New boundary variables for CQG on a null surface

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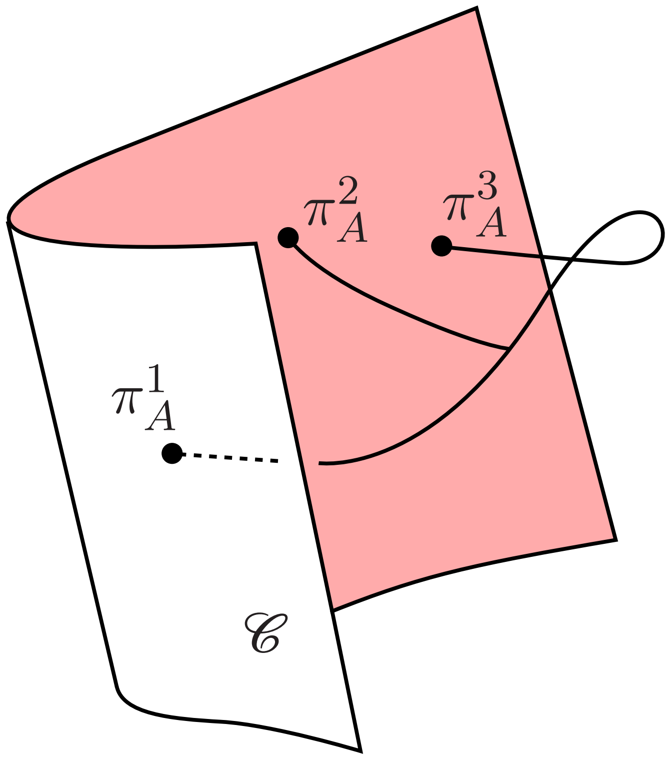
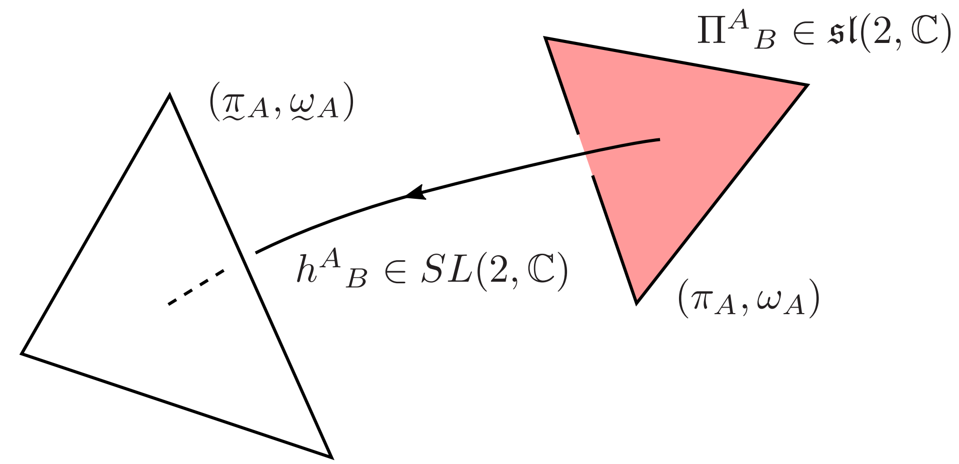
Motivation

Why spinors?

- Lesson from the standard model: Fermions couple to tetrad and connection.
  - Quantum gravity is not about quantising the metric manifold, but about the quantum geometry of the entire spin bundle.
  - The relevant gauge group is $SL(2,\mathbb{C})$.
- Lesson from GR: Spinors are useful tools for e.g. proving positivity of energy, analysing causal and asymptotic structure of spacetime. We need these tools in QG as well.
- Lesson from LQG: Bulk geometry described by spin networks. If they hit a boundary, they create a surface charge (namely a spinor).
- Lesson from QI: Spin is the fundamental unit of information
  - recent thoughts on geometry and information: [Carrozza, Hoehn, Mueller]



LQG spinor representation



- LQG in the (discrete) spinor representation:

Flux	$\Pi_{AB} = -\frac{1}{2}\pi_{(A}\omega_{B)}$
Holonomy	$h^A{}_B\omega^B = \underset{\sim}{\omega}^A, \quad h^A{}_B\pi^B = \underset{\sim}{\pi}^A$
Poissson brackets	$\{\pi_A, \omega_B\} = -\{\underset{\sim}{\pi}_A, \underset{\sim}{\omega}_B\} = \epsilon_{AB}$
Constraints	$C = \frac{\mathrm{i}}{\beta + \mathrm{i}}\pi_A\omega^A + \mathrm{cc.} = 0, \quad M = \underset{\sim}{\pi}_A\underset{\sim}{\omega}^A - \pi_A\omega^A$

- Holonomies and fluxes have a clean continuum interpretation:

$$\Pi_{AB}[\triangle] = -\frac{\mathrm{i}}{32\pi\beta G}(\beta + \mathrm{i})\int_\triangle e_{AC'} \wedge e_B{}^{C'}, \quad h^A{}_B[\gamma] = \mathrm{Pexp}\Big(-\int_\gamma A\Big)^A{}_B$$

- Question: What are then the LQG spinors in the continuum?
  Not just a technical question, crucial for understanding: (i) LQG continuum limit,
  (ii) relation between bulk+boundary in QG, (iii) black holes, null infinity and causal diamonds.
- Answer: LQG spinors are boundary variables (~edge modes) on a null surface.

Perspectives

- Based on LQG spinor representation
  - Speziale, Freidel: Twistors to twisted geometries, arXiv:1006.0199.
  - Over the last couple of years, this evolved into the spinorial representation of LQG: [Freidel, Speziale; Livine; ww; Bianchi, Guglielmon, Hackl, Yokomizo; Hnybida; Langvik; Anzà; Martín-Benito; Borja, Díaz-Polo, Garay; Tambornino; Langvik; Zhang; Dupuis, Girelli; Rennert; Chen, Banburski; Bonzom,…]
- Relevant for other approaches as well
  - Isolated Horizons, black hole entropy, relation between bulk and boundary geometry [Ashtekar, Lewandowski, Beetle, Engle,…, Perez, Pranzetti,…]
  - Quasi-local observables ("edge modes") in GR and QG […, Strominger, …, Pranzetti, Donnelly, Freidel, Hopfmüller; De Paoli, Speziale; Campiglia, Laddha;…]
  - New representations and area spectrum [Dittrich, Geiller; Bahr et. al.]
  - collapse of null shells, BH to WH transitions? [Rovelli, Haggard, Barau, Vidotto; Gambini et. al.]
  - reconstruction of geometry from information theory, see recently: [Carrozza, Mueller, Höhn]

[1] ww: Fock representation of gravitational boundary modes and the discreteness of the area spectrum, arXiv:1706.00479.

[2] ww: New boundary variables for classical and quantum gravity on a null surface, arXiv:1704.07391.

[3] ww: Discrete gravity as a TFT with light-like curvature defects, JHEP 5 (2017), arXiv:1611.02784.

Outline

- LQG spinors are the boundary variables for GR on a null surface
- Quantum geometry with null boundaries: Area quantisation without spin networks
- Proposal for the dynamics: LQG as a TFT with null defects
- Conclusion

Boundary spinors

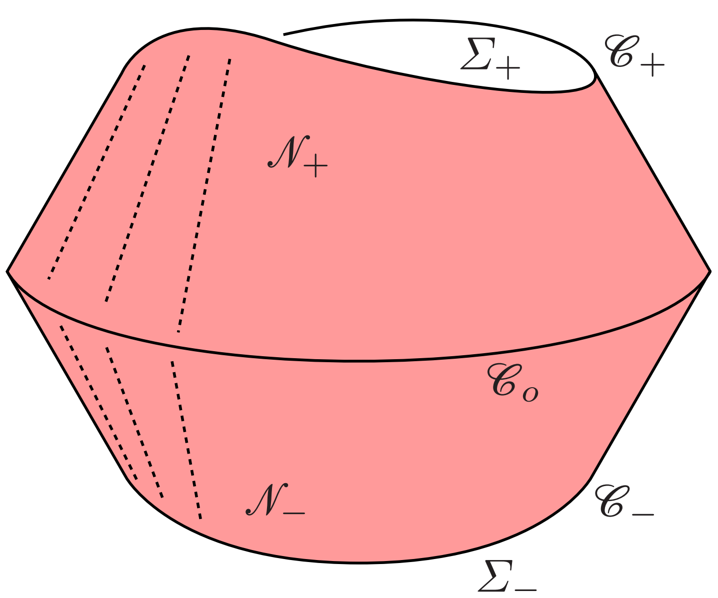
Self-dual variables in a diamond

- Goal: To understand LQG spinors from GR perspective.
- Well, now, in relativity, a spinor is the square root of a null vector.

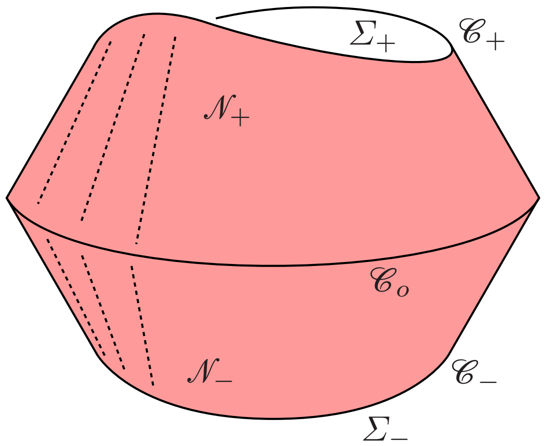
$$\ell^\alpha = -\frac{1}{\sqrt{2}}\sigma_{AA'}{}^\alpha \ell^A \bar{\ell}^{A'} = -\frac{1}{\sqrt{2}}\langle \ell|\sigma^\alpha|\ell\rangle$$

- LQG is a based on a canonical quantisation of GR in terms of Ashtekar variables. → Consider Hamiltonian GR in a situation where there are already null surfaces: Finite domains with null boundaries.
- The action consists of bulk, boundary and corner terms.
- Boundary term is needed, because we now have the additional constraint that the boundary is null.

$$q_{ab} := \underset{\leftarrow}{g_{ab}} \text{ has signature } (0{+}{+}).$$



New boundary action



- Action in the bulk:
  - a physical motion extremises the BF action

$$S[\Sigma, A] = \Big[\frac{\mathrm{i}}{8\pi G}\frac{\beta+\mathrm{i}}{\beta}\int_{\mathscr{M}} \Sigma_{AB} \wedge F^{AB}\Big] + \mathrm{cc}.$$

  - in the class of all fields that satisfy the reality conditions

$$\Sigma_{(AB} \wedge \Sigma_{CD)} = 0, \quad \Sigma_{AB} \wedge \bar{\Sigma}_{A'B'} = 0, \quad \mathfrak{Re}\big(\Sigma_{AB} \wedge \Sigma^{AB}\big) = 0.$$

- Action at the boundary:

$$\boxed{S_{\mathscr{N}}[A|\boldsymbol{\eta}, \ell|\omega, \psi] = \Big[\frac{\mathrm{i}}{8\pi G}\frac{\beta+\mathrm{i}}{\beta}\int_{\mathscr{N}} \boldsymbol{\eta}_A \wedge (D-\omega)\ell^A - \boldsymbol{\eta}_A \wedge \psi^A\Big] + \mathrm{cc}.}$$

- Boundary conditions: null surface extrinsic curvature fixed, i.e.: $\delta\omega_a = 0,\ \delta\psi^A{}_a = 0.$

Boundary variables

- What is the geometric meaning of the boundary spinors? — Similar to tetrad vs.metric: boundary spinors determine entire intrinsic geometry of $\mathscr{N}$.

$$\boldsymbol{\eta}^A{}_{ab} \in \Omega^2(\mathscr{N} : \mathbb{C}^2), \quad \ell^A \in \Omega^0(\mathscr{N} : \mathbb{C}^2)$$

- Spin (0,0) singlet: determines the area two-form

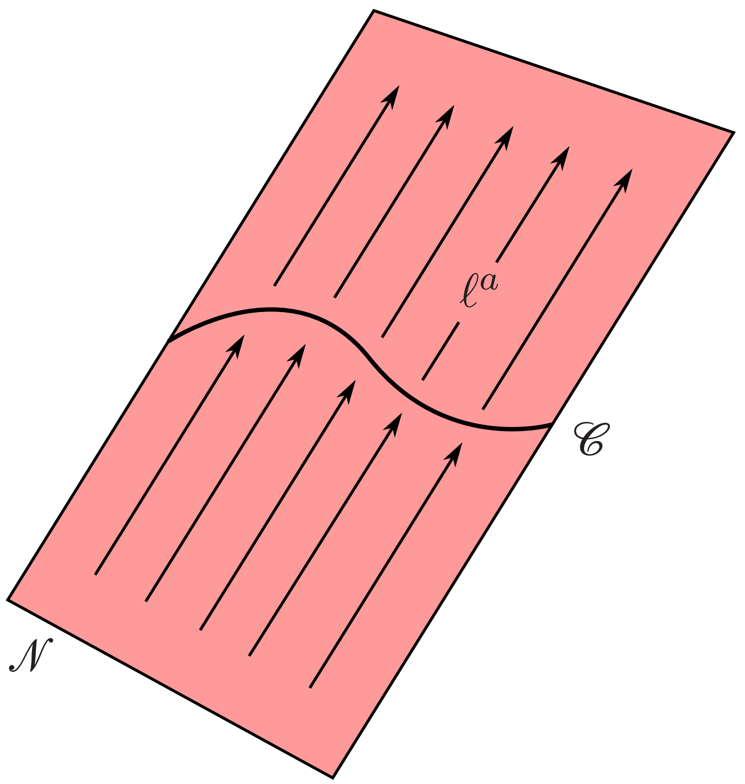
$$\mathrm{Ar}[\mathscr{C}] = \int_{\mathscr{C}} \boldsymbol{\varepsilon} = -\mathrm{i} \int_{\mathscr{C}} \boldsymbol{\eta}_A \ell^A$$

- Spin (1/2,1/2) vector component: defines null generators

$$\ell^a = e_{AA'}{}^a \ell^A \bar{\ell}^{A'} \in T\mathscr{N}$$

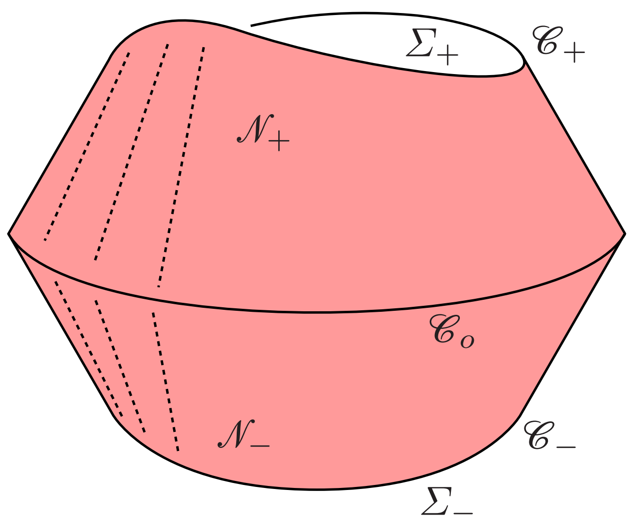
- Spin (1,0) tensor component: Plebański two-form (glueing condition between bulk+boundary)

$$\Sigma_{AB\underleftarrow{ab}} = \ell_{(A}\boldsymbol{\eta}_{B)ab}$$



NB: spinors are unique up to complexified U(1) transformations.

Symplectic structure



- Variation of the bulk+boundary action

$$\delta S = \mathrm{EOM} \cdot \delta + \Theta_{\partial \mathscr{M}}(\delta)$$

- EOM in the bulk: Einstein equations and torsionless condition
- EOM at boundary: Glueing conditions + boundary EOMs

$$D_a \ell^A = +\omega_a \ell^A + \psi^A{}_a$$
$$D \wedge \boldsymbol{\eta}_A = -\omega \wedge \boldsymbol{\eta}_A$$

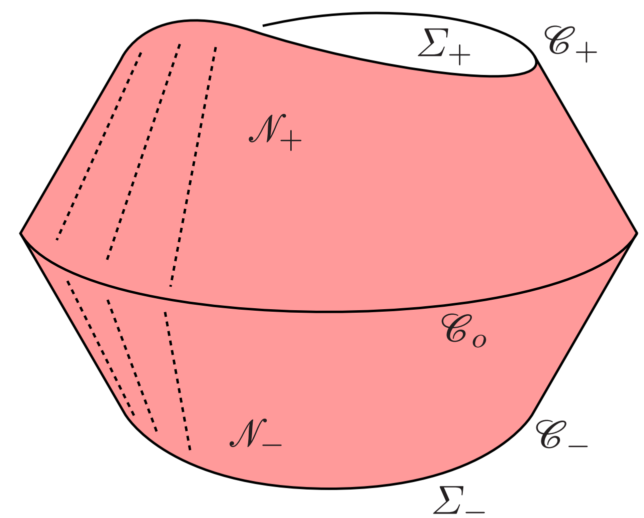
- Symplectic structure on $\boldsymbol{\Sigma}$: Contains contribution from the boundary of the (null) boundary

$$\boxed{\Theta_\Sigma = \frac{\mathrm{i}}{8\pi G} \frac{\beta + \mathrm{i}}{\beta} \Big[\int_\Sigma \Sigma_{AB} \wedge \mathbb{d} A^{AB} + \int_{\mathscr{C}} \boldsymbol{\eta}_A \mathbb{d} \ell^A \Big] + \mathrm{cc.}}$$

- Symplectic structure on $\mathscr{N}$:

$$\Theta_{\mathscr{N}} = -\frac{\mathrm{i}}{8\pi G} \frac{\beta + \mathrm{i}}{\beta} \Big[\int_{\mathscr{N}} \boldsymbol{\eta}_A \ell^A \wedge \mathbb{d}\omega + \boldsymbol{\eta}_A \wedge \mathbb{d}\psi^A \Big] + \mathrm{cc.}$$

Phase space at the corner



- Configuration variable: null flag

$$\ell^A : \mathscr{C} \rightarrow \mathbb{C}^2$$

- Canonical momentum: a spinor-valued two-surface density

$$\boxed{\boldsymbol{\pi}_A = \frac{\mathrm{i}}{16\pi G}\frac{\beta+\mathrm{i}}{\beta}\hat{\boldsymbol{\epsilon}}^{ab}\boldsymbol{\eta}_{Aab}}$$

- Canonical Poisson brackets at the corner

$$\forall z, z' \in \mathscr{C} : \big\{\boldsymbol{\pi}_A(z), \ell^B(z')\big\} = \delta^B_A \delta^{(2)}(z, z')$$

- Reality conditions = simplicity constraint in the continuum

$$\left.\begin{aligned} d^2 v = \frac{1}{2\mathrm{i}}\hat{\boldsymbol{\epsilon}}^{ab}\boldsymbol{\eta}_{Aab}\ell^A \overset{!}{\in} \mathbb{R} &\Leftrightarrow \frac{\mathrm{i}}{\beta+\mathrm{i}}\boldsymbol{\pi}_A\ell^A + \mathrm{cc.} \\ &\Leftrightarrow \boxed{K - \beta L = 0} \end{aligned}\right\} \quad \begin{aligned} L &= +\tfrac{\mathrm{i}}{2}\boldsymbol{\pi}_A\ell^A + \mathrm{cc.} \\ K &= -\tfrac{1}{2}\boldsymbol{\pi}_A\ell^A + \mathrm{cc.} \end{aligned}$$

Charges and gauge symmetries

- Gauge symmetries (degenerated directions of symplectic two-form)
  - Diffeomorphisms generated by vector fields that vanish at the two-surface corner
  - *SL(2,ℂ)* frame rotations (including transformations that do *not vanish at the corner*)
  - twisted *U(1)* transformations of the spinors, generated by reality conditions

- Hamiltonian motions
  - U(1) transformations
  - dilatations
  - diffeos preserving $\mathscr{C}$

$$L_\varphi[\mathscr{C}] = -\frac{1}{2\mathrm{i}}\int_{\mathscr{C}} \varphi\left(\boldsymbol{\pi}_A \ell^A - \text{cc.}\right)$$

$$K_\lambda[\mathscr{C}] = -\frac{1}{2}\int_{\mathscr{C}} \lambda\left(\boldsymbol{\pi}_A \ell^A + \text{cc.}\right)$$

$$K_\varphi[\mathscr{C}] - \beta L_\varphi[\mathscr{C}] = 0$$

$$J_\xi[\mathscr{C}] = \int_{\mathscr{C}} \boldsymbol{\pi}_A \mathscr{L}_\xi \ell^A + \text{cc.}$$

- Diffeomorphisms along null generators not integrable (unless first law is satisfied)

$$\not{\delta} W_\ell[\mathscr{C}] = \frac{1}{8\pi G}\int_{\mathscr{C}} \kappa_{(\ell)} \mathbb{d}\boldsymbol{\varepsilon} - \frac{1}{2}\vartheta_{(\ell)} \mathbb{d}\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}\mathbb{d}\vartheta_{(\ell)} + \mathrm{i}\sigma_{(\ell)} \bar{m} \wedge \mathbb{d}\bar{m} - \mathrm{i}\bar{\sigma}_{(\ell)} m \wedge \mathbb{d} m$$

$$\boldsymbol{\varepsilon} = \mathrm{i}\, m \wedge \bar{m}$$

Quantisation

LQG Landau operators

- Canonical Poisson commutation relations at two-dimensional corner

$$\{\boldsymbol{\pi}_A(z), \ell^B(z')\} = \delta^B_A \delta^{(2)}(z,z')$$

- Strategy: define harmonic oscillators, and quantise them — this requires additional fiducial structures.

- fiducial (unphysical) area density	$d^2\Omega = \Omega^2(\vartheta,\varphi)\sin^2\vartheta\,\mathrm{d}\vartheta\wedge\mathrm{d}\varphi$
- hermitian metric (in spin bundle)	$\delta_{AA'} = \sigma_{AA'\alpha}n^\alpha$

- Landau operators

<u>NB:</u> essentially a 'frequency'

$$\left.\begin{aligned} a^A &= \frac{1}{\sqrt{2}}\left[\sqrt{d^2\Omega}\,\delta^{AA'}\bar{\ell}_{A'} - \frac{\mathrm{i}}{\sqrt{d^2\Omega}}\boldsymbol{\pi}^A\right] \\ b^A &= \frac{1}{\sqrt{2}}\left[\sqrt{d^2\Omega}\,\ell^A + \frac{\mathrm{i}}{\sqrt{d^2\Omega}}\delta^{AA'}\bar{\boldsymbol{\pi}}_{A'}\right] \end{aligned}\right\} \quad \{a^A(z), a^*_B(z')\} = \{b^A(z), b^*_B(z')\} = \mathrm{i}\,\delta^A_B\,\delta^{(2)}(z,z')$$

$$a^*_A = \delta_{AA'}\bar{a}^{A'}$$

In QG area is quantised

- Fock vacuum

$$a^A(z)\big|0,\{d^2\Omega,n^\alpha\}\big\rangle = b^A(z)\big|0,\{d^2\Omega,n^\alpha\}\big\rangle = 0$$

- Reality conditions
  - U(1) generator: $$\hat{L}(z) = \frac{1}{2}\left[a^\dagger_A(z)a^A(z) - b^\dagger_A(z)b^A(z)\right]$$
  - squeeze operator: $$\hat{K}(z) = \frac{1}{2\mathrm{i}}\left[a_A(z)b^A(z) - \mathrm{hc.}\right]$$
  - physical states: $$\big(\hat{K}(z) - \beta\hat{L}(z)\big)\Psi = 0$$

- Oriented area

$$\begin{aligned}\mathrm{Ar}[\mathscr{C}] &= 8\pi\beta\, G\int_{\mathscr{C}}\hat{L} = \\ &= 4\pi\beta\, G\sum_{J=1/2}^{\infty}\sum_{M=-J}^{J}\sum_{s=\pm}\Big(a^\dagger_{JMs}a_{JMs} - b^\dagger_{JMs}b_{JMs}\Big)\end{aligned}$$

eigenvalues:

$$a_n = \frac{4\pi\beta\,\hbar G}{c^3}n, \quad n\in\mathbb{Z}$$

Relation to LQG

- Fock vacuum gauge equivalent to a totally squeezed state

$$\hat{C}(z) := \hat{K}(z) - \beta\hat{L}(z), \quad \Psi \sim \exp\Big(\int_{\mathscr{C}} \varepsilon\, \hat{C}\Big)\Psi$$

$$\Big|0, \{d^2\Omega, n^\alpha\}\Big\rangle \sim \lim_{t\to\infty}\Big|0, \Big\{\mathrm{e}^{-t\lambda^2} d^2\Omega, n^\alpha\Big\}\Big\rangle, \quad \lambda : \mathscr{C} \to \mathbb{R}$$

- Such a totally squeezed state satisfies formally (spinorial analogue of AL vacuum)

$$\frac{\delta}{\delta\ell^A(z)}\Psi_\emptyset[\ell^A] = \frac{\delta}{\delta\bar{\ell}^{A'}(z)}\Psi_\emptyset[\ell^A] = 0$$

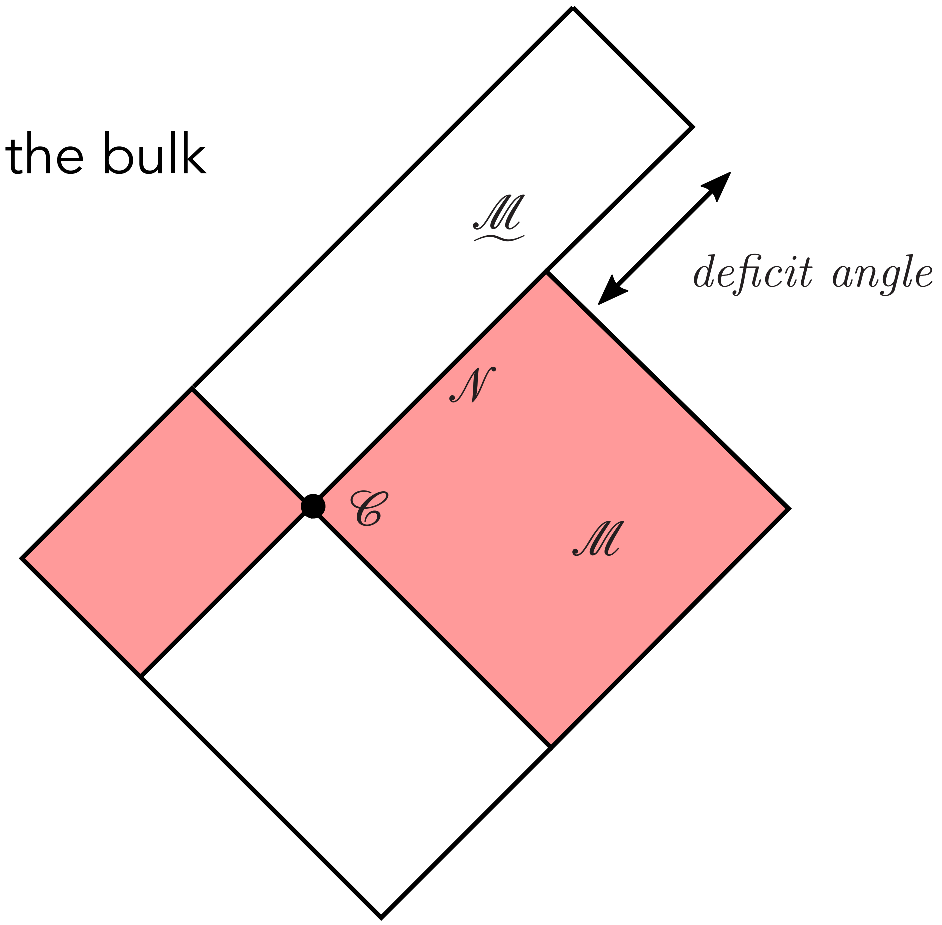
- Relation to LQG: Excite the Fock vacuum only over a certain number of punctures

$$\Psi_f[\ell^A] = f\big(\ell^A(z_1), \dots, \ell^A(z_N)\big)$$

$$\begin{aligned}\forall\zeta \in \mathbb{C} - \{0\} : \quad & f_{\underline{\rho},\underline{j}}\big(\ell^A(z_1), \dots, \zeta\ell^A(z_i), \dots, \ell^A(z_N)\big) = \\ & = \zeta^{-\mathrm{i}\rho_i + j_i - 1}\bar{\zeta}^{-\mathrm{i}\rho_i - j_i - 1}\, f_{\underline{\rho},\underline{j}}\big(\ell^A(z_1), \dots, \ell^A(z_N)\big)\end{aligned}$$

Dynamics: LQG as a TFT

LQG as a 4d TFT with defects

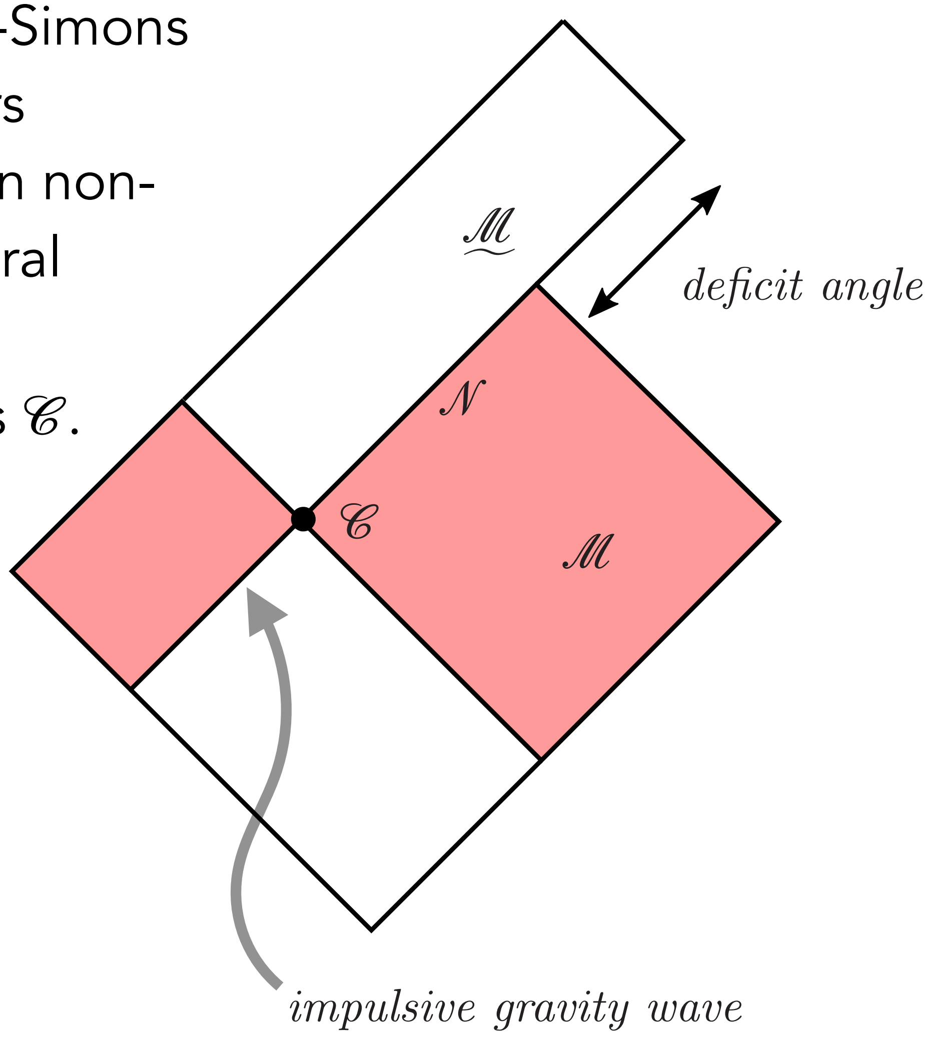


- Basic idea:
  - keep the variables at the boundary, but modify gravity in the bulk
- More explicilty:
  - decompose spacetime into four-dimensional cells
  - impose that the connection be flat (or constantly curved) in every four-cell
  - the internal boundaries be null
  - the intrinsic three-geometries match across the interface
- Equations of motion and glueing conditions:
  - Λ-flatness in the bulk: $$F^A{}_{Bab} = \frac{\Lambda}{3}\Sigma^A{}_{Bab}, \quad D_{[a}\Sigma^A{}_{Bbc]} = 0$$
  - bulk to boundary: $$\Sigma^A{}_{B\underleftarrow{ab}} = \ell_{(A}\boldsymbol{\eta}_{B)ab}$$
  - boundary to boundary: $$\left.\begin{aligned} \boldsymbol{\eta}_{Aab}\ell^A &= \underset{\sim}{\boldsymbol{\eta}}_{Aab}\underset{\sim}{\ell}^A \\ \boldsymbol{\eta}_A \otimes \boldsymbol{\eta}^A &= \underset{\sim}{\boldsymbol{\eta}}_A \otimes \underset{\sim}{\boldsymbol{\eta}}^A \end{aligned}\right\} \Leftrightarrow q_{ab} = \underset{\sim}{q}_{ab}$$

pp-waves as special solutions

- Action consists of bulk, boundary and corner terms:
  - Hamiltonian analysis: No local DOF
  - Integrating out the fields in the bulk yields *SL(2,ℂ)* Chern-Simons action at internal boundaries coupled to boundary spinors
  - All physical degrees of freedom can only appear in certain non-local moduli (at the corners). Manifest themselves as central charges in the algebra of constraints?
  - Einstein equations hold everywhere except at the corners $\mathscr{C}$.
- Solutions represent impulsive gravitational waves
  - Spin (2,0) (Weyl) and spin (1,1) curvature spinors:

$$\Psi_{ABCD} = \delta(v)\,\partial_z f(z)\,\ell_A \ell_B \ell_C \ell_D$$
$$\Phi_{ABA'B'} = \delta(v)\,\big(\partial_{\bar{z}} f + \partial_z \bar{f}\big)\,\ell_A \ell_B \bar{\ell}_{A'} \bar{\ell}_{B'}$$



[ww: Discrete gravity as a TFT with defects, JHEP 5 (2017), arXiv:1611.02784]

Conclusion

Summary

- The LQG spinors are the canonical boundary variables for gravity on a null surface
  - realises a version of quasi-local holography.
  - recasts LQG into a quantum theory of spin bundles $\mathbb{S}(\mathcal{N},\mathbb{C}^2)$ over null boundaries.
- LQG area quantisation
  - compatible with local $SL(2,\mathbb{C})$ gauge invariance
  - compatible with fundamental structure of the light cone (suggesting no modification of the dispersion relations due to quantum discreteness).
  - No discrete structures such as spin-networks or triangulations enter the derivation. LQG area spectrum is robust under change of representation (AL v. Fock).
- Dynamics: LQG as a four-dimensional TFT with null defects
  - basic idea: Keep the same null boundary variables as in GR (namely the new boundary spinors), but modify theory in the bulk by imposing that the geometry be locally flat (departure from GR as in Regge calculus).
  - special solutions: pp-wave spacetimes.