Edge modes, symmetry and subsystems in Gauge theories

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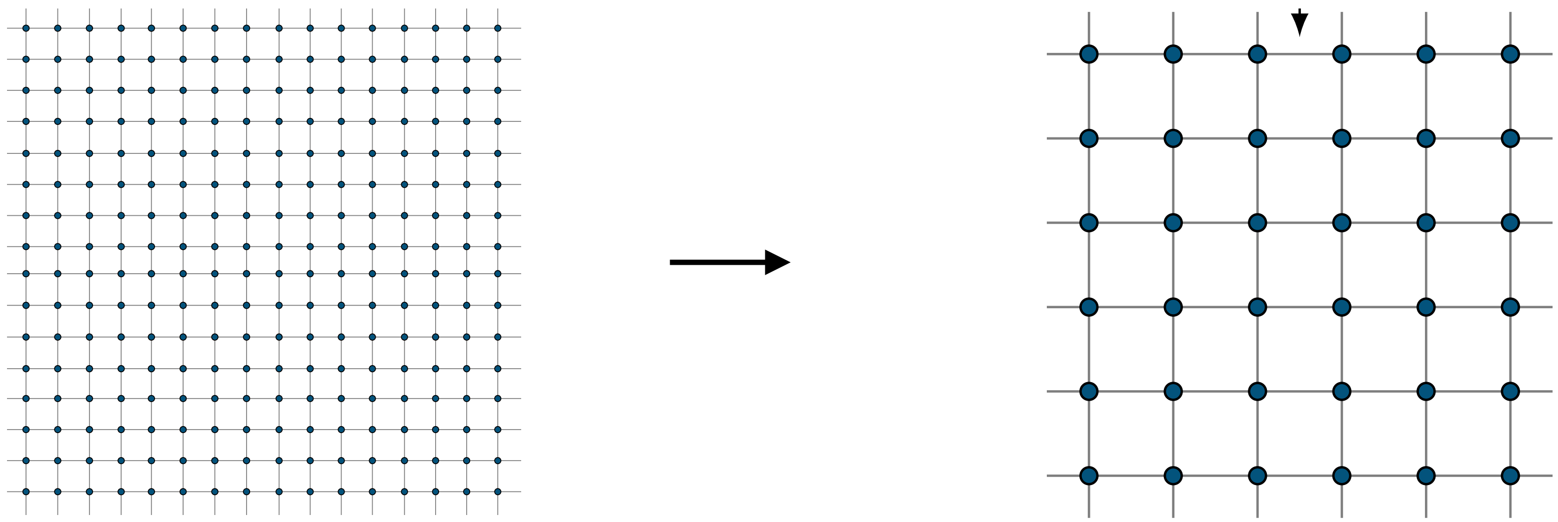
based on arXiv:1601.04744
with William Donnelly (Santa-Barbara)

Alejandro Perez, Daniele Pranzetti, [1507.02573,1611.03668]
Florian Hopfmueller, [1611.03096]
Ugo Moschella, Aldo Riello [to appear]

Quantum gravity

One way to understand the quantum nature of a complex system like Gauge theory or gravity is to break the system into its components.
To introduce a scale and then renormalise [Dittrich, Riello,…]

- Non perturbative Renormalisation

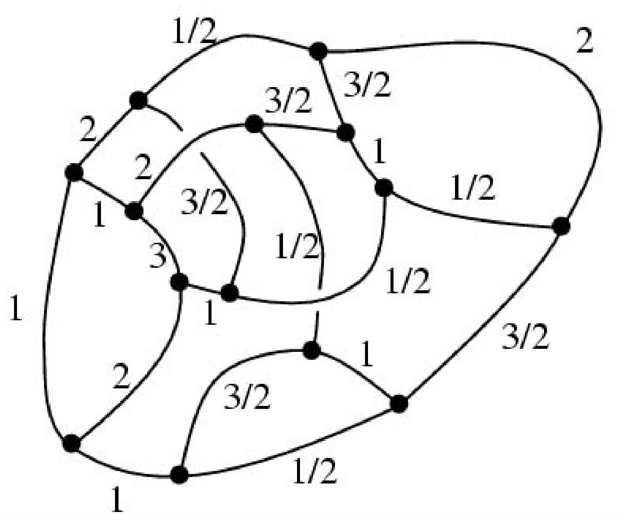


- Discretisation of quantum system needs the breaking of a big system into subsystem (nodes) and the understanding of the nature of their relationship (edges)

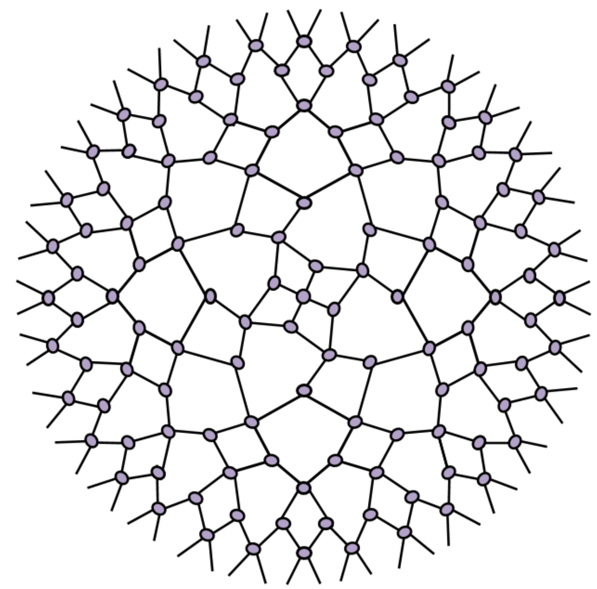
Renormalisation is about the nature of quantum degree of freedom (dof) across different separation scale .
Can we reconcile this with gauge invariance ? Yes
With general covariance ? We don't know.

Quantum gravity

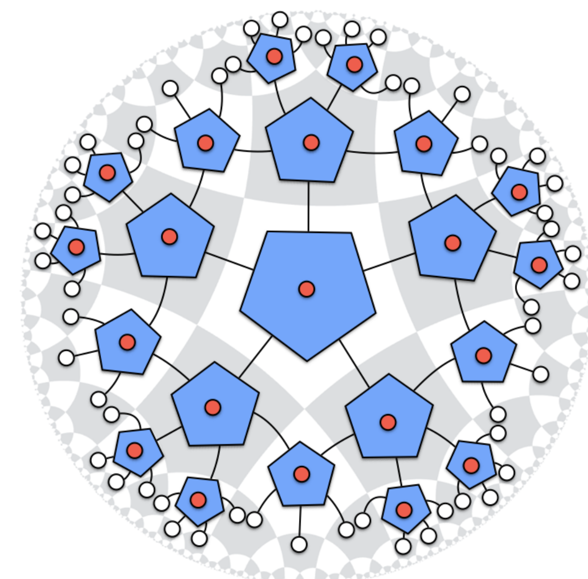
Space is a network of entangled subsystems.



[Rovelli & Smolin]



[Vidal]



[Pastawski, Yoshida, Harlow, & Preskill]

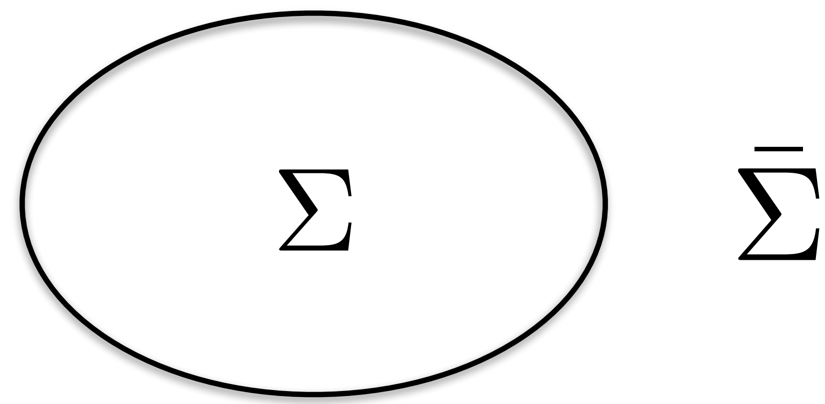
What are the subsystems for gravity?

What are the symmetries governing the gluing of these subsystems?

What is the nature of entanglement between subsystems?

Local subsystems

- Can we can define the notion of local subsystem in Gravity ? That is given a Hilbert space H can we decompose it in terms of Hilbert spaces associated with subregions.
- For a scalar field theory or spin system



$$\boxed{\mathcal{H} = \mathcal{H}_\Sigma \otimes H_{\bar{\Sigma}}}$$

Locality insures that operators associated with different factors commute

$$[A \otimes I, I \otimes B] = 0$$

Technically this is expressed as the triviality of the center:

$$Z_\Sigma = \mathcal{A}_\Sigma \cap \mathcal{A}'_\Sigma = \mathbb{C}$$

No longer true in gauge and gravity!

Local subsystems

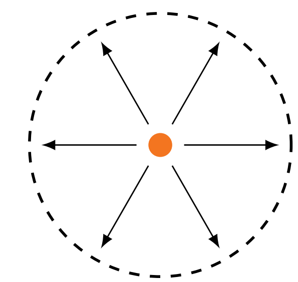
In gauge and gravity states have to satisfy constraints equations:
Gauss Law or
Hamiltonian and diffeomorphism constraints.

$$\boxed{\nabla^2\Phi = 4\pi G\,\rho.}$$

These are elliptic constraints that implies that initial data cannot be specified independently.

$$\mathcal{H} \neq \mathcal{H}_\Sigma \otimes H_{\bar{\Sigma}}$$

Fundamental non locality of gauge invariant observables



More precisely if we denote by $\mathcal{H}_\Sigma$ the Hilbert space of gauge invariant states obtained by acting on the vacua by gauge invariant operator supported on Σ

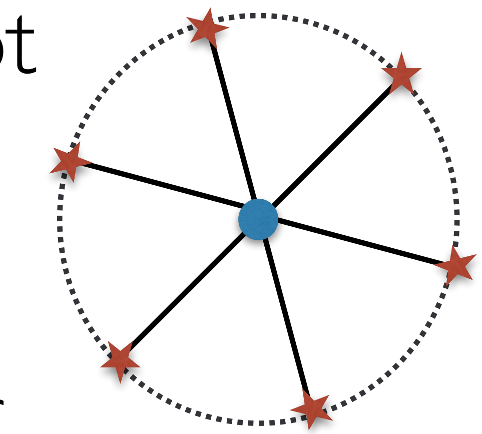
We have that $[O_\Sigma, O_{\bar{\Sigma}}] = 0$

but $\mathcal{H}_{\Sigma\cup\bar{\Sigma}} \supset \mathcal{H}_\Sigma \otimes \mathcal{H}_{\bar{\Sigma}}$ is only a subset.

What are we missing?

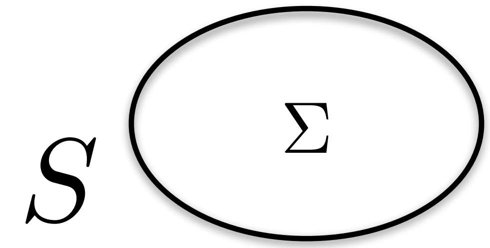
Local subsystems

$$\mathcal{H}_{\Sigma\cup\bar{\Sigma}} \supset \mathcal{H}_{\Sigma} \otimes \mathcal{H}_{\bar{\Sigma}}$$



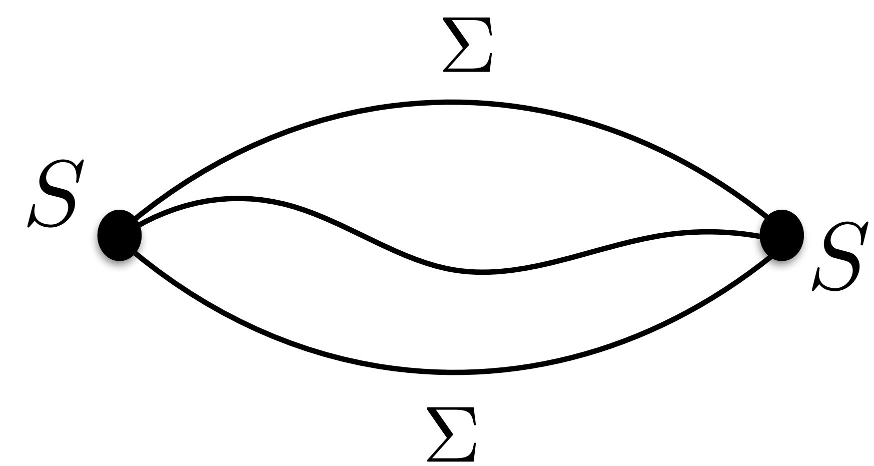
- We are missing gauge invariant observables that are not entirely supported within a region.
- Open end of Wilson lines in gauge theory
- metric degree of freedom that encodes the location of the separation surface in gravity
- The main idea is to understand that the Hilbert space associated with Σ needs to be extended
- We have to give up the commutativity of observables associated with different regions, non-locality is built-in.

Extended Hilbert space



- Given a separating surface S
- Extended Hilbert space

$$\hat{\mathcal{H}}_\Sigma = \mathcal{H}_\Sigma \otimes \mathcal{H}_S$$



New degrees of freedom associated with the boundary $S \longrightarrow (\varphi, X^a)$

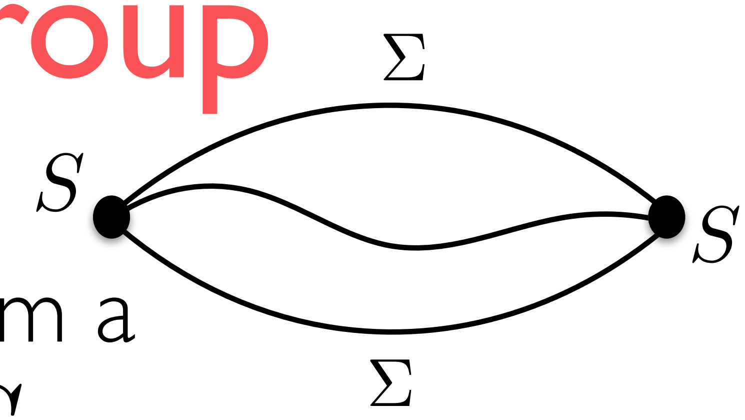
They are a necessity in order to preserve gauge invariance and there is a unique minimal way to introduce them.

They are physical, they are not pure gauge: possess a non-zero gauge invariant charge

eg: edge state in Quantum Hall.

But now we have too many: $\mathcal{H}_{\Sigma\cup\bar{\Sigma}} \subset \hat{\mathcal{H}}_\Sigma \otimes \hat{\mathcal{H}}_{\bar{\Sigma}},$

Boundary symmetry group



L.F, Donnelly

- Key result : The boundary degrees of freedom form a representation of a boundary symmetry group G_S

$$G_S : \mathcal{H}_S \to \mathcal{H}_S$$

We can devise a fusion product that allow us to reconstruct the full Hilbert space from the ones associated with subregions

$$\boxed{\mathcal{H}_{\Sigma \cup \bar{\Sigma}} = \hat{\mathcal{H}}_{\Sigma} \otimes_{G_S} \hat{\mathcal{H}}_{\bar{\Sigma}}}$$

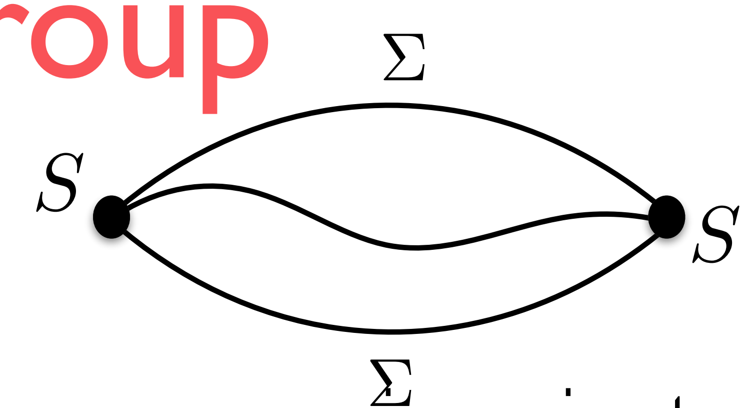
This products entangles physical states across regions. Physical states are singlets, one out of two boundary copies survives

$$(|\Psi\rangle, |\bar{\Psi}\rangle) \simeq (g|\Psi\rangle, g|\bar{\Psi}\rangle)$$

Works both for gauge theories and gravity,
How do we implemented this exactly?

Boundary symmetry group

$$\mathcal{H}_{\Sigma\cup\bar{\Sigma}} = \hat{\mathcal{H}}_{\Sigma} \otimes_{G_S} \hat{\mathcal{H}}_{\bar{\Sigma}}$$



In gauge theory we can put regularize the theory in a gauge invariant manner by putting it on a lattice and we can identify what are the boundary degree of freedom: Non abelian Aharonov-Bohm phases

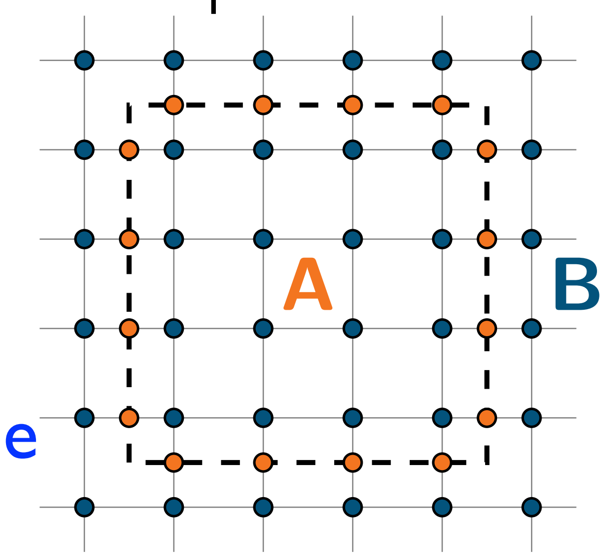
Surface degree of freedom

at every point in the boundary.

$$(E_{\perp}, \varphi)$$

Boundary charge

Boundary phase



$$[E^a_{\perp}, E^b_{\perp}] = f^{ab}{}_c E^c_{\perp}$$

Charge algebra

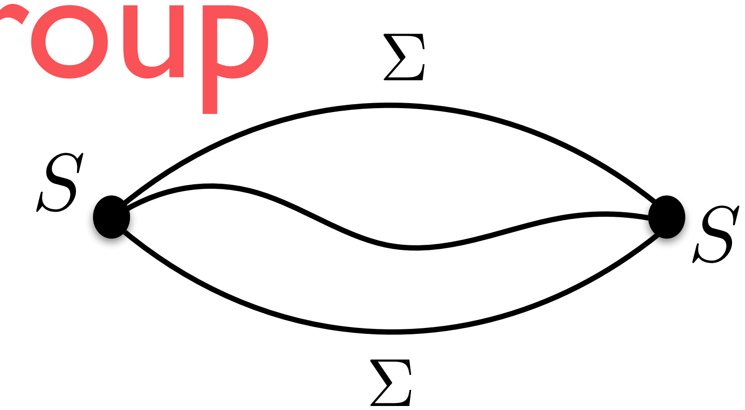
$$[E^a_{\perp}, \varphi] = \varphi T^a$$

Phases carry local charge
= dressing modes

These dof are necessary in order to account for the covariant entanglement entropy. [Donelly,Wall]

Boundary symmetry group

$$\boxed{\mathcal{H}_{\Sigma\cup\bar{\Sigma}} = \hat{\mathcal{H}}_{\Sigma} \otimes_{G_S} \hat{\mathcal{H}}_{\bar{\Sigma}}}$$



at every point in the boundary on a lattice.

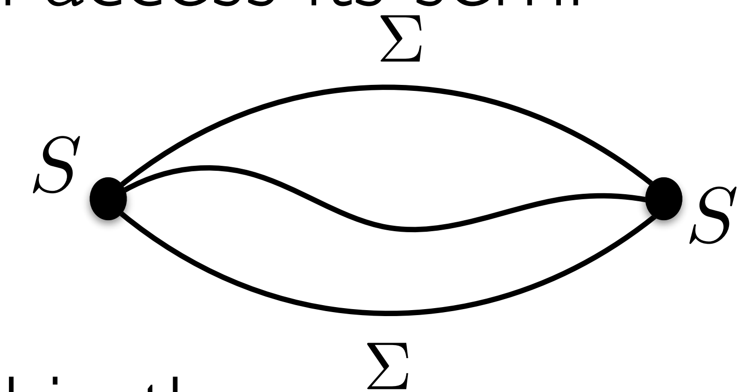
$$[E^a_\perp, E^b_\perp] = f^{ab}{}_c E^c_\perp \qquad [E^a_\perp, \varphi] = \varphi T^a$$

Is there a derivation in the continuum ?

- Discretisation puzzle: In QED We know that (A,E) are conjugate variables and that $$[E,E]=0$$
- The electric field algebra is non-commutative when discretised? Why? Is it a discretization artefact? Can it be derived from the continuum? What does it mean? What is the gravitational analog?
  - After all a discretization is a decomposition into elementary subsystems so a covariant discretization is the answer we are looking for . Can we derive it ?

Extended phase space

In gravity we cannot construct $\mathcal{H}_\Sigma$ yet, but we can access its semi-classical analog:
The Phase space of gauge and gravity $\mathcal{P}_\Sigma$



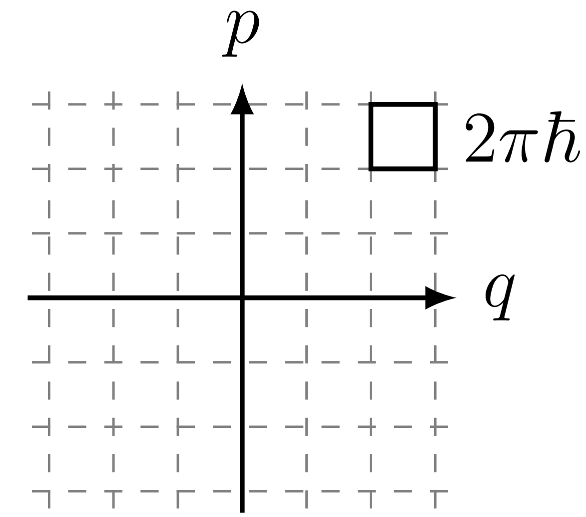
- We are going to show that it has to be extended in the presence codimension 2 boundary $\hat{\mathcal{P}}_\Sigma = \mathcal{P}_\Sigma \times \mathcal{P}_S$
- That there exist non vanishing charges of symmetry $Q_S(V)$
- That the Full Phase space is given by $\boxed{\hat{\mathcal{P}}_{\Sigma\cup\bar{\Sigma}} = \hat{\mathcal{P}}_\Sigma \times_{G_S} \hat{\mathcal{P}}_{\bar{\Sigma}}}$

$$\{Q_S(V), \cdot\} = \delta_V$$

Hamiltonian reduction

- These boundary charges are new type of gauge-invariant observables that represents the soft modes when S = infinity.

Phase space



The key structure is the **symplectic potential**

$$\Theta = p\,\delta q$$

$\Theta(\phi, \delta\phi)$ is a one form on field space. It encodes all of the the structure of classical mechanics and key elements of quantum mechanics

- It relates symmetry to conserved charges
- It tells us what is physical and what is gauge
- It determines the Poisson bracket (Commutator)
- It gives the density of states in Phases spaces
- It determines the Aharonov-Bohm phases

$$\Omega = \delta\Theta$$

$$\dim(\mathcal{H}_\Sigma) = \frac{\mathrm{Vol}_\Omega(\Sigma)}{(2\pi\hbar)^d}$$

$$J(V) := \theta(\phi, \mathcal{L}_V\phi, \phi) - \imath_V L$$

- In Field theory one uses the covariant phase space techniques

- Kijowski, Gawedzki, Crnkovic, Ashtekar, Wald, L.F, Donnelly,….

Gravity symplectic potential

We start with the Lagrangian $L = \frac{1}{2}\epsilon_g R(g)$

with pre-symplectic potential form

$$\boxed{\Theta[g, \delta g] = \frac{1}{2}\nabla_b \left(\delta g^{ab} - g^{ab}\delta g\right) \epsilon_a .}$$

Gauge symmetry : the Noether current associated with diffeomorphisms is a pure boundary term: Gravitational Gauss Law.

$V = V^a \partial_a$ a vector field, infinitesimal generator of diffeomorphisms

$$\boxed{J[V] = C_V + \mathrm{d}Q[V]} \qquad C_V = \epsilon^a G_{ab} V^b$$

The energy associated with a region is a pure boundary term
it is quasi-local

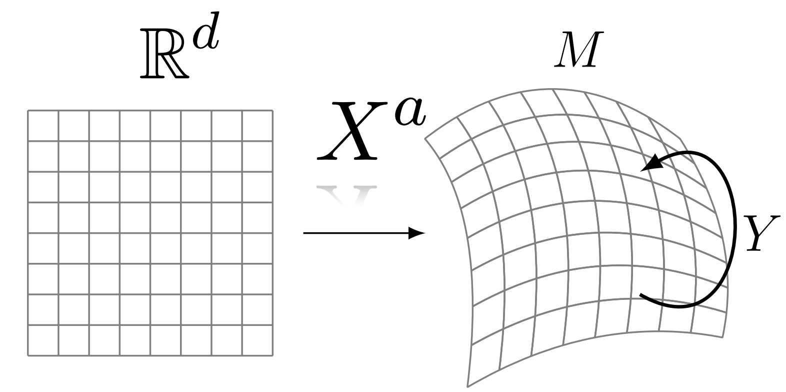
$$J_\Sigma(V) \hat{=} Q_{\partial\Sigma}(V) = \int_{\partial\Sigma} \nabla^{[a} V^{b]} \epsilon_{ab}.$$

Reference Frame

- P1: Θ is not gauge invariant.
- P2: Θ is ambiguous, we can add any boundary term to it.

Colluding these two issues we can resolve one by the other:
We can add boundary degree of freedom such that they restore gauge invariance. Not too few and not too many.

What are these degrees of freedom?:
A choice of boundary reference frame:



gravitational fluid elements
Goldstone' mode

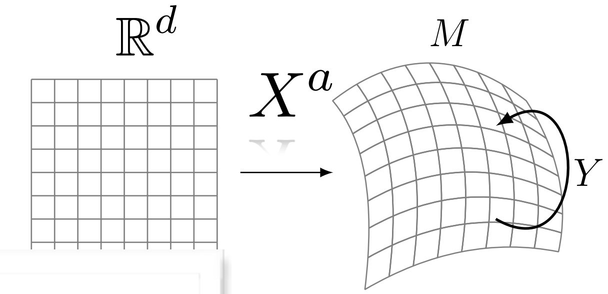
A phase $\varphi : S \to G$ in Yang-Mills
and a map $X^a(x) : U \subset \mathbb{R}^d \to M$ in gravity.

A collection of scalar fields, invertible. They are necessary in order to locates boundaries inside M and parametric the boundary frame.
Boundaries at $X^0 = X^1 = 0$

In gauge theory these would be a choice of boundary section of the principal bundle: Ehresman connection.

Covariant symplectic potential

With these we can construct a gauge invariant symplectic potential



$$\hat{\Theta}_\Sigma[\delta g, \delta_X] = \int_{X(\sigma)} (\theta[\delta g] + i_{\delta_X} L) + \int_{X(s)} Q[\delta_X]$$

$$\boxed{\delta_X^a = \delta X^a \circ X^{-1}}$$

Quasi-local aspect

vector valued differential on field

It modifies the symplectic form by a boundary term only, so only the boundary frame X becomes physical and carries additional degrees of freedom

$$\hat{\Omega}_\Sigma = \Omega_\Sigma(\delta g) + \hat{\Omega}_S(\delta g, \delta_X)$$

This fixes uniquely the boundary ambiguity and restore G invariance

Gauge invariance

Gauge invariance is restored in the presence of a boundary:

$$L_V \hat{\Theta} = 0$$

Restoration of gauge invariance requires
new boundary degrees of freedom.
Not so surprising after all. [Teitelboim, Balachandran et al., Carlip,. . .]

But these have remarkable consequences: It implies the presence of
new boundary symmetries.

These boundary symmetry are the finite analogs of asymptotic
symmetries [BMS, Ashtekar, Strominger, Campaglia. . .]

Gauge versus symmetry

The difference between gauge and symmetry lies in the value associated with the Noether charges.

Gauge: The Noether charge vanish on shell.

$$C[V] = 0$$

phases conjugate to C are:

- Unphysical
- labels redundancies

Diffeomorphisms are gauge

Symmetry : The Noether charge doesn't vanish

$$Q_S(W) \neq 0$$

phases conjugate to H are

- Physical
- labels different physical states

Change of frame are symmetries

Symmetries are not necessarily isometries

Gauge versus symmetry

Diffeomorphisms are gauge

Change of frame are surface symmetries

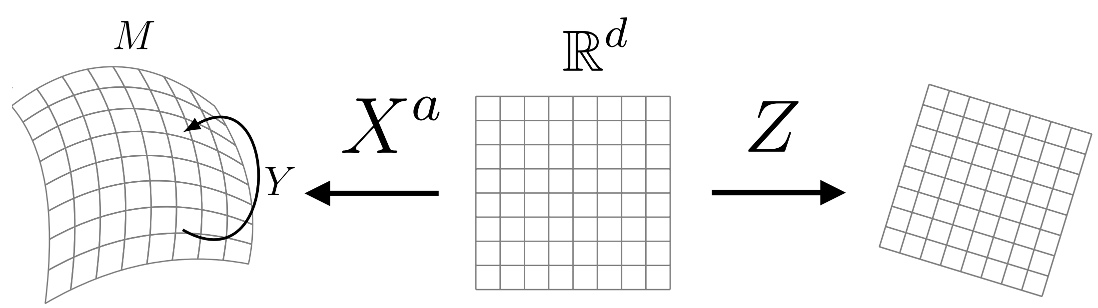
$$\boxed{\begin{array}{l} g \to Y^* g \\ X \to Y^{-1} \circ X \end{array}}$$

Integrated this gives

$$\boxed{\begin{array}{l} g_{ab} \to g_{ab}, \\ X \to X \circ Z \end{array}}$$

left action

$$C[V] = 0.$$



right action

$$Q_S(W) \neq 0$$

These actions commute

$$[\delta_V, \Delta_w] = 0$$

The hamiltonians are gauge invariant observables

$$\delta_V Q_S(W) = 0.$$

what's the boundary symmetry algebra?

Surface symmetry algebra

- **Super/surface boost** : Boosts that transform the normal plane of S in a position dependent manner.
- **Super/surface rotation** : Diffeomorphism of S that move S tangent to itself.
- **Super/surface translation** : Translations of the surface along a normal direction.

$$G_S = \left(\mathrm{Diff}(S) \ltimes \mathrm{SL}(2,\mathbb{R})^S\right) \ltimes (\mathbb{R}^{d-2})^S$$

Gravity in the presence of finite boundary possesses an infinite dimensional symmetry group!
This group is a generalization of the **BMS** symmetry group. $S \to \infty$

LF and Aldo Riello, to appear

Gravity charges

In order to compute the surface charges we introduce frame fields adapted to the entangling surface (x^i, σ^A)

(x^0, x^1) coordinates normal to S
σ^A coordinates tangent to S

$$ds^2 = h_{ij} dx^i dx^j + q_{AB}(d\sigma^A + A_i^A dx^i)(d\sigma^B + A_i^B dx^i)$$

Normal metric — Tangential metric — Normal connection form

$$\boxed{F_A := \frac{\sqrt{q}}{\sqrt{h}} q_{AB} \left(\partial_0 A_1^B - \partial_1 A_0^B + [A_0, A_1]^B\right)}$$

normal curvature generates surface diffeos

$$\boxed{H^i{}_j = \frac{\sqrt{q}}{\sqrt{h}} \left(\epsilon^{ik} h_{kj}\right)}$$

normal metric generates surface boosts

Non commutative symmetry algebra:

$$\{H^i{}_j(\sigma), H^k{}_l(\sigma')\} = (\delta^i_l H^k{}_j - \delta^k_j H^i{}_l)\delta^{(2)}(\sigma - \sigma')$$

$$\{F_A(\sigma), F_B(\sigma')\} = (F_A(\sigma)\partial'_B - F_B(\sigma')\partial_A)\delta^{(2)}(\sigma - \sigma')$$

Conclusion

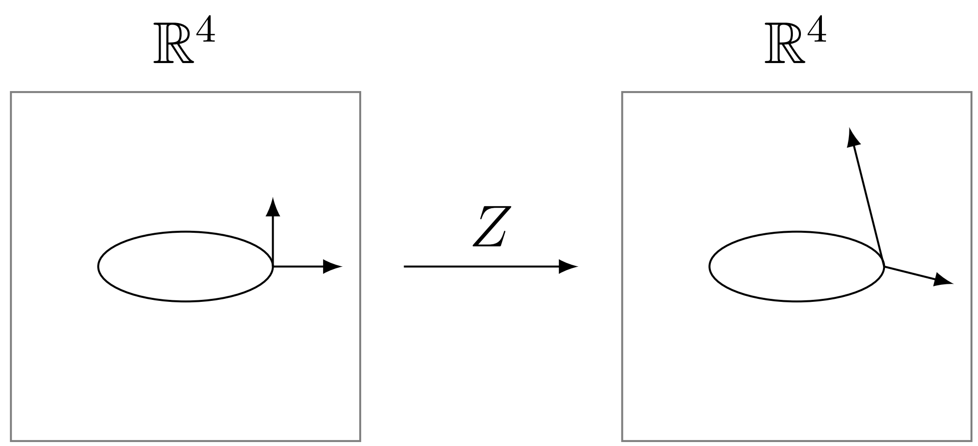
- Gauge invariance in the presence of a boundary requires new dof: These are boundary phases or boundary frames, that generalizes Aharanov-Bohm phases in a FT context.
- They organize themselves in terms of a boundary symmetry algebra
- They explain why and how the link data in a discretization has to be non-commutative.
- This resolves a long standing question: Gauge invariance is not just a redundancy, it is associated with an infinite dimensional symmetry group

recent developments

- In first order gravity, understanding that the combination of edge modes for gauge + diffeos reconstruct the boundary coframe field e. Proof that the boundary symmetry group includes centrally extended Virasoro symmetry: Gravity $\rightarrow$ CFT Pranzetti, Perez
- New understanding of edge modes in CS theory M. Geiller
- new perspective on Inclusion of Null boundaries Hopfmueller, Wieland
- New take on BRST = covariantisation of Bdy symmetry Riello, Gomes
- The target of compactified string is Non-commutative Minic, Leigh

Surface boosts

- **Super/surface boost** : Boosts that transform the normal plane of S in a position dependent manner

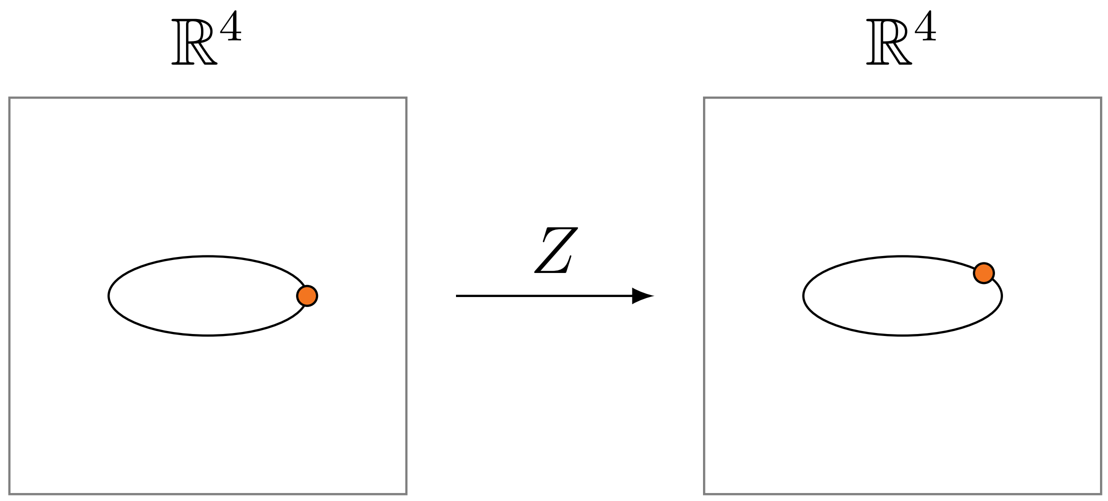


Infinitesimal boost are vector fields $W = \mathrm{d}X(w)$ such that

$$W|_S = 0 \qquad \text{and} \qquad \partial_i W_\perp^j \neq 0.$$

Surface Diffeomorphisms

- **Super/surface boost** : leave the entangling surface invariant but move points along it

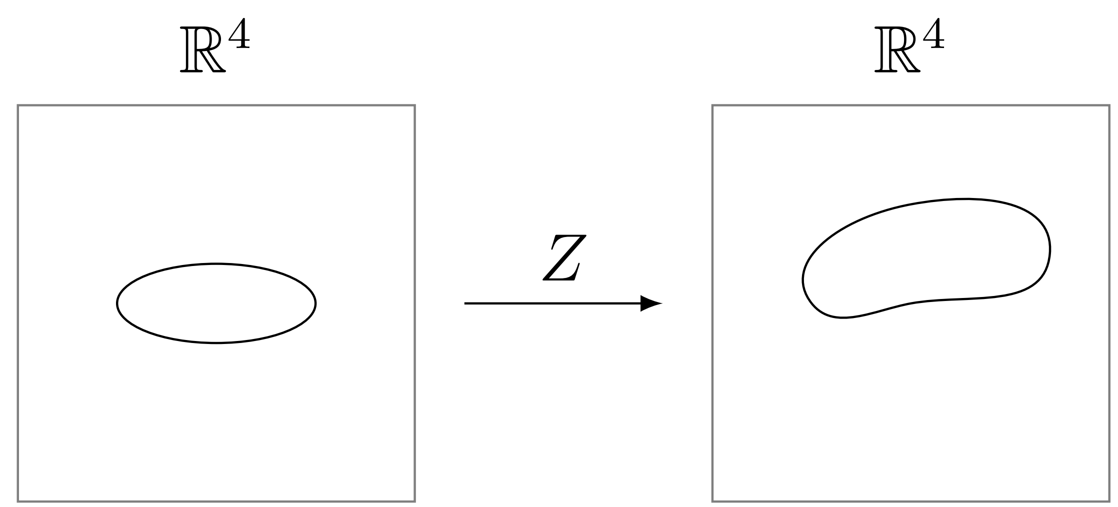


Infinitesimal diffeos are vector fields such that

$$W_\perp|_S = 0 \qquad \text{and} \qquad W_{||}|_S \neq 0.$$

Surface translations

- **Surface translations** : move the entangling surface



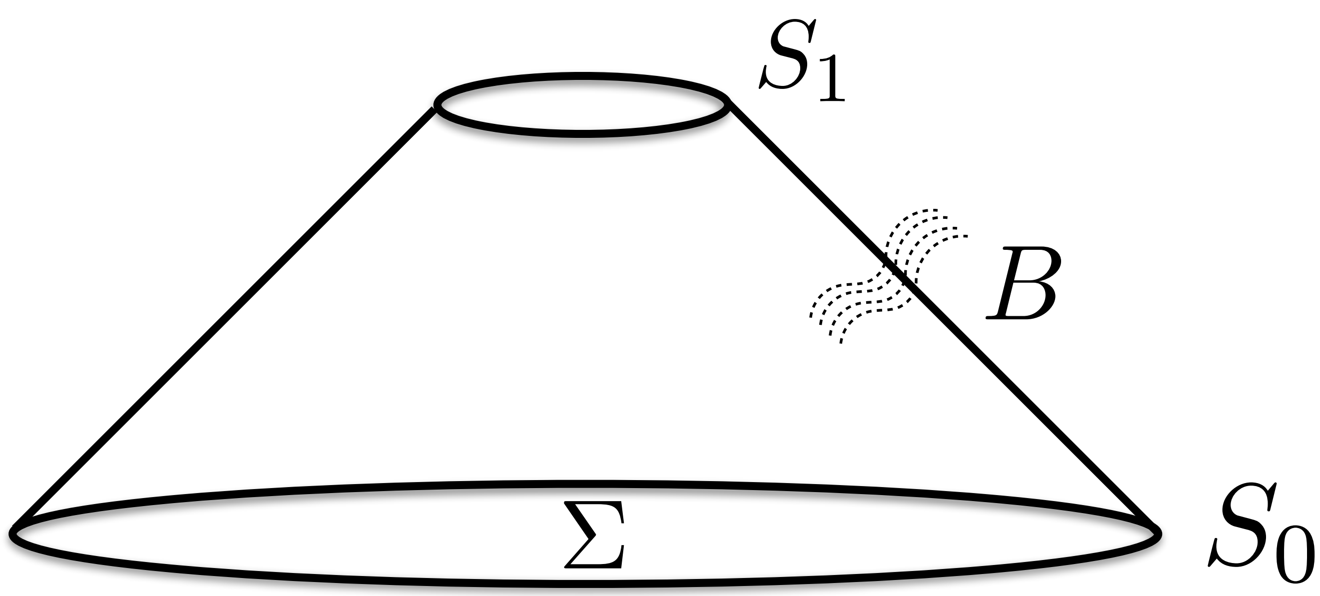
Infinitesimal translations are vector fields such that

$$W_{\perp}|_S \neq 0.$$

They are not canonical symmetry of phase space when there is symplectic flux through the boundary.

Observables or not?

The boundary degrees of freedom are definitely observables.



$$\Delta R[W_{||}] =$$ Gravitational wave flux along null B

L.F, F. Hopfmuller

Memory effect

A gravitational flux will induce a change of frame (local boost, rotation or translation) that registers in the boundary gravitational charges

Symmetry and degree of freedom

Do we understand the nature of degree of freedom in the context of gauge symmetries? Not really, there are many paradoxes…

- Black Hole information paradox.
- Firewall paradox.
- Is the newtonian potential classical? Is the vacuum unique?

- All these paradoxes are about what is gauge and what is symmetry? Are observables observer dependent or observer independent?

- Discretisation puzzle: In QED We know that (A,E) are conjugate variables and therefore [E,E] = 0. When we discrtise however we chose instead that

$$[E_e^a, E_{e'}^b] = i\delta_{ee'}C^{ab}{}_c E_e^c.$$

- The electric field algebra is non-commutative? Why? Is it a discretization artefact? Can it be derived from the continuum? What does it mean? What is the gravitational analog?