Strings and Loops:

What can they learn from each other?

Herman Verlinde
Princeton University

Loops17, Warsaw
July 5, 2017

S. Jackson, L. McGough, HV, NPB 901 (2015) 382, arXix:1412.5205
T. G. Mertens, G. J. Turiaci, HV, arXiv:1705.08408

Our de Sitter Universe

Does it have a dual holographic description?

$S_{visible} \simeq 10^{90}$

Atoms
Photons
Neutrinos
Gravitons

$S_{dark} \simeq 10^{123}$

black holes
dark energy
S = ¼ x Horizon Area

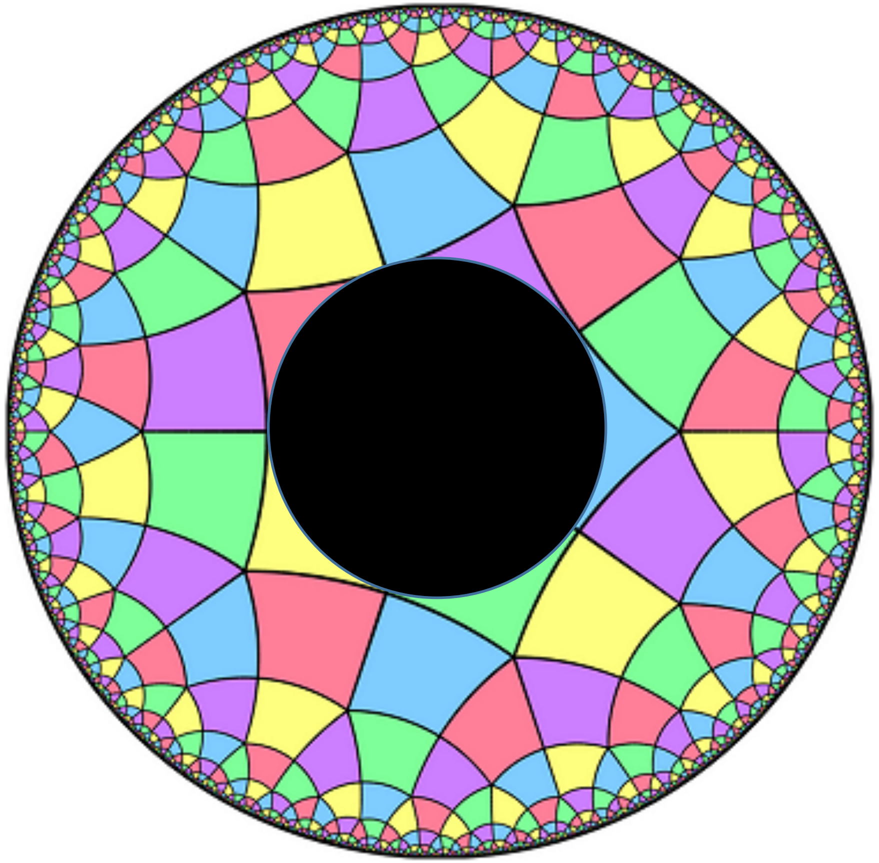
Most of the stuff in our universe is invisible!

AdS/CFT

Gauge theory/gravity correspondence

Bulk

Black Hole

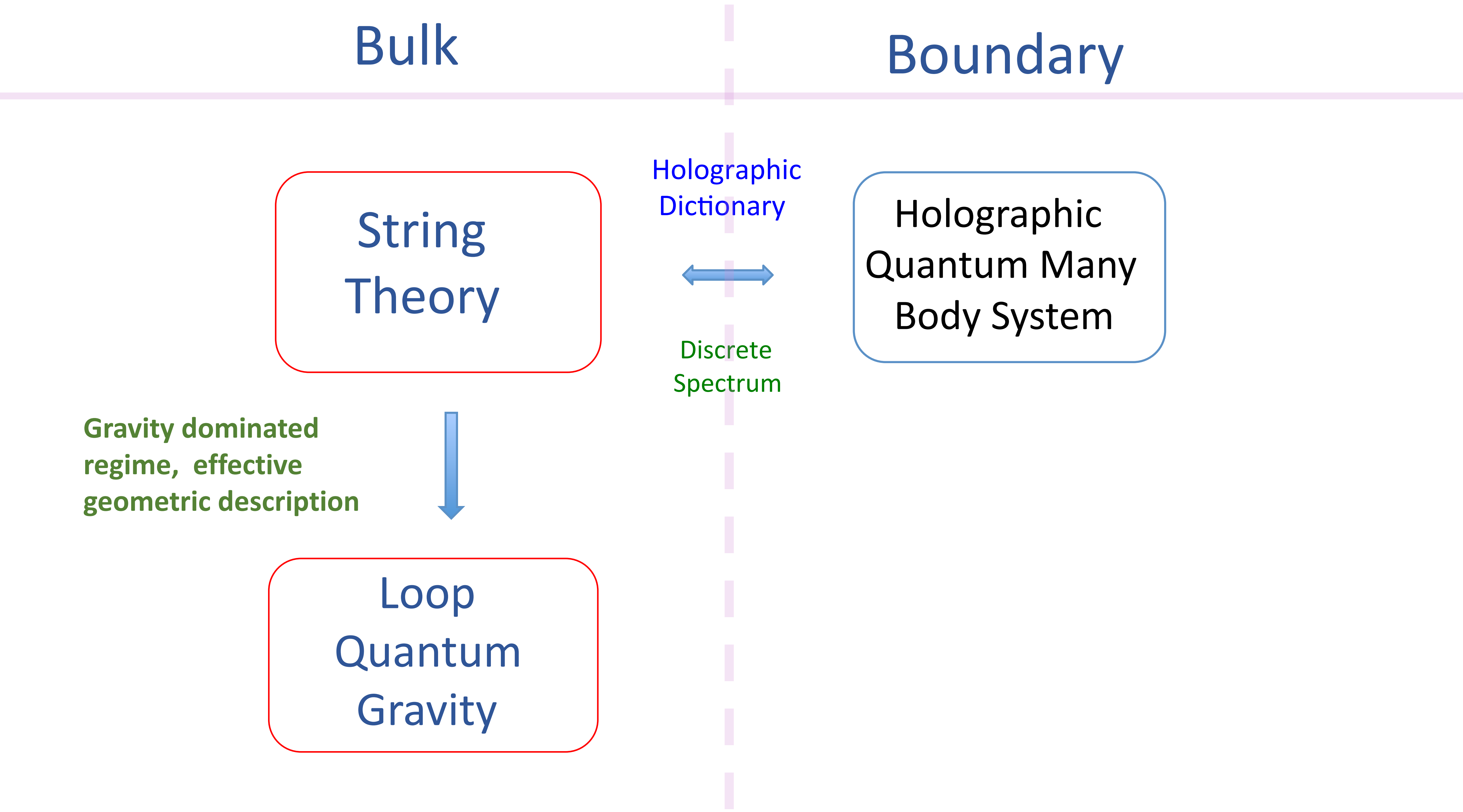


Boundary

Finite Temperature

How do we extract local bulk physics from the CFT?

Bulk		Boundary
String Theory	Holographic Dictionary ⟷ Discrete Spectrum	Holographic Quantum Many Body System


Bulk
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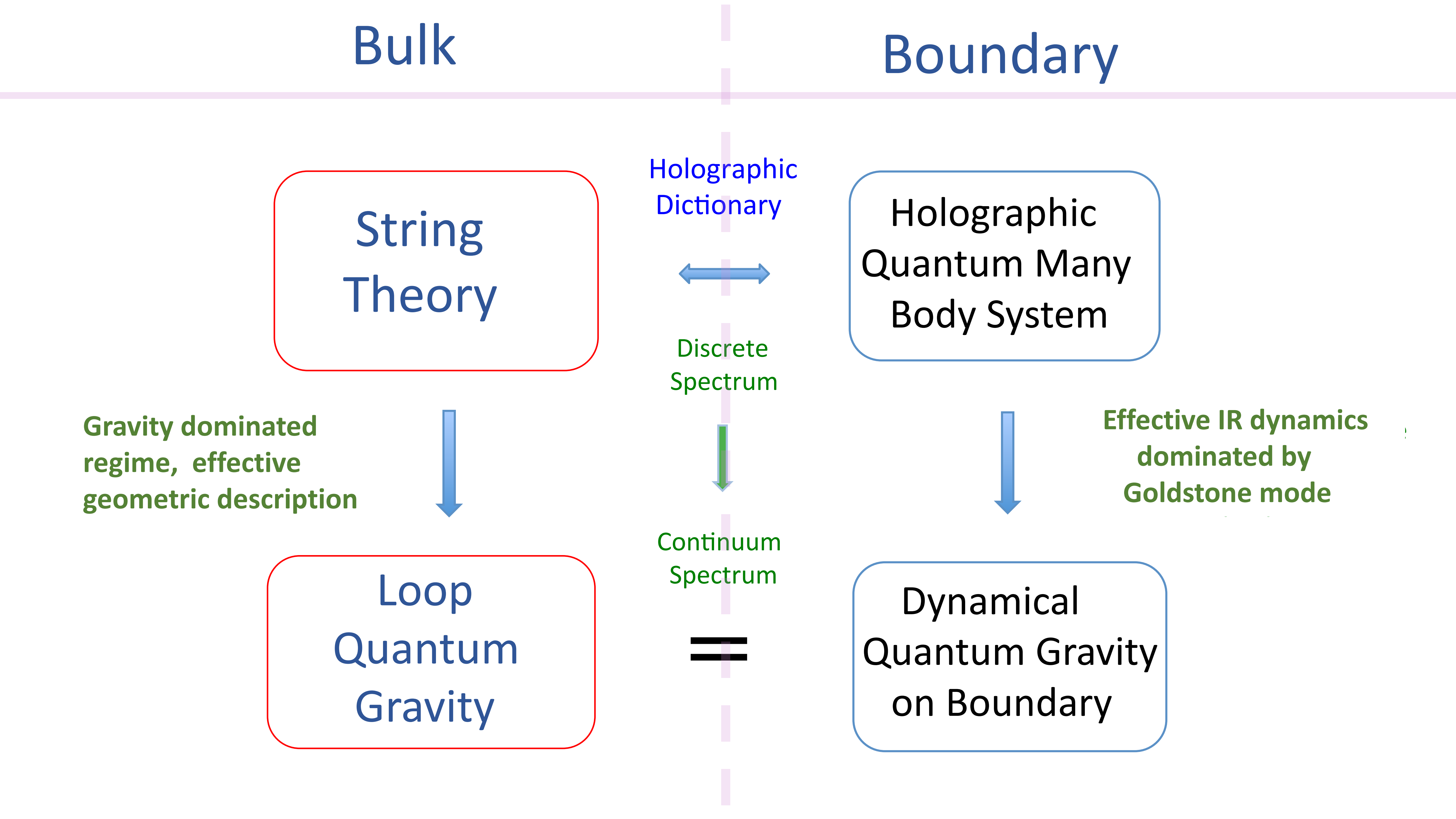
Gravity dominated regime, effective geometric description
Loop Quantum Gravity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Gravity dominated regime, effective geometric description
Effective IR dynamics dominated by Goldstone mode
Continuum Spectrum
Loop Quantum Gravity
=
Dynamical Quantum Gravity on Boundary

This paradigm is motivated by low dimensional examples

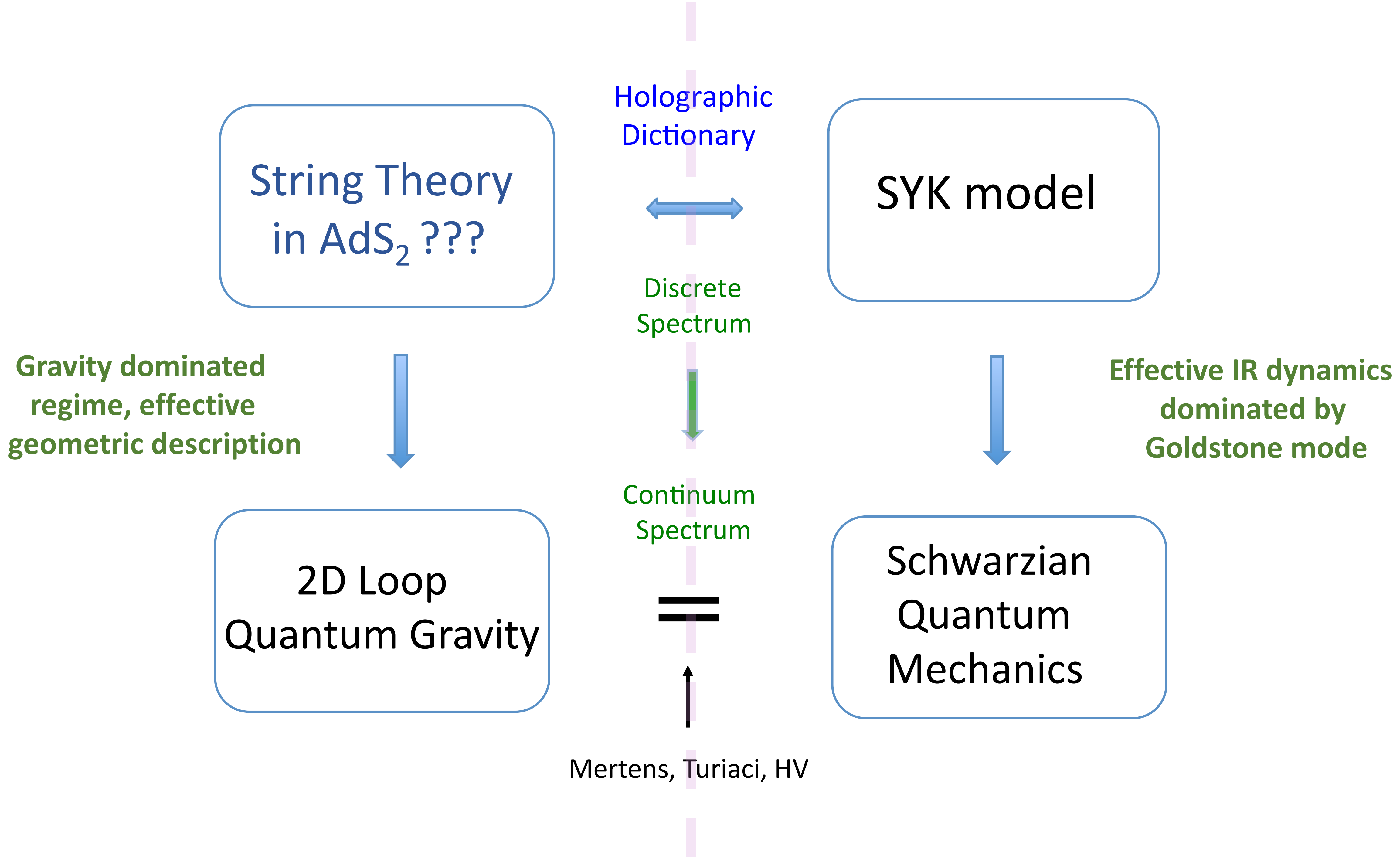
2D dilaton gravity ←→ SYK model

$$S_{2D} = \int d^2x \sqrt{-g}\, \Phi(R+\Lambda) \; + \; S_{\rm matter}$$

3D AdS gravity ←→ 2D CFT

$$S_{3D} = \int d^3x \sqrt{-g}\, (R+\Lambda) \; + \; S_{\rm matter}$$

Both gravity models are exactly quantizable as LQG theories!


String Theory
in AdS_2 ???
Holographic
Dictionary
SYK model
Discrete
Spectrum
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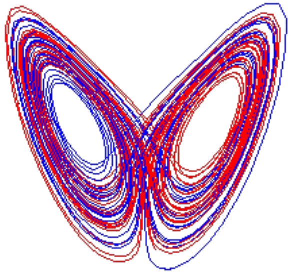
Continuum
Spectrum
2D Loop
Quantum Gravity
=
Schwarzian
Quantum
Mechanics
Mertens, Turiaci, HV

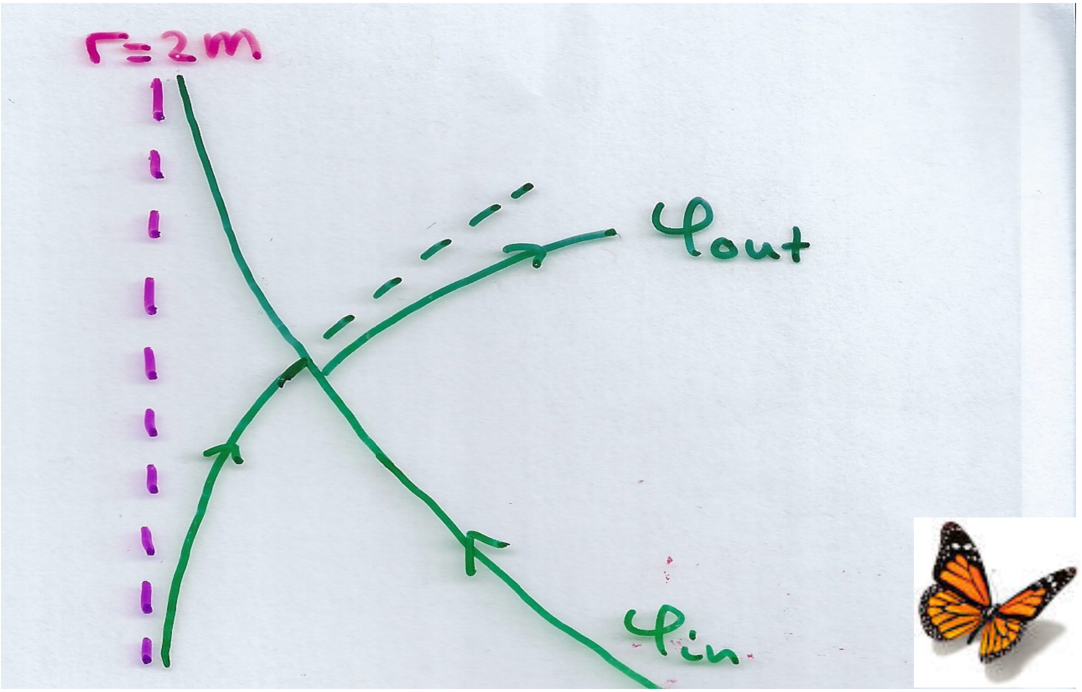
Holography relates shockwave interaction to butterfly effect

$$\lambda = 2\pi/\beta.$$



Lyapunov behavior

Quantum Chaos



Holography relates shockwave interaction to butterfly effect

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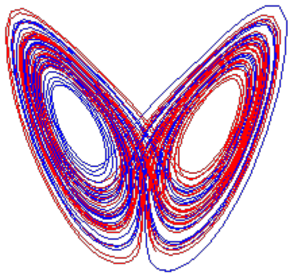

Lyapunov behavior

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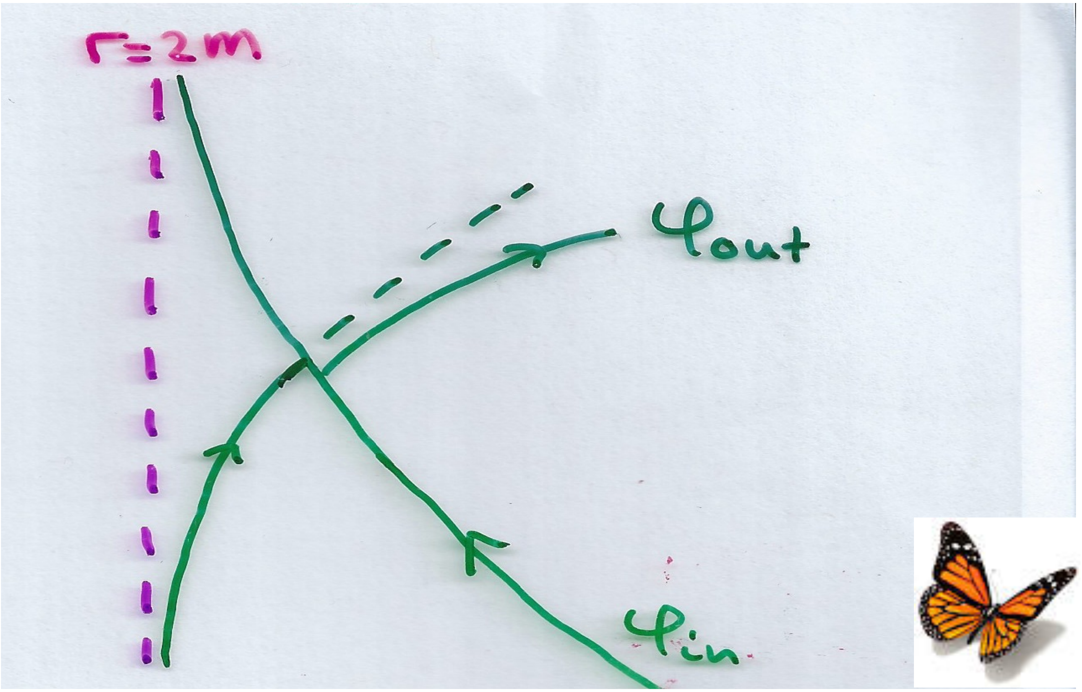


$$\phi_{in}(t_2)\phi_{out}(t_1) \;=\; e^{i\hbar e^{\lambda(t_2-t_1)}\partial_1\partial_2}\phi_{out}(t_1)\phi_{in}(t_2)$$

Exchange algebra:

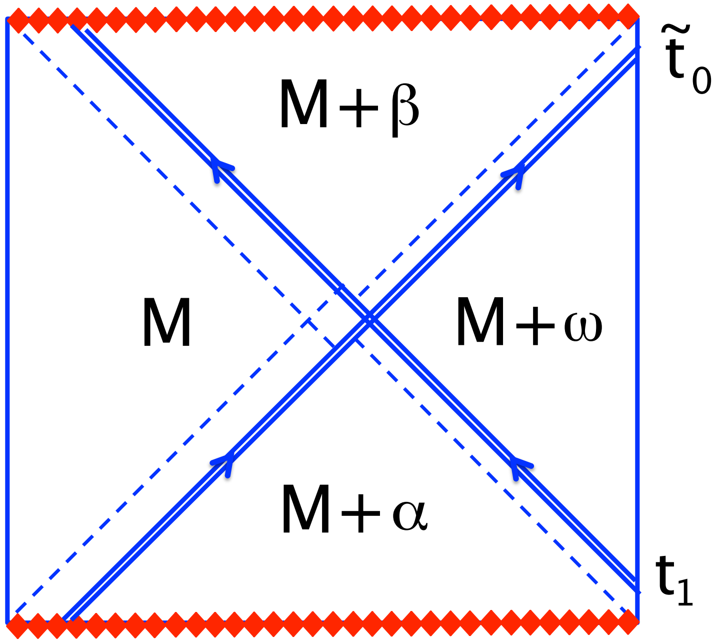
when an *in* and *out*-wave cross, each will undergo an exponentially growing displacement

$$\phi_{\omega-\alpha}(t_1)\,\phi_\alpha(t_0) \;=\; e^{\frac{i}{\hbar}S_{\alpha\beta}}\,\phi_{\omega-\beta}(\tilde{t}_0)\,\phi_\beta(\tilde{t}_1).$$

Exchange relation for localized wave-packets

→ contains the gravitational scattering amplitude

→ scattering phase determined via geometric optics



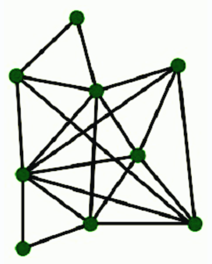
$$\mathcal{R}_{\alpha\beta} \;=\; \exp\left(\frac{i}{\hbar}\,S_{\alpha\beta}\right)$$

R-matrix

SYK model = 1D many body QM with maximal chaos

$$H = \sum_{ijk\ell} J_{ijk\ell} \psi^i \psi^j \psi^k \psi^\ell$$

random couplings



$$\{\psi^i, \psi^j\} = \delta^{ij}$$

N majorana variables

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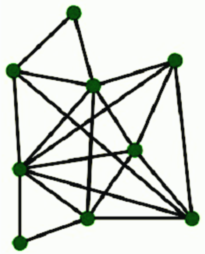

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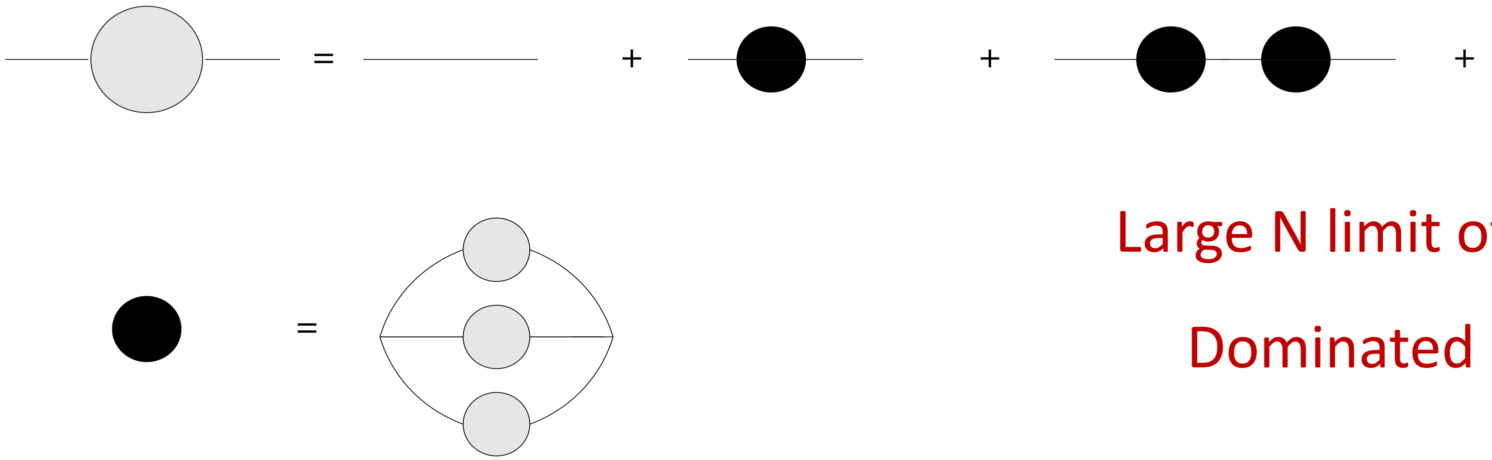

Large N limit of SD equations = soluble

Dominated by `melonic' diagrams

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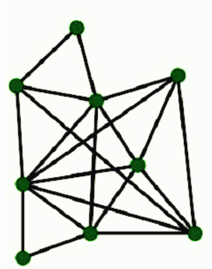

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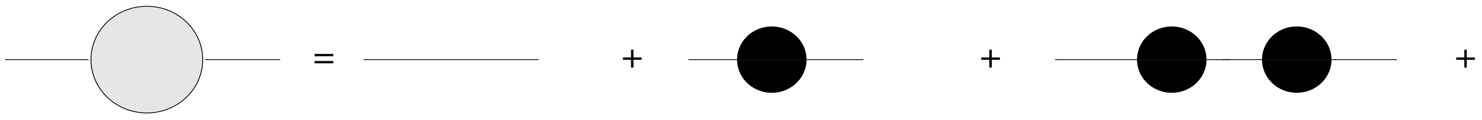
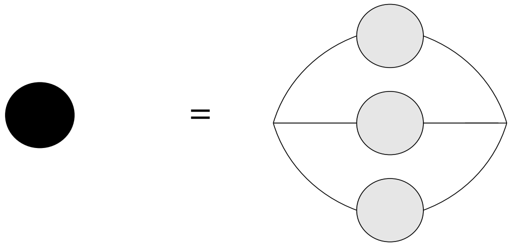




Large N limit of SD equations = soluble

Dominated by `melonic' diagrams

Relation to tensor models and group field theory

Gurau, Witten

$$H = \int \prod_{i,j} dh_{i,j}\ \psi^0_{h_{03}h_{02}h_{01}}\psi^1_{h_{10}h_{13}h_{12}}\psi^2_{h_{21}h_{20}h_{23}}\psi^3_{h_{32}h_{31}h_{30}}$$

IR limit of SD equations

$$\int d\tau' G(\tau,\tau')\Sigma(\tau',\tau'') = -\delta(\tau-\tau'') \ , \qquad \Sigma(\tau,\tau') = J^2 \left[G(\tau,\tau')\right]^3$$

are invariant under 1D diffeomorphisms

$$G(\tau,\tau') \to [f'(\tau)f'(\tau')]^{\Delta} G(f(\tau),f(\tau')) \ , \qquad \Sigma(\tau,\tau') \to [f'(\tau)f'(\tau')]^{3\Delta} \Sigma(f(\tau),f(\tau'))$$

→ IR effective theory is dominated by a dynamical

Goldstone mode = 1D reparametrizations $f(\tau)$

$$S[f] = -C\int_0^\beta d\tau \left(\{f, \tau\} + \frac{2\pi^2}{\beta^2} f'^2 \right) \qquad \{f, \tau\} = \frac{f'''}{f'} - \frac{3}{2}\left(\frac{f''}{f'}\right)^2$$

IR theory = Schwarzian theory = exactly solvable

➔ should be able to compute

$$S[f] = -C\int_0^\beta d\tau\left(\{f,\tau\} + \frac{2\pi^2}{\beta^2}f'^2\right) \qquad \{f,\tau\} = \frac{f'''}{f'} - \frac{3}{2}\left(\frac{f''}{f'}\right)^2$$

IR theory = Schwarzian theory = exactly solvable

➔ should be able to compute

$$Z(\beta) = \int_{\mathcal{M}} \mathcal{D}f\, e^{-S[f]}$$

Partition function

$$\mathcal{M} = \mathrm{Diff}(S^1)/SL(2,\mathbb{R})$$

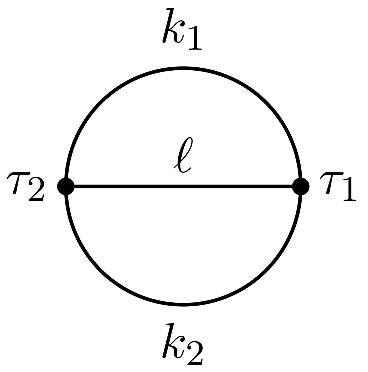
$$\langle \mathcal{O}_1 \ldots \mathcal{O}_n \rangle = \frac{1}{Z}\int_{\mathcal{M}} \mathcal{D}f\, e^{-S[f]}\, \mathcal{O}_1 \ldots \mathcal{O}_n$$

Correlation functions

$$\mathcal{O}_\ell(\tau_1,\tau_2) \equiv \left(\frac{\sqrt{f'(\tau_1)f'(\tau_2)}}{\frac{\beta}{\pi}\sin\frac{\pi}{\beta}[f(\tau_1)-f(\tau_2)]}\right)^{2\ell}$$

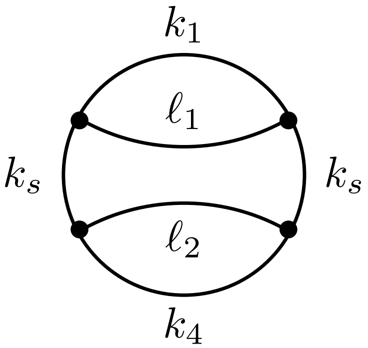
Two-point function

$$\langle \mathcal{O}_\ell(\tau_1, \tau_2) \rangle = \int \prod_{i=1}^{2} d\mu(k_i)\ \mathcal{A}_2(k_i, \ell, \tau_i).$$



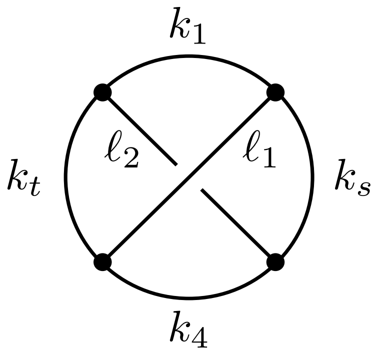
Four-point function

$$\langle \mathcal{O}_{\ell_1}(\tau_1, \tau_2)\, \mathcal{O}_{\ell_2}(\tau_3, \tau_4) \rangle = \int \prod_{i=1}^{3} d\mu(k_i)\ \mathcal{A}_4\big(k_i, \ell_i, \tau_i\big).$$

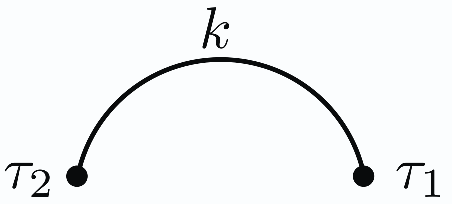


OTO four-point function

$$\langle \mathcal{O}_{\ell_1}(\tau_1, \tau_2)\, \mathcal{O}_{\ell_2}(\tau_3, \tau_4) \rangle_{\mathrm{OTO}} = \int \prod_{i=1}^{4} d\mu(k_i)\ \mathcal{A}_4^{\mathrm{OTO}}\big(k_i, \ell_i, \tau_i\big)$$

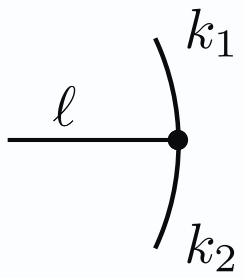


The exact non-perturbative answer for the 2n-point functions can be summarized via a simple set of diagrammatic rules:



$$= \; e^{-\,k^2\,(\tau_2-\tau_1)}$$

\`propagator’

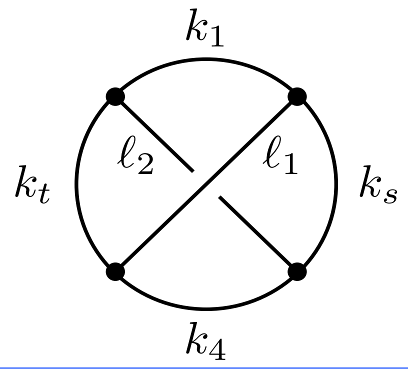


$$= \; \gamma_\ell(k_1, k_2)$$

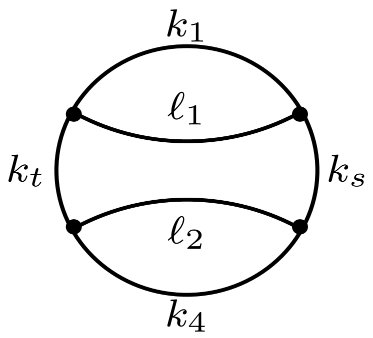
\`vertex’

$$\gamma_\ell(k_1, k_2) \; = \; \sqrt{\frac{\Gamma(\ell \pm ik_1 \pm ik_2)}{\Gamma(2\ell)}}.$$

R-matrix



$$= R_{k_s k_t} \left[\begin{smallmatrix} k_4 & \ell_2 \\ k_1 & \ell_1 \end{smallmatrix} \right]$$



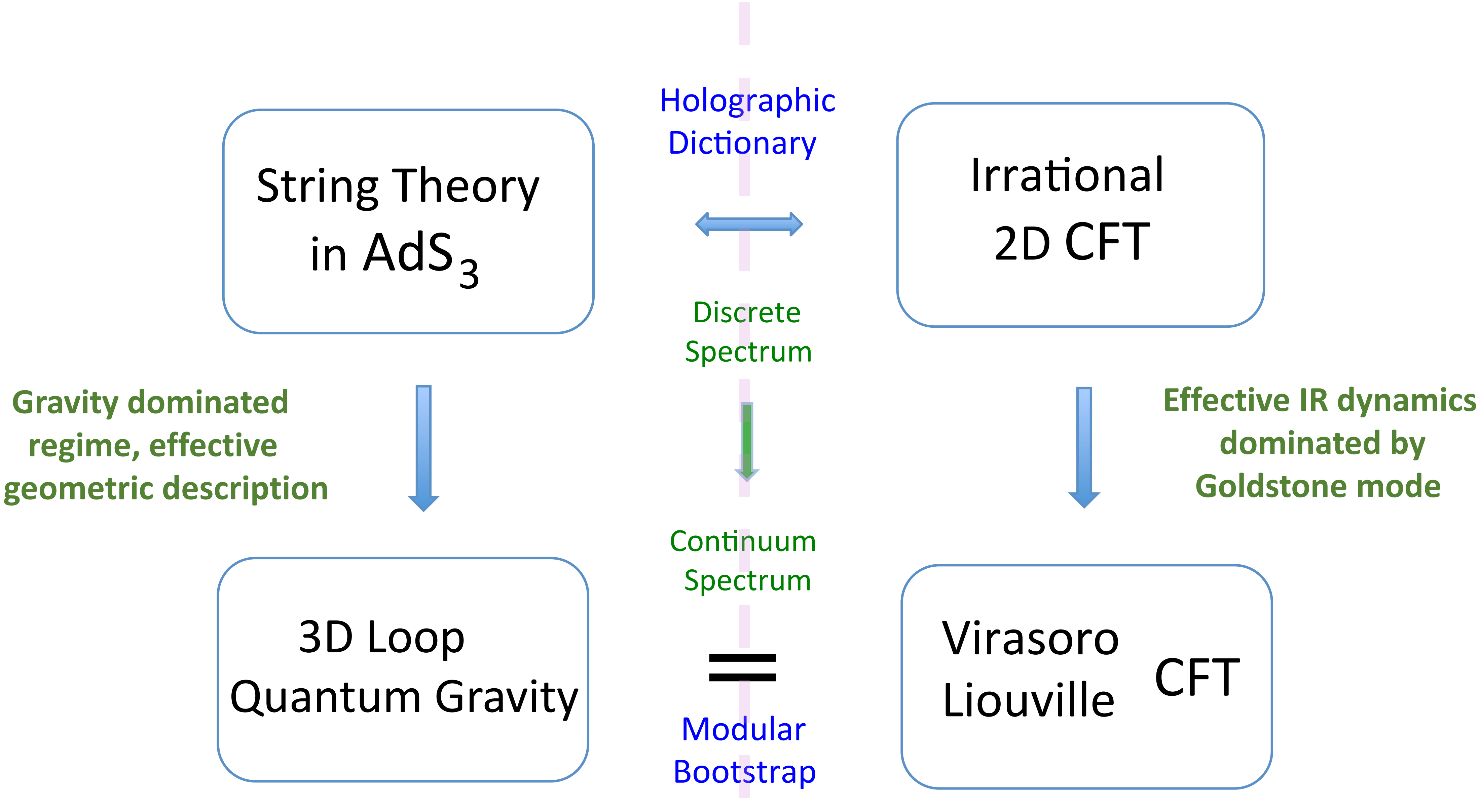
The R-matrix of the Schwarzian is found to be equal to a classical 6j-symbol of SU(1,1)

$$R_{k_s k_t} \left[\begin{smallmatrix} k_4 & \ell_2 \\ k_1 & \ell_1 \end{smallmatrix} \right] = \begin{Bmatrix} \ell_1 & k_4 & k_s \\ \ell_2 & k_1 & k_t \end{Bmatrix} = \sqrt{\Gamma(\ell_1 \pm ik_2 \pm ik_s)\Gamma(\ell_3 \pm ik_2 \pm ik_t)\Gamma(\ell_1 \pm ik_4 \pm ik_t)\Gamma(\ell_3 \pm ik_4 \pm ik_s)}$$
$$\times \mathbb{W}(k_s, k_t; \ell_1 + ik_4, \ell_1 - ik_4, \ell_3 - ik_2, \ell_3 + ik_2),$$

$\mathbb{W}$ = Wilson function

Matches with the gravitational shockwave amplitude

Groenevelt



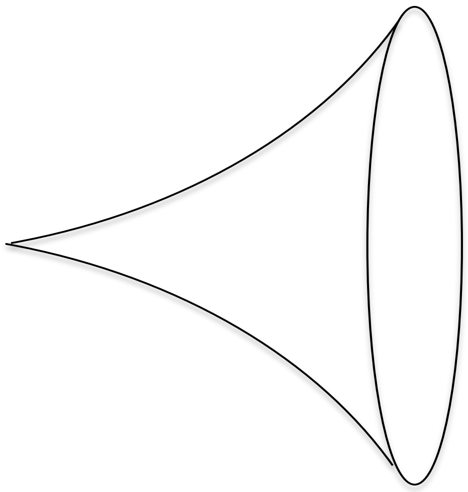
HV, NPB 337 (1990) 652

S. Jackson, L. McGough, HV, NPB 901 (2015) 382 [arXix:1412.5205]

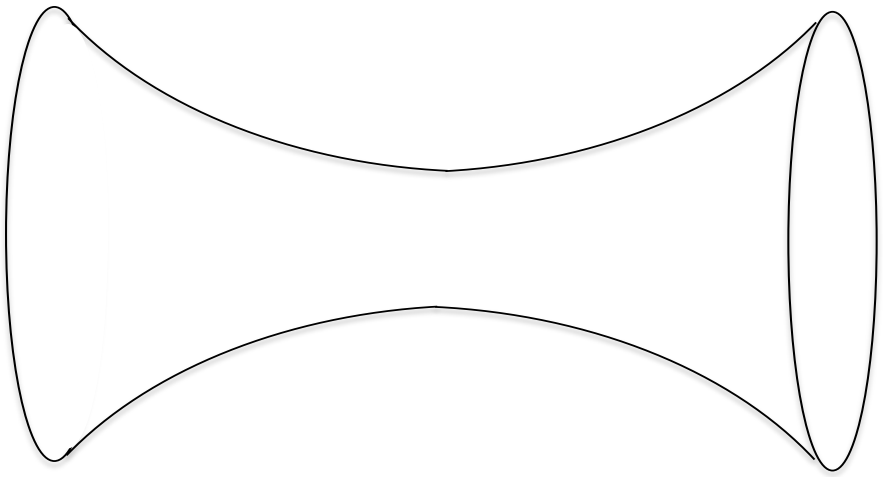
2D Virasoro CFT = 2D Quantum Hyperbolic Geometry

$$T(z) = \sum_{i=1}^{n-1} \left(\frac{\Delta_i}{(z-z_i)^2} + \frac{c_i}{z-z_i} \right)$$

Stress-energy tensor

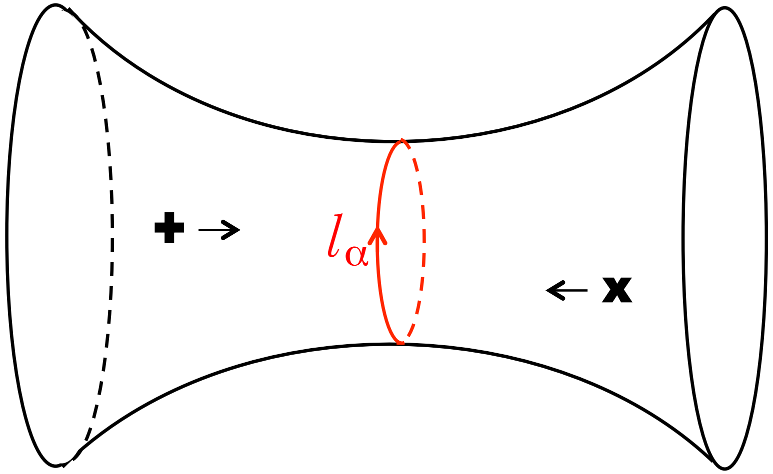


Elliptic

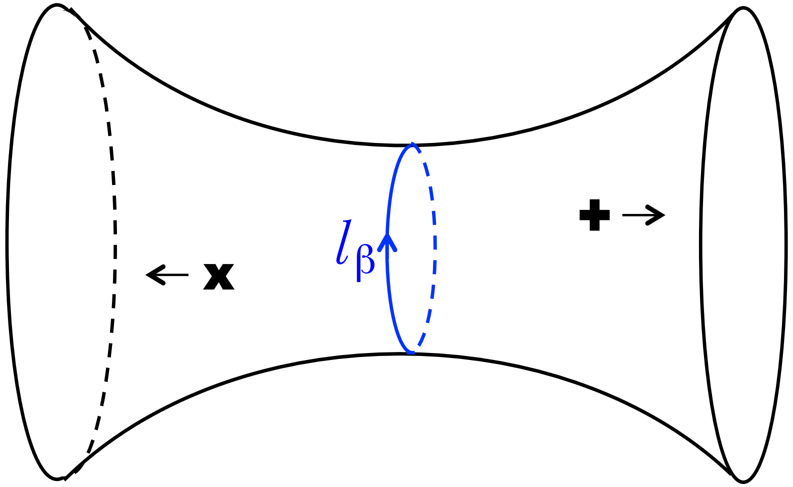


Hyperbolic

2+1-D AdS Gravity = 2D Quantum Hyperbolic Geometry



$$\hat{l}_\alpha \, |\alpha\rangle = l_\alpha |\alpha\rangle.$$

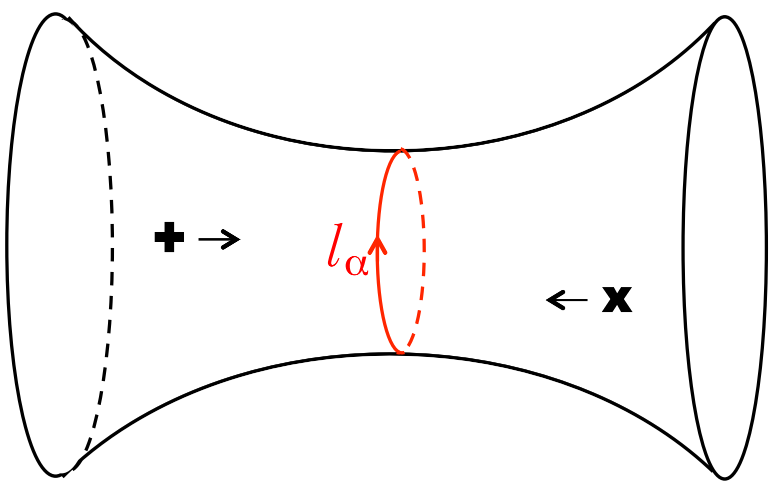


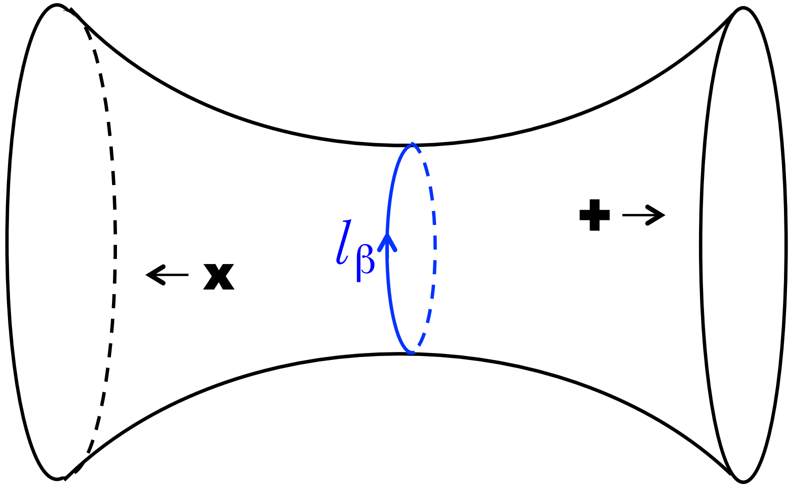
$$\hat{l}_\beta \, |\beta\rangle = l_\beta |\beta\rangle \, .$$

$$\Omega_{\mathrm{WP}} = dl_\alpha \wedge d\tau_\alpha = dl_\beta \wedge d\tau_\beta$$

$$\{l_\alpha, \tau_\alpha\}_{\mathrm{WP}} = \{l_\beta, \tau_\beta\}_{\mathrm{WP}} = 1$$

2+1-D AdS Gravity = 2D Quantum Hyperbolic Geometry





$$\hat{l}_\alpha \,|\alpha\rangle \;=\; l_\alpha |\alpha\rangle.$$

$$\hat{l}_\beta \,|\beta\rangle \;=\; l_\beta |\beta\rangle\,.$$

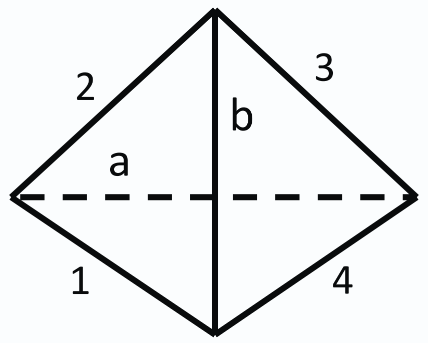
$$\mathcal{R}_{\alpha\beta} \;=\; \exp\!\left(\frac{i}{\hbar}\, S_{\alpha\beta}(l_\alpha, l_\beta)\right) \;=\; \langle\beta|\alpha\rangle$$

Scattering phase

$$\mathcal{R}_{\alpha\beta} = \exp\left(\frac{i}{\hbar} S_{\alpha\beta}(l_\alpha, l_\beta)\right).$$

$$\tau_\alpha = \frac{\partial S_{\alpha\beta}}{\partial l_\alpha}, \qquad \tau_\beta = -\frac{\partial S_{\alpha\beta}}{\partial l_\beta}.$$

= generating function of a canonical transformation

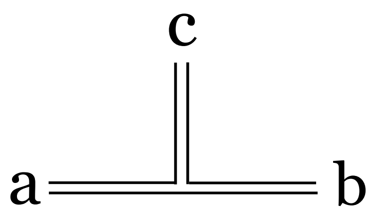


$$S_{\alpha\beta} = \mathrm{Vol}\left(T\begin{bmatrix} 1 & 2 & \alpha \\ 3 & 4 & \beta \end{bmatrix}\right)$$

Volume of a hyperbolic tetrahedron

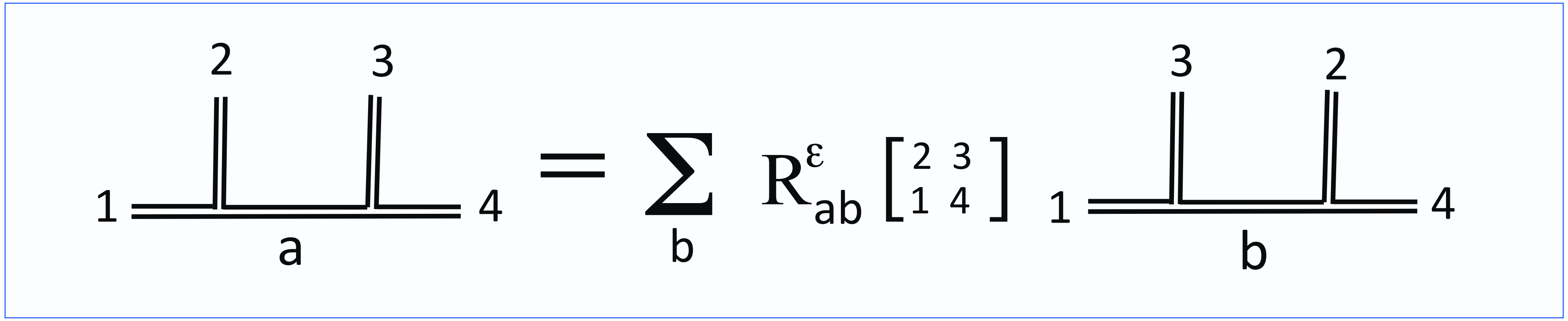
Can we derive this exchange algebra from the CFT?

$$\psi\begin{bmatrix} & c & \\ a & & b \end{bmatrix}(z) \quad =$$



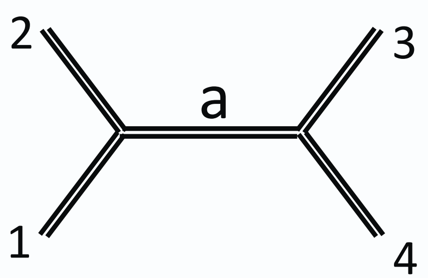
chiral vertex operators

satisfy a braid relation:

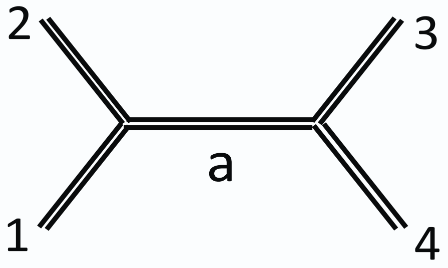


Can we derive this exchange algebra from the CFT?

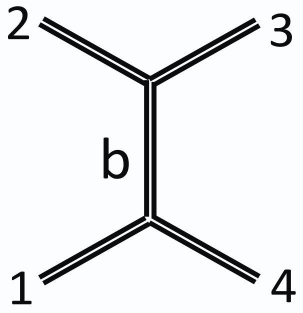
$$\left\langle \mathcal{O}_1(0)\,\mathcal{O}_2(1)\,\mathcal{O}_3(z,\bar{z})\,\mathcal{O}_4(\infty)\right\rangle = \sum_a \Big| \quad \Big|^2$$



Conformal blocks



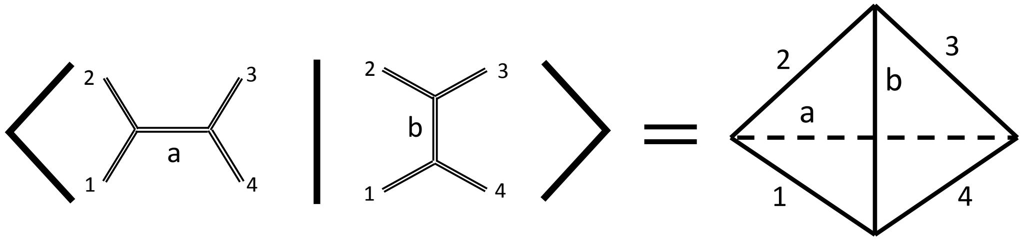
$$= \sum_b F_{ab}\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$$



F = "Fusion matrix"

Ponsot-Teschner

Fusion category of Virasoro CFT = Fusion Category of $U_q(SL_2)$

$F_{ab}\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} =$ 

quantum 6j-symbol

$$\mathcal{R}_{\alpha\beta}^{U_q(\mathfrak{sl}_2)} \simeq \begin{Bmatrix} j_1 & j_2 & j_\alpha \\ j_3 & j_4 & j_\beta \end{Bmatrix}_q \simeq \exp\left\{\frac{i}{2\pi b^2}\mathrm{Vol}(\mathrm{T})\right\}$$

Volume of a hyperbolic tetrahedron

The search for the theory on quantum gravity is guided by basic principles:

- holography: $S = \frac{1}{4}$ Area
- quantum information theory
- thermodynamics
- locality and causality
- space-time dynamics
- ....

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- holography: S = ¼ Area
- quantum information theory
- thermodynamics
- locality and causality
- space-time dynamics
- ....

Physical lesson: gravitational dynamics of event horizons ←→ quantum chaos

Some helpful tools: many body QM, CFT bootstrap, large N, tensor networks, ...