2D dilaton gravity theories in polar-type variables: A review of some classical and quantum results

Tatjana Vukašinac (UMSNH, Mexico)

In collaboration with
Alejandro Corichi (UNAM, Mexico)
Asieh Karami (IPM, Iran)
Saeed Rastgoo (UAM, Mexico)

Motivation and outline

- Study of lower dimensional models in order to explore the technique of loop quantum gravity "a la Dirac".
- Classical general 2D dilaton gravity theories can be recast in Ashtekar-type variables.
- The classical Dirac algebra can be made a Lie algebra, by an appropriate redefinition of the Hamiltonian constraint.
- 2D dilaton gravity theories have black hole solutions and it is important to analyze the possibility of singularity resolution.
- Loop quantization has been performed in two special cases: CGHS theory and spherically symmetric sector of general relativity.
- In the corresponding quantum theories the question of singularity resolution can be addressed.

The generic 2D action in metric variables

The generic 2D DG action, that leads to 2nd order differential equations for the metric g_{ab} and a scalar (dilaton) field Φ, coupled to a scalar matter field f is

$$S[g,\Phi,f] = S_{\mathrm{dg}}[g,\Phi] + S_{\mathrm{m}}[g,\Phi,f]\,,$$

where

$$S_{dg}[g,\Phi] = \int \mathrm{d}^2x\sqrt{-g}\left(Y(\Phi)R(g) + \frac{1}{2}g^{ab}\partial_a\Phi\partial_b\Phi + V(\Phi)\right)$$

$$S_m[g,\Phi,f] = -\int \mathrm{d}^2x\sqrt{-g}\,W(\Phi)g^{ab}\partial_a f\partial_b f$$

where $Y(\Phi)$, $V(\Phi)$ and $W(\Phi)$ are model dependent functions of the dilaton field.

Cartan variables in 2D

- We start by writing the metric in terms of diads

$$g_{ab} = \eta_{IJ} e^{I}{}_{a} e^{J}{}_{b}$$

where η_{IJ} is the Minkowski metric, $e^{I}{}_{a}$ are the diads and $I, J = \{0, 1\}$ are the internal Lorentz indices while a, b are the abstract (spacetime) ones.

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- In 2D the spin connection only has two components, $\omega_a{}^{IJ} = \omega_a \epsilon^{IJ}$
- The curvature tensor as well as the torsion tensor only have one independent component.

$$\begin{aligned} R &= 2\partial_{[a}\omega_{b]}\epsilon^{IJ} e^a{}_I e^b{}_J \\ T^I_{ab} &= 2\partial_{[a} e_{b]}{}^I + 2\epsilon^I{}_J \omega_{[a} e_{b]}{}^J \end{aligned}$$

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- We impose as an additional condition the vanishing of torsion, by adding the term $X_I \epsilon^{ab} T^I_{ab}$ to the Lagrangian density.

1+1 decomposition

- $\mathcal{M} \approx \Sigma_t \times \mathbb{R}$: Induced metric on Σ_t is $q_{ab} = g_{ab} + n_a n_b$, with n^a the unit normal to Σ_t.
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- Decompositon of diads: $e^a{}_I = E^a{}_I + n^a n_I$.
- Configuration variables are $({}^*X^I, \omega_1, \Phi, f)$ and the corresponding momenta are:

$$
\begin{aligned}
P_I =& \frac{\partial L}{\partial {}^*\dot{X}^I} = 2\sqrt{q} n_I \\
P_\omega =& \frac{\partial L}{\partial \dot{\omega}_1} = 2Y(\Phi) \\
P_\Phi =& \frac{\partial L}{\partial \dot{\Phi}} = \frac{\sqrt{q}}{N}\left(N^1 \Phi' - \dot{\Phi}\right) \\
P_f =& \frac{\partial L}{\partial \dot{f}} = -\frac{2W(\Phi)\sqrt{q}}{N}\left(N^1 f' - \dot{f}\right)
\end{aligned}
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Canonical variables

The set of canonical variables is

$$\{({}^{*}X^{I}, P_{I}),\ (\omega_{1}, P_{\omega}),\ (\Phi, P_{\Phi}),\ (f, P_{f})\}$$

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The total Hamiltonian density:

$$H = N\mathcal{H}_0 + N^1\mathcal{H}_1 + \omega_0\mathcal{G} + B\mu_1$$

where $\mathcal{H}_0$ is the Hamiltonian constraint, $\mathcal{H}_1$ is the diffeomorphism constraint, $\mathcal{G}$ is the Gauss constraint and

$$\mu_1 := P_\omega - 2Y(\Phi) \approx 0$$

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The consistency condition of μ_1 leads to a secondary constraint

$$\mu_2 \approx 0$$

There are no tertiary constraints and the multiplier B can be determined from $\dot{\mu}_2 \approx 0$.

Polar-type variables

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- Since $|P| = 2\sqrt{q} = 2E_1$, we change from (P_I, P_ω) to (E_1, E_2, η):

$$P_0 = 2E_1 \cosh\eta, \quad P_1 = 2E_1 \sinh\eta, \quad P_\omega = E_2$$

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(Bojowald and Swiderski, 2006; Bojowald and Reyes, 2009; Gambini, Pullin and Rastgoo, 2010; Corichi, Karami, Rastgoo and T.V.,2016)

Second class procedure

- Constraints (μ_1, μ_2) form a second class system, such that

$$\mu_1 = 0 \Rightarrow E_2 = 2Y(\Phi)$$
$$\mu_2 = 0 \Rightarrow P_\Phi = -K_1 E_1 Z(E_2)$$

with $Z(E_2) = \frac{\mathrm{d}}{\mathrm{d}E_2} Y^{-1}\left(\frac{E_2}{2}\right)$.

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- The corresponding Dirac brackets are not in canonical form,

$$\{K_2(x), K_1(y)\}_D = K_1 Z^2 \delta(x-y), \quad \{K_2(x), E_2(y)\}_D = -E_1 Z^2 \delta(x-y)$$

so, one introduces, $U_2 = K_2 + E_1 K_1 Z^2(E_2)$.

- The remaining independent canonical variables are: $\{(K_1, E_1), (U_2, E_2), (f, P_f)\}$.

Vacuum Hamiltonian constraint

$$\mathcal{H}_0^g = \frac{E_2''}{E_1} - \frac{E_1' E_2'}{E_1^2} - K_1 U_2 - \frac{Z^2 E_2'^2}{2E_1} + \frac{K_1^2 E_1 Z^2}{2} - E_1 V$$

- There are no terms of higher than 2nd spatial derivatives and no terms proportional to E_1'' or $E_1'^2$.
- U_2 appears in a lineal form, so that one can redefine the shift in order to eliminate it from $\mathcal{H}_0^g$.

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We rescale the shift and the lapse

$$\bar{N}^1 = N^1 + N\frac{K_1}{E_2'}, \qquad \bar{N} = N\frac{E_1}{A(E_2)E_2'}$$

with

$$A(E_2) = C_0 \exp\left\{-\int \mathrm{d}E_2\, Z^2(E_2)\right\}$$

These rescalings are well defined in the regions of the phase space where $E_2' \neq 0$, as well as $A(E_2) \neq 0$.

Abelianization of the Hamiltonian constraint

The new Hamiltonian constraint takes the following form:

$$\tilde{\mathcal{H}}_0 = \tilde{\mathcal{H}}_0^g + \tilde{\mathcal{H}}_0^m$$

$$\tilde{\mathcal{H}}_0^g = \left[(E_2'^2 - K_1^2)A - 2\int AV\,\mathrm{d}E_2\right]'$$

$$\tilde{\mathcal{H}}_0^m = A(WE_2'f'^2 + \frac{P_f^2 E_2'}{4W} - \frac{f'P_f K_1}{E_1})$$

such that

$$\{\tilde{H}_0(N), \tilde{H}_0(M)\} = 0$$

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The diffeomorphism constraint has the standard form

$$\mathcal{H}_1 = E_1 K_1' - U_2 E_2' + f' P_f$$

Details of the Abelianization

The rescaling of the lapse and shift functions corresponds to the redefining of the Hamiltonian constraint. The new one is given by

$$\bar{\mathcal{H}}_0 := \frac{A(E_2)}{E_1}\left(E_2'\mathcal{H}_0 - K_1\mathcal{H}_1\right)$$

On the other hand $\bar{\mathcal{H}}_0 = \bar{\mathcal{H}}_0^g + \bar{\mathcal{H}}_0^m$, where $\bar{\mathcal{H}}_0^g$ is the pure gravitational-dilaton part and $\bar{\mathcal{H}}_0^m$ is the scalar field contribution. We have

$$\{\bar{H}^g(N), \bar{H}^g(M)\} = 0$$

but the matter part of the new constraint does not become Abelian. Nevertheless, the whole smeared Hamiltonian constraint is Abelian

$$\{\bar{H}(N), \bar{H}(M)\} = 0$$

Sideremark: Abelianization in modified theories

A recent study of modified 3+1 SS theories, where the curvature terms in the Hamiltonian constraint are modified by bounded functions, shows that

- These modifications preserve the Abelian nature of a new vacuum Hamiltonian constraint. The modificaction leads to a consistent gauge theory, that, in the limit when the deformation vanishes, reproduces the standard hypersurface deformation algebra.
- The Hamiltonian constraint coupled to a scalar field cannot be made Abelian. In this case, due to holonomy corrections, the Poisson brackets of the constraints do not close.
- These results can be extended to a more general class of 2D deformed dilaton-gravity theories.

(Bojowald, Brahma and Reyes, 2016; Bojowald and Brahma, 2017)

3+1 SS vacuum gravity

The spacetime metric is

$$ds^2 = -(Ndt)^2 + \frac{(E^\varphi)^2}{|E^x|}(dx + N_r dt)^2 + |E^x| d\Omega^2$$

where $E^x(x)$ and $E^\varphi(x)$ are the independent components of a densitized triad reduced to spherical symmetry. E^x is a scalar, while E^φ is a scalar density of weight 1.

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The triads are canonically conjugate to components of extrinsic curvature

$$\{K_x(x), E^x(y)\} = G\delta(x-y), \qquad \{K_\varphi(x), E^\varphi(y)\} = \frac{1}{2}G\delta(x-y)$$

After the Abelianization and integration by parts the smeared Hamiltonian constraint is

$$H_0^g(\tilde{N}) = -\int dx \tilde{N}(-\sqrt{|E^x|}(1 + K_\varphi^2 - \Gamma_\varphi^2) + 2GM)$$

with $\tilde{N} = N'$ and $\Gamma_\varphi = -(E^x)'/2E^\varphi$.

3+1 SS vacuum gravity viewed as a DG theory

On the other hand

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu + \Phi^2 d\Omega^2$$

with $\mu, \nu = 0, 1$, and the EH action can be written as

$$S_{\text{spher}} = \frac{1}{G} \int d^2x \sqrt{-|g|} \left(\frac{1}{4} \Phi^2 R + \frac{1}{2} g^{ab} \partial_a \Phi \partial_b \Phi + \frac{1}{2} \right)$$

where R is a scalar curvature of a 2D spacetime.

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In order to construct the Hamiltonian in polar-type variables we follow the general procedure and identify

$$\begin{aligned} P_\omega =& E^x, \\ P_0 =& \frac{2E^\varphi}{(E^x)^{\frac{1}{4}}} \cosh \eta, \qquad P_1 = \frac{2E^\varphi}{(E^x)^{\frac{1}{4}}} \sinh \eta. \end{aligned}$$

CGHS vacuum model

CGHS model is a special case of the general 2D dilaton-gravity model:

$$Y(\Phi) = \frac{1}{8}\Phi^2, \quad V(\Phi) = \frac{1}{2}\Phi^2\lambda^2$$

with λ the cosmological constant.

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with λ the cosmological constant.
After the identification: $E_1 := E^\varphi$ and $E_2 := E^x$ (and some additional redefinition of the lapse), the new Hamiltonian constraint can be written as:

$$\mathcal{H} = E^x\left[4\left(E^x\right)^2 E^\varphi\lambda^2 + K_\varphi^2 E^\varphi - 2\lambda G_2 M E^\varphi E^x - \frac{\left(E^{x\prime}\right)^2}{E^\varphi}\right]$$

where M is the ADM mass of the CGHS black hole and G_2 is the dimensionless Newton's constant in 2D spacetimes.

Relation between volume and singularity

Classically the volume of a region R in a spatial hypersurface Σ,

$$V(\mathrm{R}) = \int_{\mathrm{R}} dx \sqrt{\det(q)}$$

So, if at some region $\det(q) = 0 \quad \Rightarrow \quad V(\mathrm{R}) = 0$.

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So, if at some region $\det(q) = 0 \quad \Rightarrow \quad V(\mathrm{R}) = 0$.
If $\det(q) = q_{11} = E^{\varphi} = 0$, then for that region, a metric

$$g_{\mu\nu} = \begin{pmatrix} -N^2 & 0 \\ 0 & 0 \end{pmatrix}$$

The Riemann invariants of this metric (in that region) diverge. So, in 2D, a vanishing volume in a region means existence of singularity in that region.

The kinematical Hilbert space: 3+1 SS and CGHS

The kinematical Hilbert space is the direct product

$$H_{\mathrm{kin}} = H^M_{\mathrm{kin}} \otimes \left(\bigoplus_g H^g_{\text{kin-spin}} \right)$$

where $H^M_{\mathrm{kin}} = L^2(\mathbb{R}, dM)$, is associated to the global degree of freedom of the mass of the black hole M and $H^g_{\text{kin-spin}}$ corresponds to a given graph (spin network) g.

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The states in $H^g_{\rm kin\text{-}spin}$ are

$$\langle U_x, K_\varphi | g, \vec{k}, \vec{\mu} \rangle = \prod_{e_j \in g} \exp\left(\frac{i}{2} k_j \int_{e_j} dx\, U_x(x) \right) \prod_{v_j \in g} \exp\left(\frac{i}{2} \mu_j K_\varphi(v_j) \right).$$

Here e_j are the edges of the graph, v_j are its vertices, $k_j \in \mathbb{Z}$ is the edge color, and $\mu_j \in \mathbb{R}$ is the vertex color.

(Gambini, Olmedo and Pullin, 2014; Corichi, Olmedo and Rastgoo, 2016)

Representation of operators: 3+1 SS and CGHS

The mass and the momenta act as multiplicative operators:

$$\hat{M}|g, \vec{k}, \vec{\mu}, M\rangle = M|g, \vec{k}, \vec{\mu}, M\rangle$$

$$\widehat{E^{\varphi}}(x)|g, \vec{k}, \vec{\mu}, M\rangle = \hbar G_2 \sum_{v_j \in g} \delta(x - x_j)\mu_j |g, \vec{k}, \vec{\mu}, M\rangle$$

$$\widehat{E^{x}}(x)|g, \vec{k}, \vec{\mu}, M\rangle = \hbar G_2 k_j |g, \vec{k}, \vec{\mu}, M\rangle$$

where k_j is either the color of the edge if $x \in e_j$, or if x is a vertex, the color of the edge to the right of the vertex and x_j is the position of the vertex v_j.

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Representation of point holonomies of lenght ρ is:

$$\widehat{N^{\varphi}_{\pm n\rho}}(x)|g,\vec{k},\vec{\mu},M\rangle = |g,\vec{k},\vec{\mu}'_{\pm n\rho},M\rangle, \quad n \in \mathbb{N}$$

where $\vec{\mu}'_{\pm n\rho}$ either has the same components as $\vec{\mu}$ up to $\mu_j \to \mu_j \pm n\rho$, if $x = x_j$, or it will be $\vec{\mu}$ but with a new component $\pm n\rho$, i.e. will be $\{\ldots,\mu_j,\pm n\rho,\mu_{j+1},\ldots\}$, if $x_j < x < x_{j+1}$.

Hamiltonian constraint: 3+1 SS and CGHS

The action of the Hamiltonian constraint

$$\hat{\mathcal{H}}(N)|g,\vec{k},\vec{\mu},M\rangle = \sum_{v_j \in g} N(x_j)\hat{C}_j|g,\vec{k},\vec{\mu},M\rangle$$

where $\hat{C}_j$ are difference operators (whose form is different in 3+1 SS and CGHS cases).

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- The restriction of $\hat{\mathcal{H}}(N)$ to any vertex v_j acts as a difference operator mixing the colors μ_j, it relates states which have μ_j's that belong to a semi-lattices of step 4ρ.

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- The restriction of $\hat{\mathcal{H}}(N)$ to any vertex v_j acts as a difference operator mixing the colors μ_j, it relates states which have μ_j's that belong to a semi-lattices of step 4ρ.
- Starting from a state for which none of μ_j's are zero, the result of the action of the constraint never reach a state with any of μ_j's being zero.

Singularity resolution in quantum theory: CGHS

The quantum volume operator of the CGHS satisfies

$$\hat{\mathcal{V}}|g, \vec{k}, \vec{\mu}, M\rangle \propto \sum_{v_j \in g} |\mu_j| |g, \vec{k}, \vec{\mu}, M\rangle$$

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- In the classical theory, the geometry possesses a singularity whenever the determinant of the metric q vanishes at some point.
- In the quantum theory there is an analogous situation, for instance if a graph g has $\mu_j = 0$ at one or some given vertices.
- In the quantum theory there is an invariant domain of the scalar constraint free of such states with vanishing μ_j. In this way, the solutions to the constraints will have support only on them, preventing the vanishing of μ_j at any vertex.

Singularity resolution in quantum theory: CGHS

The quantum volume operator of the CGHS satisfies

$$\hat{\mathcal{V}}|g, \vec{k}, \vec{\mu}, M\rangle \propto \sum_{v_j \in g} |\mu_j||g, \vec{k}, \vec{\mu}, M\rangle$$

- In the classical theory, the geometry possesses a singularity whenever the determinant of the metric q vanishes at some point.
- In the quantum theory there is an analogous situation, for instance if a graph g has $\mu_j = 0$ at one or some given vertices.
- In the quantum theory there is an invariant domain of the scalar constraint free of such states with vanishing μ_j. In this way, the solutions to the constraints will have support only on them, preventing the vanishing of μ_j at any vertex.
- The restriction to this invariant domain implies the resolution of the classical singularity. (Corichi, Olmedo and Rastgoo, 2016)

Singularity resolution in quantum theory: 3+1 SS

The quantum volume operator of the 3+1 SS satisfies

$$\hat{\mathcal{V}}|g, \vec{k}, \vec{\mu}, M\rangle \propto \sum_{v_j \in g} |\mu_j| \sqrt{k_j} |g, \vec{k}, \vec{\mu}, M\rangle$$

Singularity resolution in quantum theory: 3+1 SS

The quantum volume operator of the 3+1 SS satisfies

$$\hat{\mathcal{V}}|g,\vec{k},\vec{\mu},M\rangle \propto \sum_{v_j\in g} |\mu_j|\sqrt{k_j}|g,\vec{k},\vec{\mu},M\rangle$$

- In this case, in the classical theory, the vanishing of volumen in a region does not imply the singularity in that region.

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- In this case, in the classical theory, the vanishing of volumen in a region does not imply the singularity in that region.
- Again, there is an invariant domain of the quantum Hamiltonian constraint, formed by the states with $\mu_j > 0$ and $k_j > 0$.
- The triad components cannot vanish on the space of solutions of the Hamiltonian constraint, suggesting that the classical singularity is resolved.

(Gambini, Olmedo and Pullin, 2014)

The physical Hilbert space: 3+1 SS and CGHS

A generic solution, $\langle\Psi_g|$, to the Hamiltonian constraint satisfies

$$\langle\Psi_g|\hat{\mathcal{H}}(N)^\dagger = \sum_{v_j\in g}\langle\Psi_g|N_j\hat{C}_j = 0$$

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The states in the Hilbert space should be invariant under the Hamiltonian and diffeomorphism constraints and in order to construct them one applies group averaging technique.

- The states averaged by the group associated to the Hamiltonian constraint are

$$\langle\Psi_g^{\mathcal{H}}| = \int_{-\infty}^{\infty}\prod_{n=1}^{V} d\mathbf{g}_n\langle\Psi_g|$$

  with $\mathbf{g_n} = \exp\left(i\hat{\mathcal{H}}(N_n)\right)$.
- A physical, diffeomorphism invariant state, is defined as

$$\langle\Psi^{\mathrm{phys}}| = \sum_{g\in[g]}\langle\Psi_g^{\mathcal{H}}|$$

Conclusions and outlook

- A new set of variables can be introduced for a wide class of 2D dilaton-gravity models coupled to a scalar field, that make them suitable for loop quantization.
- Redefinition of the Hamiltonian constraint makes it Abelian in the general model with scalar field.
- Based on this classical reformulation, the quantization of the vacuum 3+1 SS and CGHS models has been performed suggesting the singularity resolution in the corresponding quantum theories.

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There are still some open questions, as, for example,

- Relation between solutions of the constraints and spin foams.
- Study of the classical limit.
- Reduced phase space quantization with a true Hamiltonian.

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