New representations for quantum gravity

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Introduction

New momentum

- Recent focus in quantum gravity on concepts which have been "neglected" in the past: entanglement, boundaries, holography, dualities, phases, vacua, renormalization, . . .
- Leads to technical tools, conceptual developments, and reaches out to other communities

This talk: dualities, phases, vacua

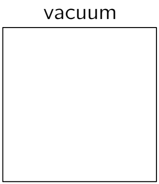
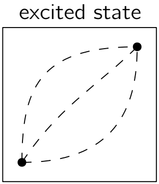
- We expect the continuum to have a **non-trivial phase diagram** (Carrozza, Dittrich)
- One can already characterize **phases at the kinematical level**
- This leads to **new representations** which are **inequivalent** to the "standard" one
- Vacua and excitations are given by **TQFTs and defects**
- In $2+1$ with $\Lambda > 0$, this unravels **properties akin to that of condensed matter systems**
- It also physically motivates the introduction of a **new basis adapted to coarse-graining**

Outline

AL representation

Vacuum and excitations (Ashtekar, Isham, Lewandowski)

- Vacuum is a degenerate state with no geometrical excitations: $X_S|\emptyset\rangle_{\text{AL}} = 0,\ \forall\, S$
- Excitation operators are holonomies

$\hat{h}_1 \hat{h}_2 \hat{h}_3 \triangleright$  $=$  $\in \mathcal{H}_\Gamma$

- Graph Hilbert space $\mathcal{H}_\Gamma = L^2\big(G^L/G^N, \mathrm{d}\mu_{\text{Haar}}\big)$
- Embedding of Hilbert spaces based on embedding of graphs with refinement $j = 0$
- Kinematical continuum limit $\mathcal{H}_\infty = \cup_\Gamma \mathcal{H}_\Gamma / \sim = L^2\big(\overline{\mathcal{A}/\mathcal{G}}, \mathrm{d}\mu_{\text{AL}}\big)$
- Smooth geometries should be described by highly excited states

Is LQG discrete or continuous?

- "Both", since we work on finite graphs but states live in $\mathcal{H}_\infty$
- Field theory with arbitrary finite # of degrees of freedom = TQFT with defects
- Classically: spin network phase space $T^*G \simeq T^*$(space of piecewise-flat connections) (Bianchi, Freidel, MG, Ziprick)

AL representation and beyond

Which TQFT and which defects for quantum gravity? (Dittrich, MG)

- AL representation: vacuum is $E = 0$ and defects carry geometry (∼ strong coupling limit (confining phase) of lattice gauge theory)
- BF representation: vacuum is $F[A] = 0$ and defects carry curvature (∼ zero coupling limit (deconfining phase) of lattice gauge theory)
- BF_Λ representation: vacuum is $F[A] = \Lambda E^2$ and defects carry curvature
- Need a Hilbert space $\mathcal{H}_\infty$ supporting arbitrarily many excitations and the generating algebra

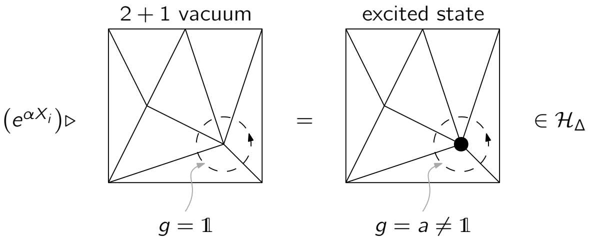
A priori obstruction (Fleischhack, Lewandowski, Okołów, Sahlmann, Thiemann)

- Uniqueness of AL representation under technical assumptions, including
  - diffeomorphism invariance
  - existence of the fluxes as operators (i.e. weak continuity of exponentiated fluxes)
- Very different from QFT, where there exist inequivalent representations
- Need to violate some assumptions in order to have a new representation

BF representation

Vacuum and excitations

- Vacuum is a discretization of $F[A] = 0$, i.e. flat connections: $\psi\{g_c\} = \prod_{\text{cycles}} \delta(g_c, \mathbb{1})$ (Gambini, Griego, Pullin, Bobienski, Lewandowski, Mroczek, Drobiński)
- Excitation operators are exponentiated fluxes: $\left(e^{\alpha X_i}\right) \triangleright \psi\{g_c\} = \psi(g_1, \dots, g_i a, \dots)$



Classical theory (Dittrich, MG)

- Uses simplicial (gauge-covariant) fluxes for $G = \mathrm{SU}(2)$ and $d = 2 + 1$ or $3 + 1$
- Requires careful study of refinement maps and cylindrical consistency of the (h, X) algebra
- Encodes interplay between curvature and torsion (Livine, Charles, Delcamp, Dittrich)

Quantum theory (Bahr, Dittrich, MG)

- Discrete Hilbert space topology to accommodate flat vacuum and inductive limit
- Fluxes don't exist as operators (cf. polymer QM, LQC, Bohr compactification)
- "Compactification of momentum space" (Rovelli, Vidotto, Riello)

BF representation

	AL	BF
TQFT state	$E = 0$	$F[A] = 0$
vacuum state	$\lvert\emptyset\rangle$ = nothing	$\lvert\emptyset\rangle = \prod_{\text{cycles}} \delta(g_c, \mathbb{1})$
excitations	holonomies $h_\ell \triangleright \lvert\emptyset\rangle = h_\ell$ dual graphs	exponentiated fluxes $\left(e^{\alpha X_i}\right) \triangleright \lvert\emptyset\rangle = \psi(g_1, \ldots, g_i\alpha, \ldots)$ codimension-1 simplices
defects	dual graphs	codimension-2 simplices
refinement	no geometry: $j_{\text{finer}} = 0$	flatness: $\delta(g_{\text{finer}}, \mathbb{1})$
measure	Haar	discrete
$\mathcal{H}_\infty$	$\cup_\Gamma \mathcal{H}_\Gamma / \sim$	$\cup_\Delta \mathcal{H}_\Delta / \sim$
generalization	$E_{\text{bgd}} \neq 0$ (Koslowski, Sahlman)	$F[A] \neq 0$ $(\Lambda \neq 0)$ (Dittrich, MG)

$BF_\wedge$ representation

1. AL representation
2. BF representation
3. $BF_\wedge$ representation
4. Further applications
5. Perspectives

BF_Λ representation

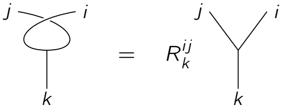
Strategy

- Adopt the viewpoint of TQFTs with defects
- In 3d Euclidean with $\Lambda > 0$, the TQFT is known and involves $SU(2)_k$ at root of unity (Reshetikhin, Turaev, Viro, Witten)
- Natural IR cutoff which replaces the Bohr compactification, and finite-dimensional $\mathcal{H}$'s
- But we need extra structure: vacuum, excitations, creation operators, i.e. extended TQFT
- Use graphical calculs, which then allows extension to 4d (Delcamp, Dittrich, Bärenz, Barrett)

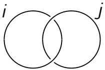
BF$_\Lambda$ representation

Graphical calculus

- Data defining $\mathrm{SU}(2)_\mathrm{k}$ at root of unity (or a certain category $\mathcal{C}$):
  - irreps: $j \in \{0, 1/2, \ldots, \mathrm{k}/2\}$ with $\mathrm{k} = \left(G\hbar\sqrt{\Lambda}\right)^{-1}$ and $v_j^2 = (-1)^{2j} \dim(j)$
  - fusion coefficients: $i \otimes j = \bigoplus_k \delta_{ijk}\, k$
  - R-matrix:

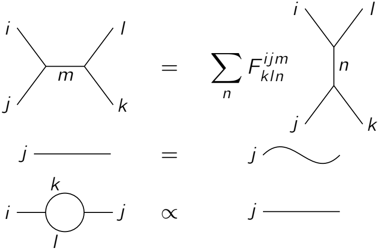


  - modular S-matrix: $S_{ij} = \dfrac{1}{\mathcal{D}}$



with $\det(S) \neq 0$ and $\mathcal{D}^2 = \sum_{j=0}^{\mathrm{k}/2} v_j^4$

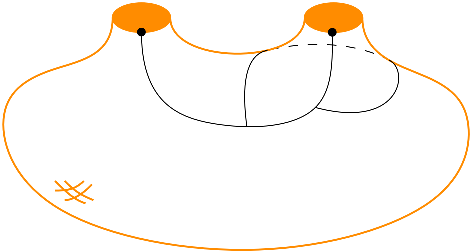
- Topological invariance encoded in local equivalence relations $\sim$



BF_Λ representation

States and vacuum

- Consider $\mathbb{S}^2$ with d defects (valency $\in \{0,1\}$) and $\mathcal{H}_d = \mathrm{span}\{\text{graphs} \ / \sim\}$ (no lattice!)
- $\dim \mathcal{H}_0 = \dim \mathcal{H}_1 = 1$, but $\dim \mathcal{H}_{d>1} > 1$
- Defects can carry excitations, i.e. generic states have
  - curvature: non-contractible loops
  - torsion: open links
- Embedding maps: add a defect in the vacuum state
- Vacuum: state with no excitations (Levin, Wen)
  - flatness constraint F: add vacuum loop (q-deformed flatness)
  - Gauss constraint G: project on gauge-invariant vertices
- Cylinder basis $\mathcal{Q}^{ij}_{rs}$
- $|\emptyset\rangle = F \triangleright G \triangleright \mathcal{Q}^{ij}_{rs}$



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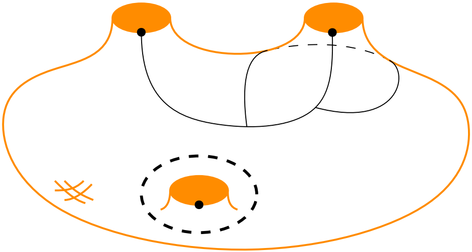

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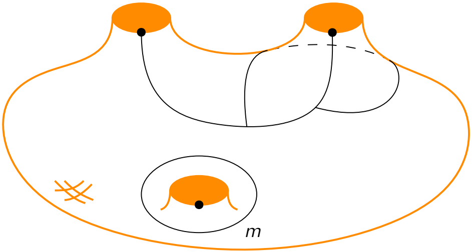
$$\frac{1}{\mathcal{D}} \sum_{m}^{k/2} v_m^2$$



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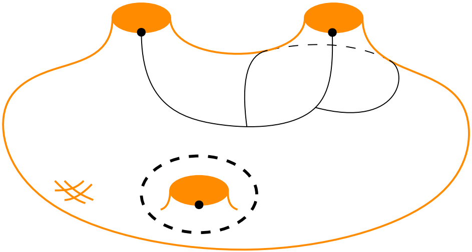

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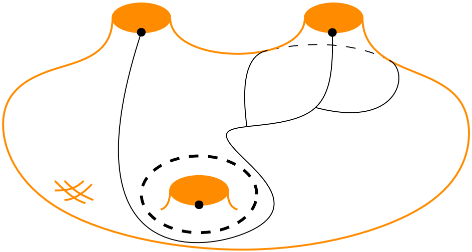

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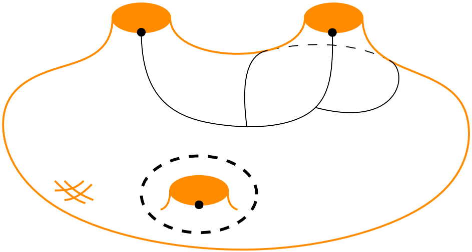

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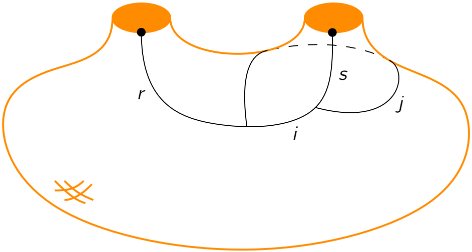

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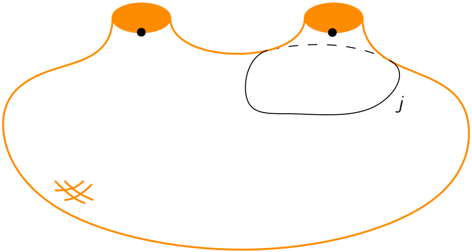
$G \triangleright \mathcal{Q}^{ij}_{rs} =$ 

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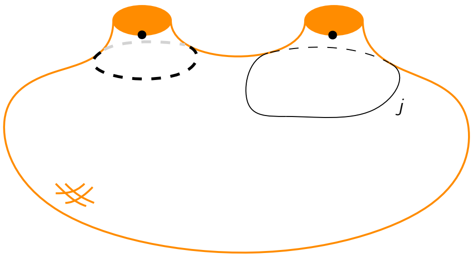

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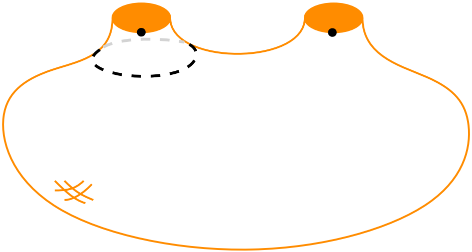
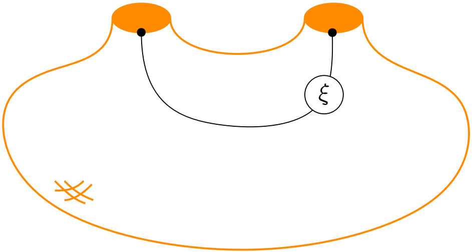


BF_Λ representation

Quasi-particle excitations

- Defects of the TQFT do not interact nor care about their relative distance
- We look for states $\xi(i,j)$ with the stability property $\mathcal{Q} \triangleright \xi = \xi$
- Quasi-particles are irreps of the tube algebra defined by $\mathcal{Q}^{ij}_{rs} \times \mathcal{Q}^{kl}_{su} = \sum_{mn} (\text{stuff}) \mathcal{Q}^{mn}_{ru}$

  (Ocneanu, Müger, Lan, Wen, Kong, Kitaev, Levin)



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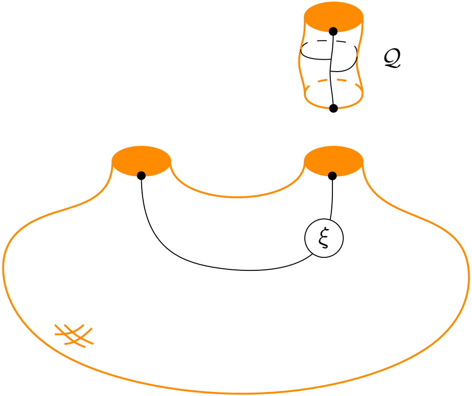

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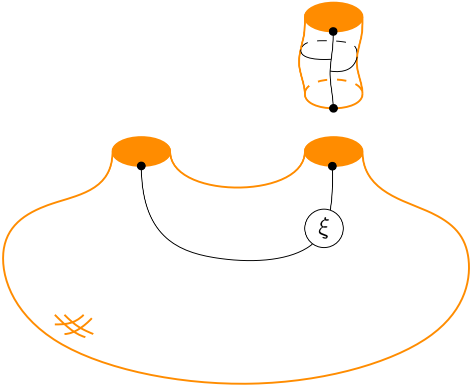

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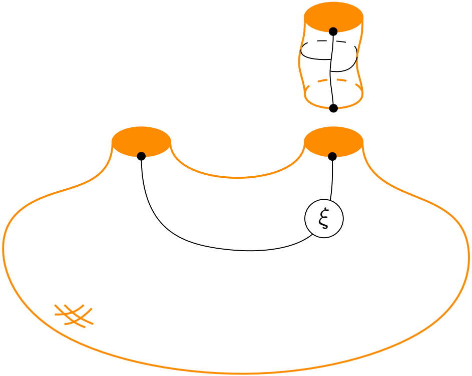

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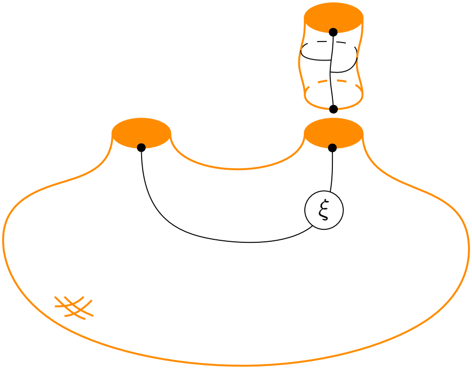

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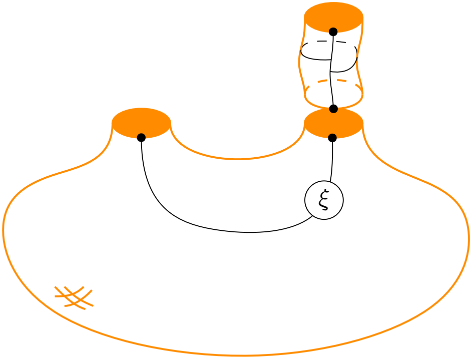

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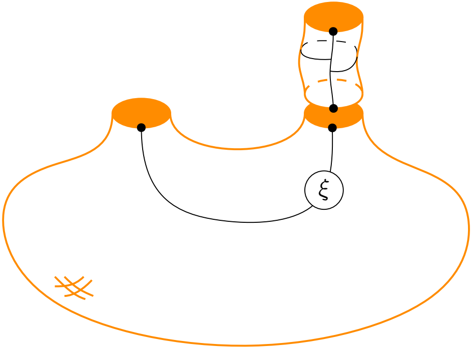

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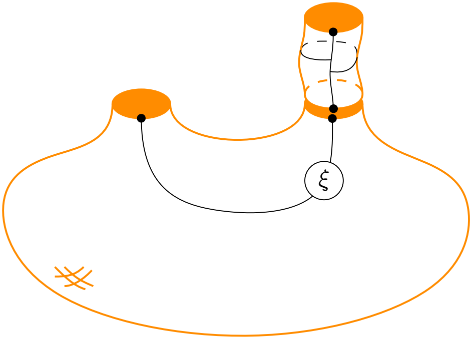

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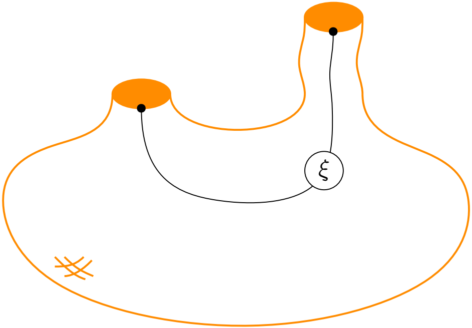
  (Ocneanu, Müger, Lan, Wen, Kong, Kitaev, Levin)



BF$_\Lambda$ representation

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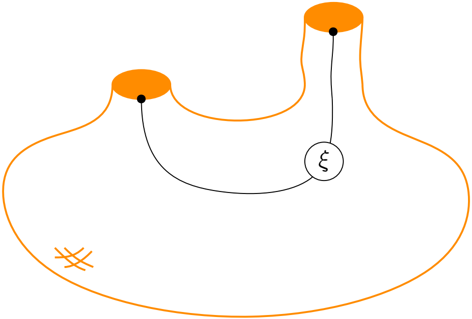

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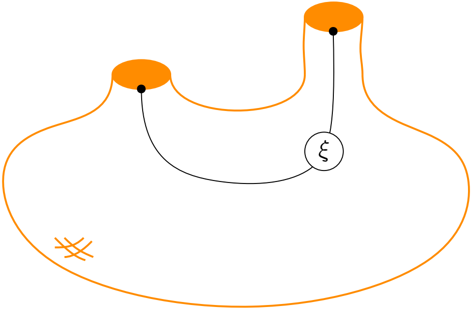

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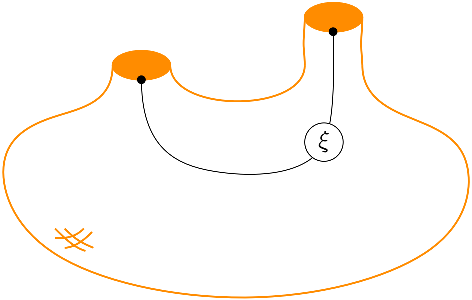

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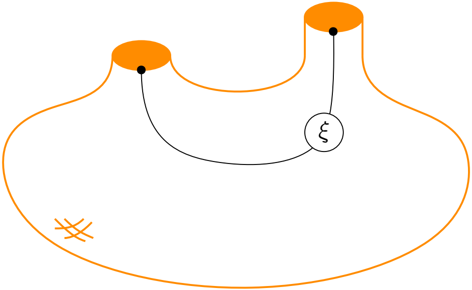

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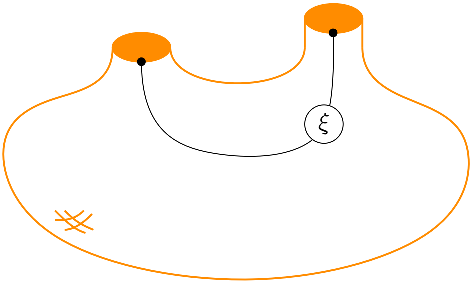

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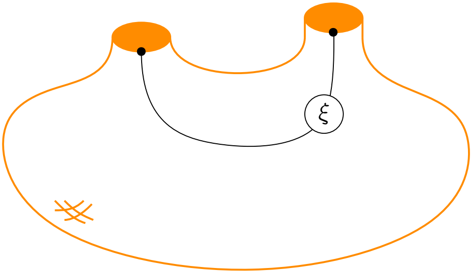

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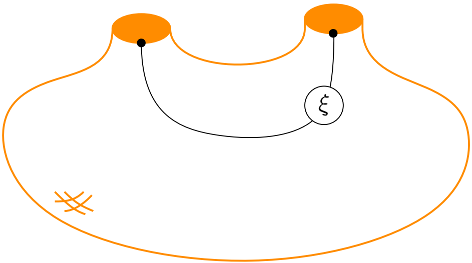

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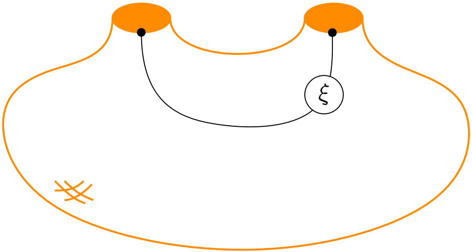
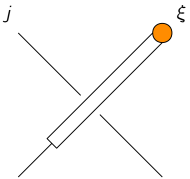


BF$_\Lambda$ representation

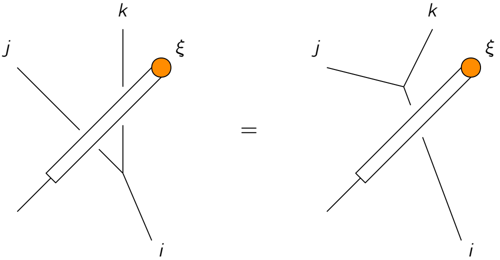
Drinfel'd center

- Such ξ's are a known linear combination of the $\mathcal{Q}$'s
- Their stability implies that they have a well-defined braiding with standard links
- The "half-braiding" tensors Ω are elements of the Drinfel'd center category $\mathcal{Z}(\mathcal{C})$
- Because $\mathcal{C} = \mathrm{SU}(2)_k$ is modular, $\mathcal{Z}(\mathcal{C}) = \mathcal{C} \boxtimes \mathcal{C}^{\mathrm{op}}$ and $\dim\big(\mathcal{Z}(\mathcal{C})\big) = (\dim \mathcal{C})^2$ (Müger)

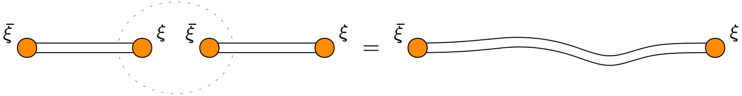
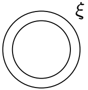
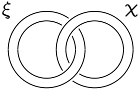
$\Omega : \xi \otimes j \to j \otimes \xi$



$\Omega = \sum RRFFF$



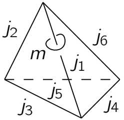
Ribbon operators

- Excitations are obtained by the action (fusion) of ribbon operators (Kitaev, Freidel, Zapata)
- Ribbons can be
  - glued 
  - closed  $= v_i^2 v_j^2$
  - linked  $= \boldsymbol{S}_{\xi\chi} = \boldsymbol{S}_{(ij)(ab)} = S_{ia}S_{jb}$
- Open ribbons $\equiv$ exponentiated fluxes
- Closed ribbons $\equiv$ generalized holonomies

BF_Λ representation

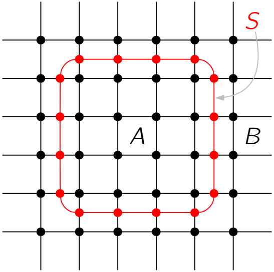
Particles and edge modes

- Drinfel'd double labels $\equiv$ mass and spin of point particles (conjugacy class and irrep for SU(2))
- Generalizes to $\Lambda > 0$ results for $\Lambda = 0$ (Bais, Meusburger, Muller, Noui, Perez, Schroers)
- Turaev–Viro state sum with massive particle insertion



$$\rightarrow \qquad \frac{S_{mj_1}}{v_{j_1}^2} F^{j_1 j_2 j_3}_{j_4 j_5 j_6}$$

- The boundary symmetry algebra of classical $2+1$ first order gravity is Kac–Moody$\big(\text{Lie}(G_{\text{CS}})\big)$ (Donnelly, Freidel, MG)
- Mathematical characterization of line defect excitations (Kitaev, Kong) $\simeq$ (Donnelly, Freidel) fusion/entangling product

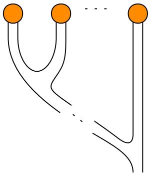


Further applications

Further applications

Thinking about the BF vacuum has lead to

- Fusion basis
  - can be thought of as spin networks for the Drinfel'd double
  - puts emphasis on the excitations instead of the lattice
  - graph independent and non-local
  - stable under coarse graining since it encodes from the onset curvature and torsion
  - useful for entanglement entropy and subsystems (Aldo Riello's talk)



- Duality between the "two halves" of the Drinfel'd double, i.e.
  - spin network basis
  - curvature basis

  by a Fourrier transform on the S-matrix (Barrett, Roche, Freidel, Noui, MG)
- Duality between spin foams and group foams
- Construction of $3+1$ TQFTs (Delcamp, Dittrich) with Turaev–Viro basis on Heegard surfaces

Perspectives

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What we have

- New kinematical vacuum and continuum Hilbert space supporting curvature excitations
- Better understanding of the canonical / spin foam / TQFT interplay
- New realization of quantum geometry with discrete and bounded spectra
- New basis adapted to coarse graining

Many exciting possibilities

- Implications for continuum limit / canonical dynamics
- Fixed points of coarse-graining flow $\rightarrow$ extended lattice TQFTs $\rightarrow$ new quantum geometries
- Condensed matter inspirations: condensation of defects, phase transitions, domain walls
- Implications for quantum cosmology and black holes
- $3+1$ fusion basis and its use for coarse graining
- $3+1$ TQFTs (e.g. Poincaré) relevant for quantum gravity (Freidel, Baratin, Barrett, Bärenz)
- Link with Hopf algebra lattice gauge theories (Meusburger, Wise)

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