonian operators

Physical Hamiltonian operators

Physical Hamiltonian: $H = \int_\Sigma d^3x\, h(C, C_a, q) = H(C, C_a, q)$

Scalar constraint functional

$$C(N) = \frac{1}{2sk\beta^2} \int_\Sigma d^3x\, N \left(\frac{\epsilon_{ijk} E^a_i E^b_j F^k_{ab}}{\sqrt{|\det[E]|}} + \left(1 - s\beta^2\right) \sqrt{|\det[E]|}\, R \right)$$

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Euclidean part (Euclidean operator)

Lorentzian part (Curvature operator)

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Constructing the operator on Cyl^*

Physical Hamiltonian operators

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- Different regularizations of C ⟶ Different dynamics

  [T. Thiemann 96', 98']
  [E. Alesci, M. Assanioussi, J. Lewandowski, I. Mäkinen 14', 15']
  [J. Yang, Y. Ma 15']

- Computational complications: spectral decomposition of H ?

  Graph changing ✗
  Approximation methods?

`` YES!!! ´´

Perturbation theory!

Physical Hamiltonian operators

$$\hat{C}(N) := \frac{1}{2sk\beta^2}\left(\hat{C}^E(N) + (1 - s\beta^2)\hat{C}^L(N)\right)$$

[M. Assanioussi, J. Lewandowski, I. Mäkinen 17']

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"Unperturbed Hamiltonian"

Recipe: split the Hamiltonian to two operators with different parameter dependence, s.t. A graph preserving part (diagonalizable) + $\epsilon(\beta)$ * second operator

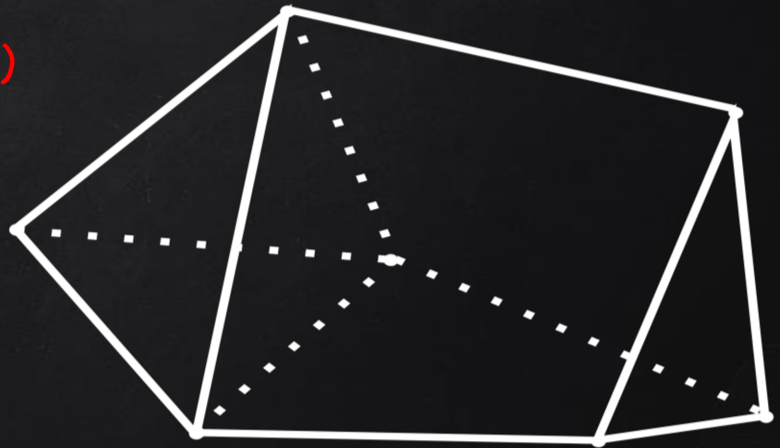
[M. Assanioussi, J. Lewandowski, I. Mäkinen 17']

Physical Hamiltonian operators

Curvature operator

$$C^L(N) := \int_\Sigma d^3x \; N\sqrt{|\det[E]|}R(E) = \lim_{\epsilon\to 0} \sum_{\Delta\in\mathscr{C}^\epsilon} N(x_\Delta) \sum_{h\in\Delta} L_h^\Delta(E)\Theta_h^\Delta(E)$$

(Regge calculus)



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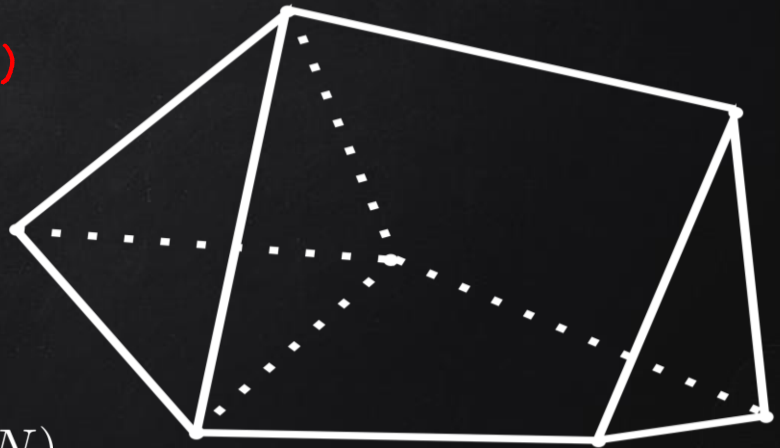

$$\hat{C}^L(N) := \text{Avg}\left[\lim_{\epsilon\to 0} \sum_{\Delta\in\mathscr{C}^\epsilon} N(x_\Delta)\hat{R}_\Delta\right] = \hat{R}(N)$$

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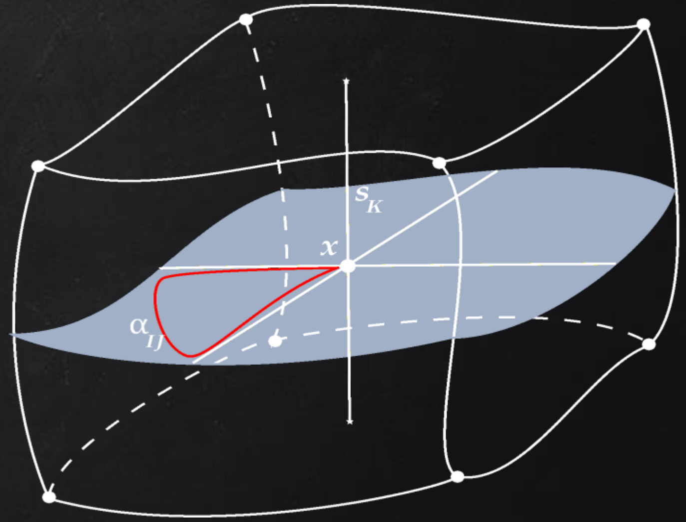
Regularization →



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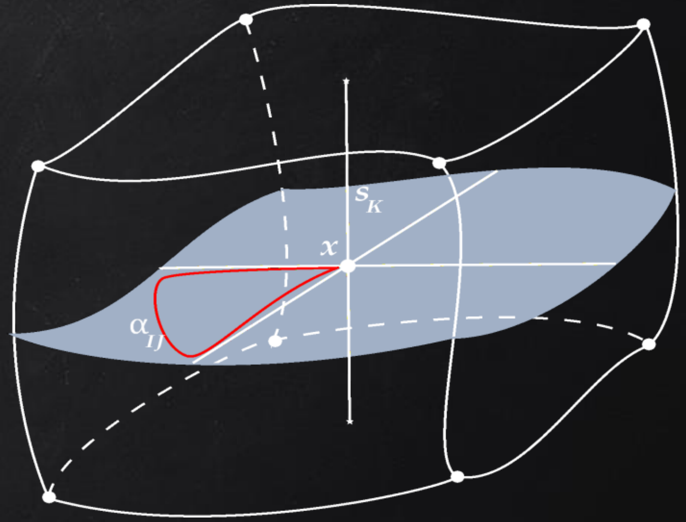

Moving to Cyl^*:

$$\hat{C}^E(N) := \lim_{\epsilon \to 0} \left[\hat{C}^E_\epsilon(N)\right]^*$$

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Approximation method for the dynamics

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Dynamics

$$\hat{U}(t) := \exp[-it\hat{H}] \ , \qquad \hat{H} = f(\beta)\left(\hat{C}^L - \frac{1}{1+\beta^2}\hat{C}^E\right)$$

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$$\hat{H} = f(\beta)(\hat{H}_0 + \epsilon\hat{V}) \qquad , \qquad |\epsilon| \ll 1 \Leftrightarrow \beta^2 \gg 1$$

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$$H_0 + \epsilon V = \sum_n \lambda_n \left|\lambda_n\right\rangle \left\langle\lambda_n\right|$$

$$\left|\lambda_n\right\rangle = \left|\lambda_n^{(0)}\right\rangle + \epsilon\left|\lambda_n^{(1)}\right\rangle + \epsilon^2\left|\lambda_n^{(2)}\right\rangle + \dots \ , \qquad \lambda_n = \lambda_n^{(0)} + \epsilon\lambda_n^{(1)} + \epsilon^2\lambda_n^{(2)} + \dots$$

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$$\mathscr{A}_{ij}(t) = \langle\Psi_j|\,\hat{U}^{(0)}(t)\,|\Psi_i\rangle + \epsilon\,\langle\Psi_j|\,\hat{U}^{(1)}(t)\,|\Psi_i\rangle + \epsilon^2\,\langle\Psi_j|\,\hat{U}^{(2)}(t)\,|\Psi_i\rangle + \dots$$

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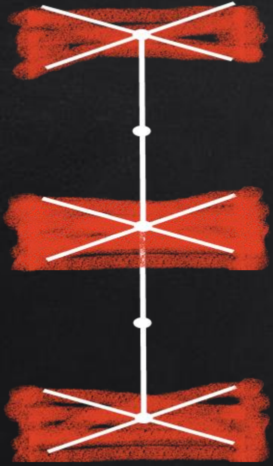
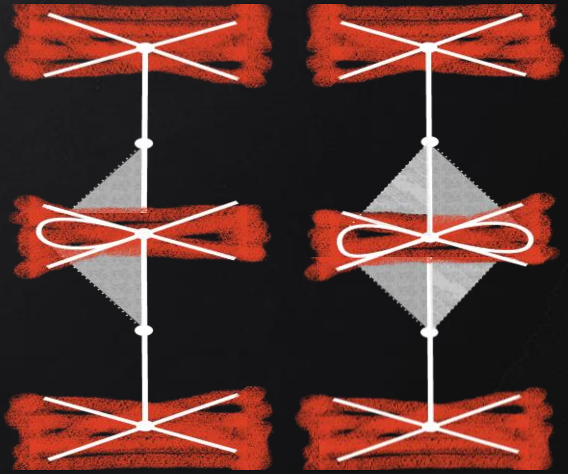
Exp.:  + 0 + $\epsilon^2(\beta)$ 

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  Concretely? For what range of β?

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⟶ Evolution of volume & curvature expectation values!

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⟶ Evolution of volume & curvature expectation values!

✓ initial states = eigenvectors of volume (4−valent vertex)
✓ β−expansion is taken up to 2nd order:

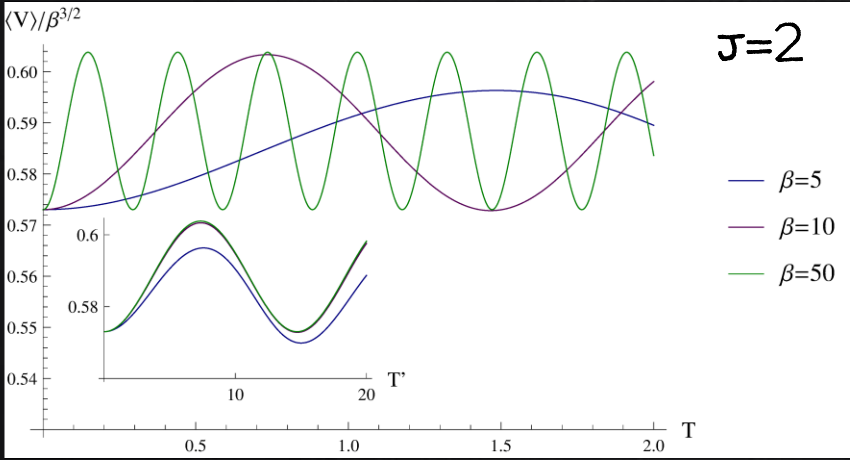
$$\langle \mathscr{O}(t) \rangle = \sum_{i,j} \exp\left[-itf(\beta)\,(\lambda_i - \lambda_j) \right] \langle \Psi_0 | \lambda_j \rangle \langle \lambda_j | \mathscr{O} | \lambda_i \rangle \langle \lambda_i | \Psi_0 \rangle$$

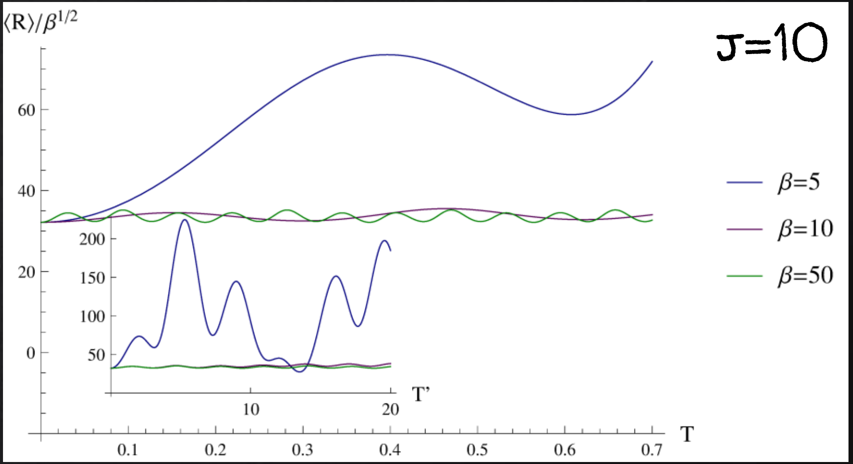
[M. Assanioussi, J. Lewandowski, I. Mäkinen 17']

Approximation method for the dynamics

Scalar field model in examples

$$T' := \sqrt{1+\beta^2}\;T$$





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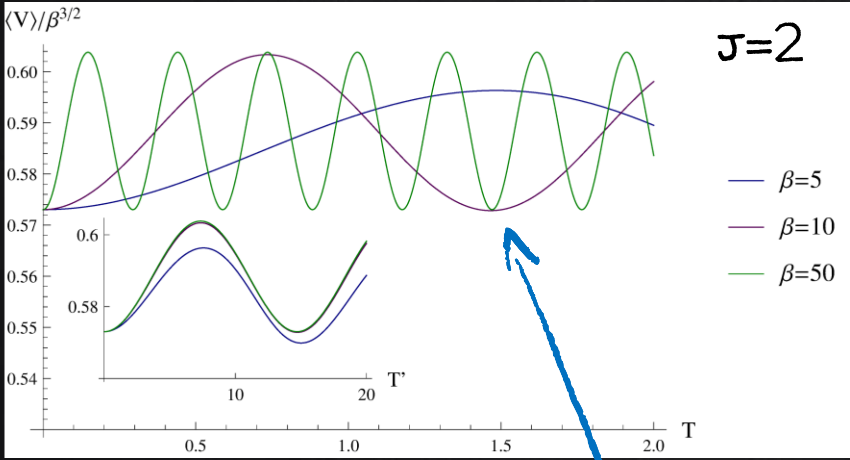
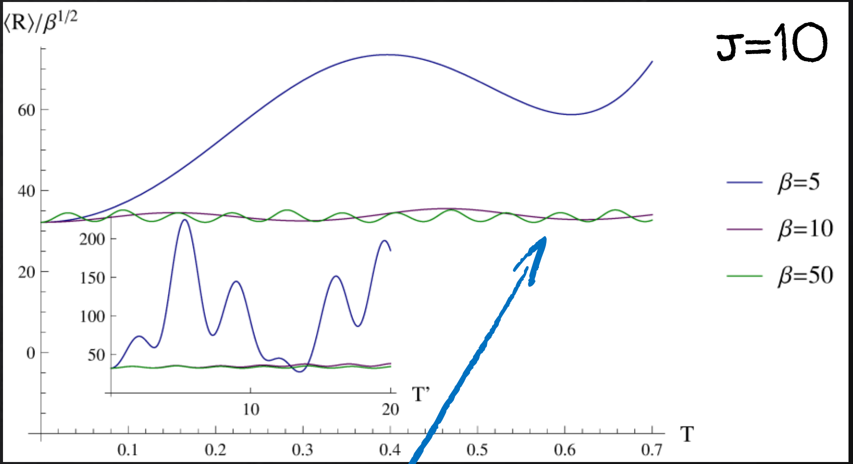
J=2



J=10



Periodic evolution at 0th order

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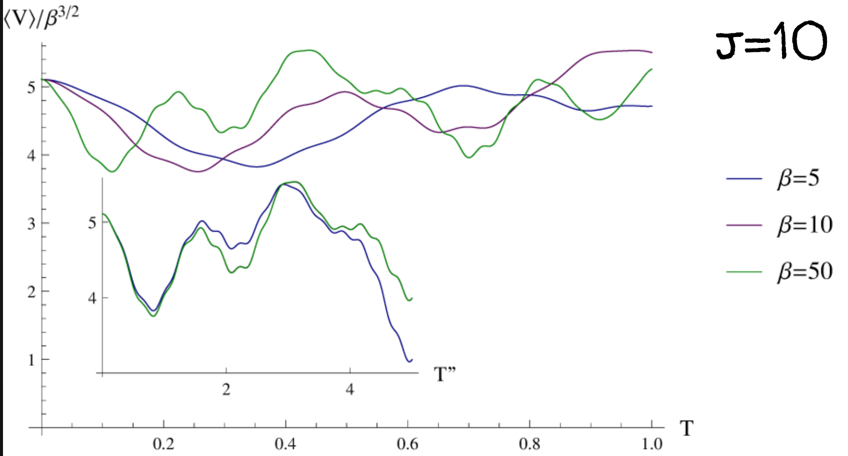
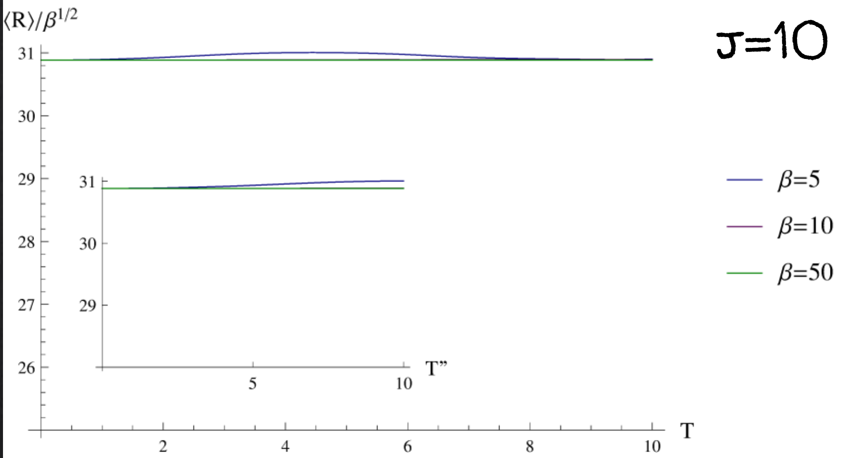
Dust field model in examples

$$T'' := \frac{1+\beta^2}{|\beta|^{3/2}}\, T$$





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+ Cosmological constant + SM matter fields ? In progress…

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- Einstein : $\beta^{\frac{1}{2}}$

- Cosmol. Const. : $H_\Lambda = \sqrt{q}\Lambda \longrightarrow \beta^{\frac{3}{2}}$

- Higgs field: $H_H = \frac{\pi_H^2}{\sqrt{q}} + \frac{E_i^a E_i^b}{\sqrt{q}}(\dots) \longrightarrow \beta^{\frac{1}{2}}$

- Yang-Mills: $H_{YM} = \frac{q_{ab}}{\sqrt{q}}(\tilde{E}_I^a \tilde{E}_I^b + \tilde{B}_I^a \tilde{B}_I^b) \longrightarrow \beta^{-\frac{1}{2}}$

- Fermions: $H_H = \frac{E_i^a}{\sqrt{q}}(\dots + O(\beta^{-1})) \longrightarrow \beta^{-\frac{1}{2}}$

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✓ Symmetric Hamiltonian operators

✓ Approximation method (pert. th.) for the Hamiltonian op.:

  * treatment of the square root in the SF model
  * explicit computation of dynamics

🔍 Work out the Perturbative approach for S.M. matter fields

🔍 Self-adjointness proofs of the total Hamiltonian

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