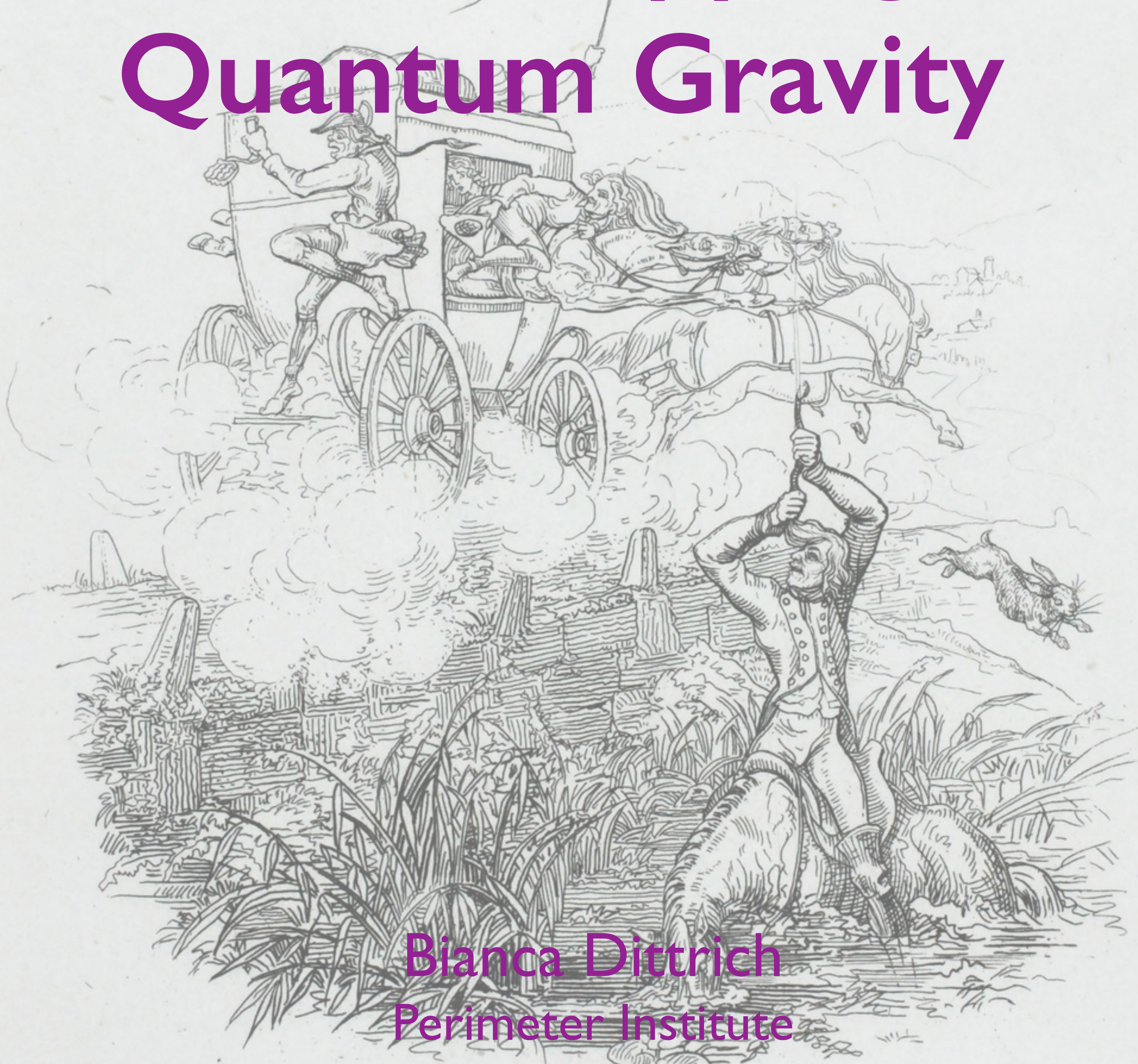
Bootstrapping Quantum Gravity



Bianca Dittrich
Perimeter Institute

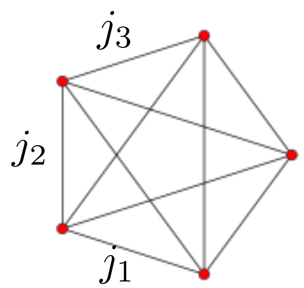
Loops 2017

Aim:

A feasible approximation scheme in which loop quantum gravity dynamics can be accessed reliably.

Constructing amplitudes

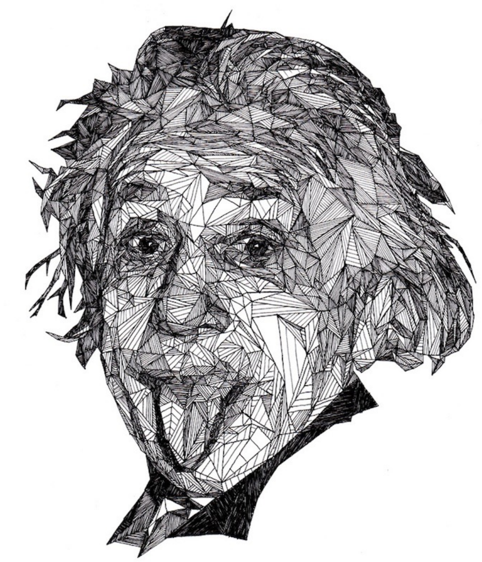
Simplex amplitude



$$\longrightarrow \mathcal{A}_\sigma(j)$$

[spin foams:

Baratin, Barrett, Crane, Dupuis, Engle, Freidel, Kaminski, Krasnov, Lewandowski, Livine, Oriti, Reisenberger, Rovelli, Speziale, ...]



In itself not a complete proposal for a theory of quantum gravity.

How to get triangulation invariant amplitudes
for more complicated boundary states?

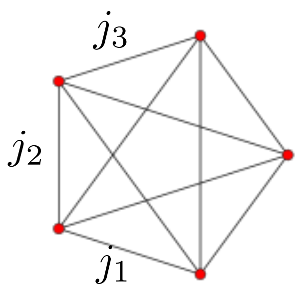
Constructing amplitudes

[spin foams:

Baratin, Barrett, Crane, Dupuis, Engle, Freidel, Kaminski, Krasnov, Lewandowski, Livine, Oriti, Reisenberger, Rovelli, Speziale, …]

[but applies also to many other

Simplex amplitude



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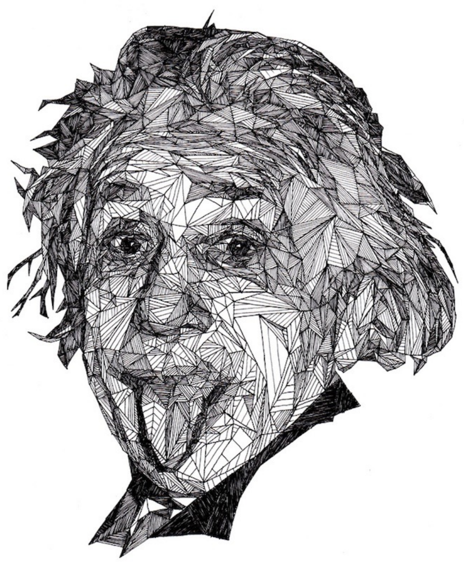

In itself not a complete proposal for a theory of quantum gravity.

How to get triangulation invariant amplitudes
for more complicated boundary states?

Refining
(only bulk?) triangulation

Summing
over (only bulk?) triangulations

e.g. GFT, see S. Carozza's talk

Key question: What are actually good boundary states?

Key: What are good boundary states?

Calculating amplitudes (or solving constraints) should be feasible.

Boundary states should actually be relevant for the description of interesting processes.
(Most states are not.)

The dynamics should determine a notion of ‘best boundary states to use’.
(that is identify relevant observables.)

The consistent boundary framework

- A framework to ‘solve quantum gravity’: construction of consistent amplitudes
- … which define a continuum dynamics
- … and can be computed in a reliable approximation scheme.

- Provides renormalization framework for background independent theories
- … with organizational principle for boundary states

How to formulate a consistent theory of quantum gravity (via amplitudes)?

The consistent boundary framework

- A framework to 'solve quantum gravity': construction of consistent amplitudes
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- … and can be computed in a reliable approximation scheme.

- Provides renormalization framework for background independent theories
- … with organizational principle for boundary states

Abandons notion of fundamental building blocks:
In diffeomorphism invariant interacting theories non-local amplitudes are unavoidable.
Opens many questions: e.g. What does triangulation invariance mean for such non-local amplitudes?

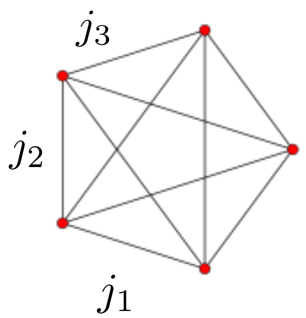
Background dependent truncation methods based on locality are not applicable.
(Graph distance does not agree with metric distance.)

How to formulate a consistent theory of quantum gravity (via amplitudes)?

- Consistency
- Feasibility

Consistency: motivation

Should (boundary) spins be small or large? Is it UV or IR?



Consistency: motivation

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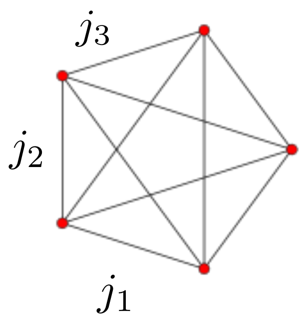


Boundary data can describe a very small or an arbitrary large simplex.
In particular a macroscopic piece of geometry.

For large boundary data we rather describe IR physics:
The amplitude should be an effective amplitude which takes into account fluctuations "on smaller scales" - which would appear if we would refine the boundary.

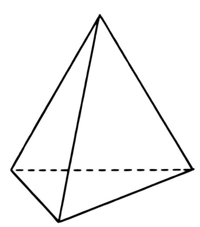
$\mathcal{A}_\sigma(j)$ should describe both, small and large scales consistently: large scales should be determined from short scales

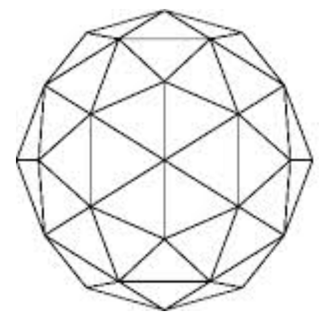
▶Need to bootstrap a consistent amplitude.
▶Need to formulate the consistency conditions.

Approximation scheme for effective amplitudes

1. Start with amplitude for simplest building block.

2. Defines amplitude for more complicated boundary via gluing principle.



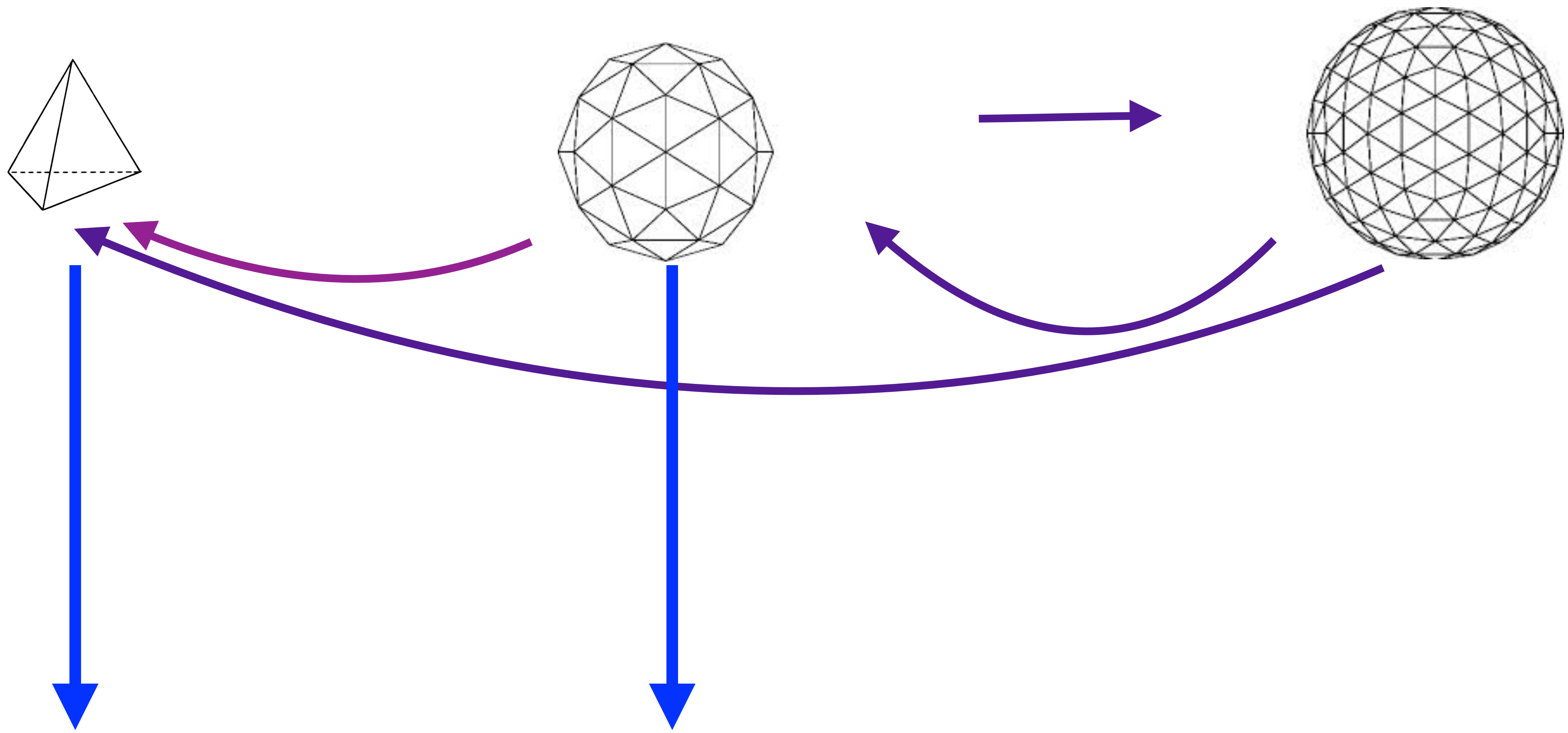


3. Find a dynamically preferred truncation back to coarse boundary. This allows an iterative coarse graining scheme leading to a fixed point.

Improved amplitude

Approximation scheme for amplitudes

4. Take more and more complicated boundaries into account.

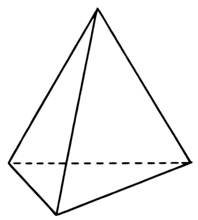


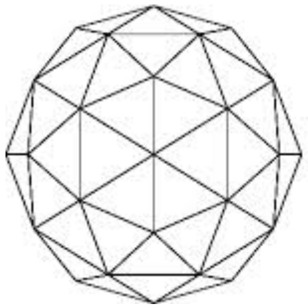
Improved amplitude

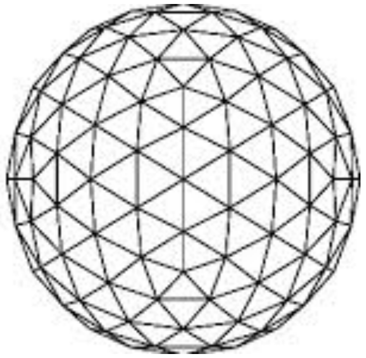
Improved amplitude

Approximation scheme for amplitudes

Seed: Initial (a priori inconsistent) amplitudes







Improved amplitude

Improved amplitude

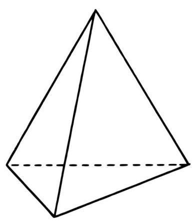
Improved amplitude

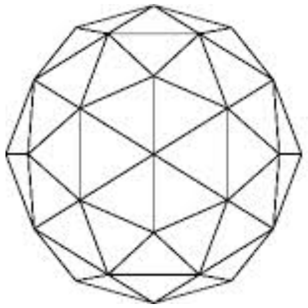
Determine consistent amplitudes with more and more complicated boundaries.

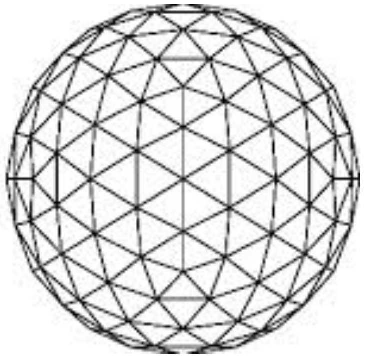
Amplitude changes across all scales (complexity classes)!

Approximation scheme for amplitudes

Seed: Initial (a priori inconsistent) amplitudes







Improved amplitude

Improved amplitude

Improved amplitude

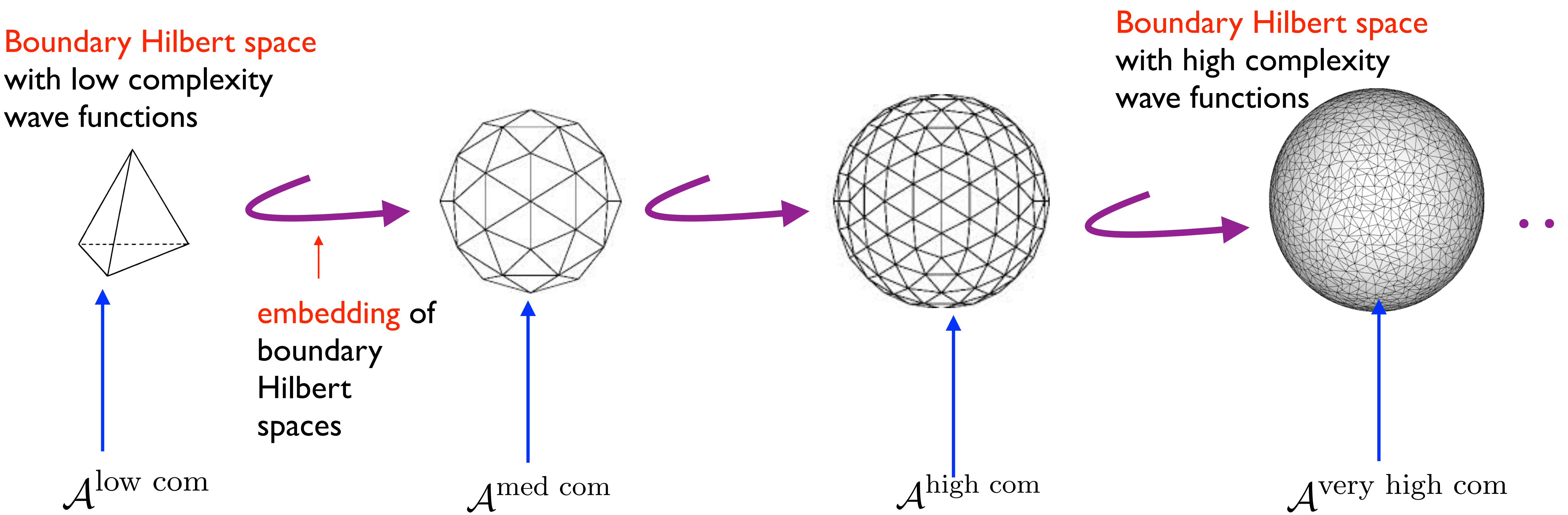
Determine consistent amplitudes with more and more complicated boundaries.

Amplitude changes across all scales (complexity classes)!

Effective amplitude:
takes into account
(arbitrary) refined
boundaries.

Consistency conditions

[BD NJP 12, BD 14 (Review)]



Consistency conditions

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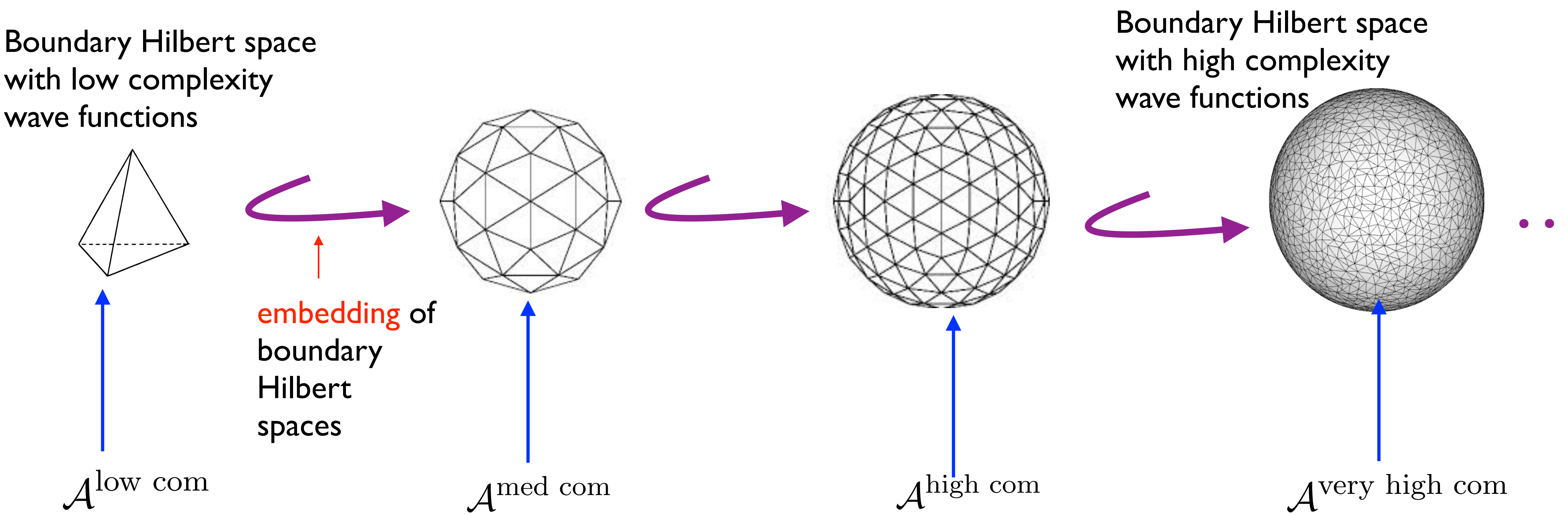

For the amplitudes we demand the consistency conditions:

$$\mathcal{A}^{\text{low com}}(\psi_{\text{low com}}) \hookleftarrow \mathcal{A}^{\text{med com}}(\psi_{\text{med com}}) \xhookleftarrow[\text{(pullbacks of embeddings)}]{\text{restricts to}} \mathcal{A}^{\text{high com}}(\psi_{\text{high com}})$$

The consistent boundary formulation

[BD NJP 12, BD 14 (Review)]

For the amplitude we demand consistency conditions:

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If this holds for arbitrary refinements: defines continuum amplitudes.

A (complete) family of consistent amplitudes defines a theory* of quantum gravity.

* Corresponds to a complete renormalization trajectory,

with scale given by complexity parameter.

A family of effective amplitudes

$$\begin{tikzcd} \mathcal{A}^{\text{low com}}(\psi_{\text{low com}}) & \mathcal{A}^{\text{med com}}(\psi_{\text{med com}}) \arrow[l] & \mathcal{A}^{\text{high com}}(\psi_{\text{high com}}) \arrow[l, "\text{restricts to}"'] \arrow[ll, bend left] \end{tikzcd}$$

Amplitudes for simplest building blocks are effective
amplitudes: do include all effects from higher modes.

Opposite to the traditional view:
Simplest building blocks defines ‘fundamental’ amplitude.

Here: Simplest building blocks defines simplest process, e.g.
homogeneous boundary states.
Corresponds rather to IR limit.

What do we gain?

- Provides criteria for a consistent quantum gravity dynamics - consistent over all scales.

- Provides definition of physical vacuum as simplest physical state. [BD, Steinhaus NJP 2014]

- Unifies discrete and continuum formulations,
  and in this way addresses
  - the issue of discretizations a priori breaking diffeomorphism symmetry [Bahr, BD, CQG 2009]
  - discretization or triangulation dependence [Bahr, BD, PRD 2009, Bahr, BD, Steinhaus PRD 11]
  - necessity for non-local amplitudes in (3+1)D [BD, Kaminski, Steinhaus, CQG 2014]
  - discretization ambiguities. [Bahr, BD, Steinhaus PRD 11]

- Consistency
- Feasibility

The consistent boundary framework

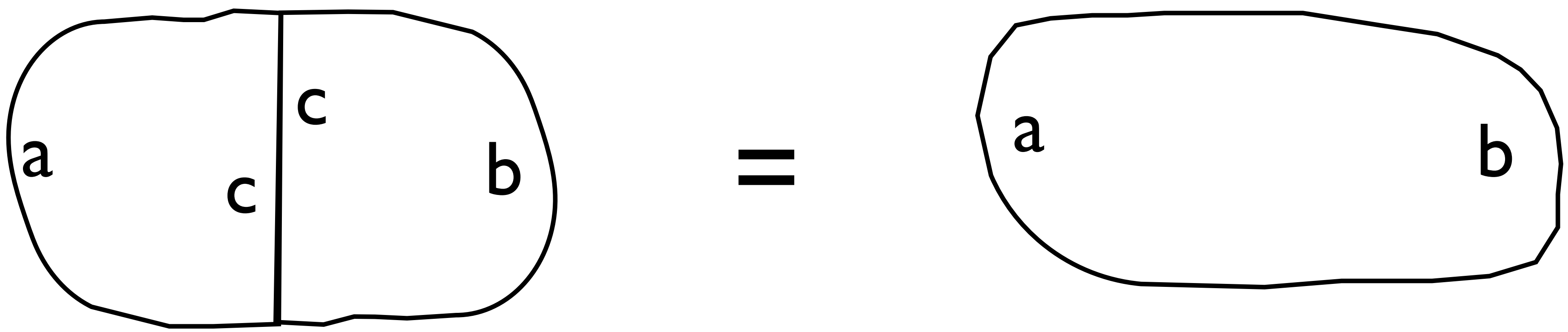
- A consistent family of effective amplitude can be obtained in an iterative approximation scheme where:

  - one starts with simplest state (low complexity) and NOT at highest energy scales.

    Definition of new kinematical `low energy' vacuum states: Marc Geiller's talk

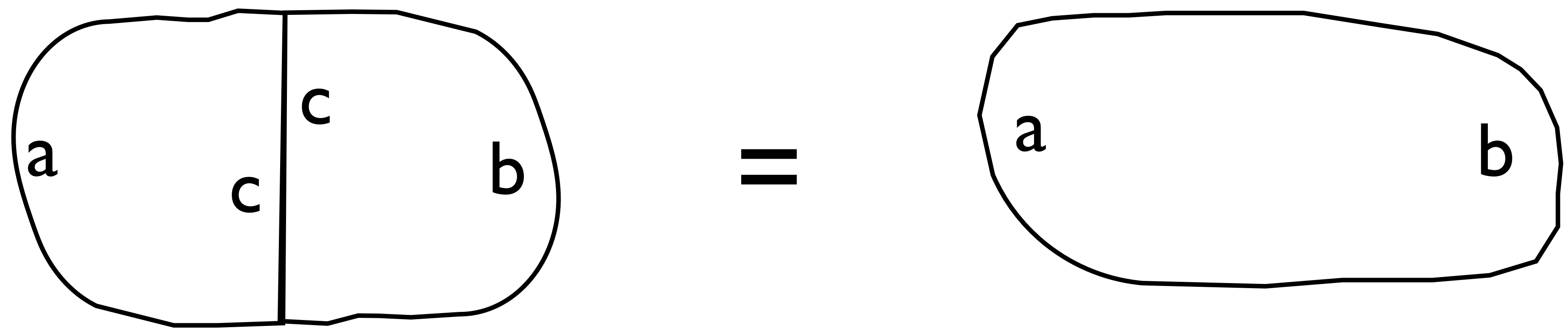
  - the truncation is determined by the dynamics of the system and not chosen by hand.

Dynamics sets conditions for the embedding map



$$\sum_c \mathcal{A}(a \cup c)\mathcal{A}(c \cup b) = \mathcal{A}_{\text{finer}}(a \cup b)$$

Dynamics sets conditions for the embedding map



$$\sum_c \mathcal{A}(a\cup c)\mathcal{A}(c\cup b) \;=\; \mathcal{A}_{\text{finer}}(a\cup b)$$

(transpose of)
embedding map

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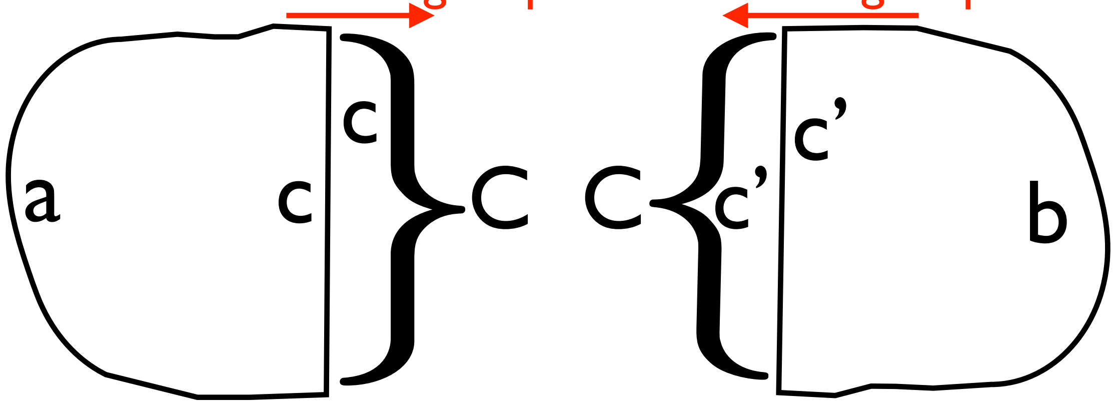


truncation,
ideally just a
compression

Choose embedding maps,
so that truncation error
is minimized.

$$\sum_C\sum_{c'}\sum_c \mathcal{A}(a\cup c)\iota^t(c\lhd C)\;(C\rhd c')\iota^t\mathcal{A}(c'\cup b) \;=\; \mathcal{A}_{\text{trunc}}(a\cup b)$$

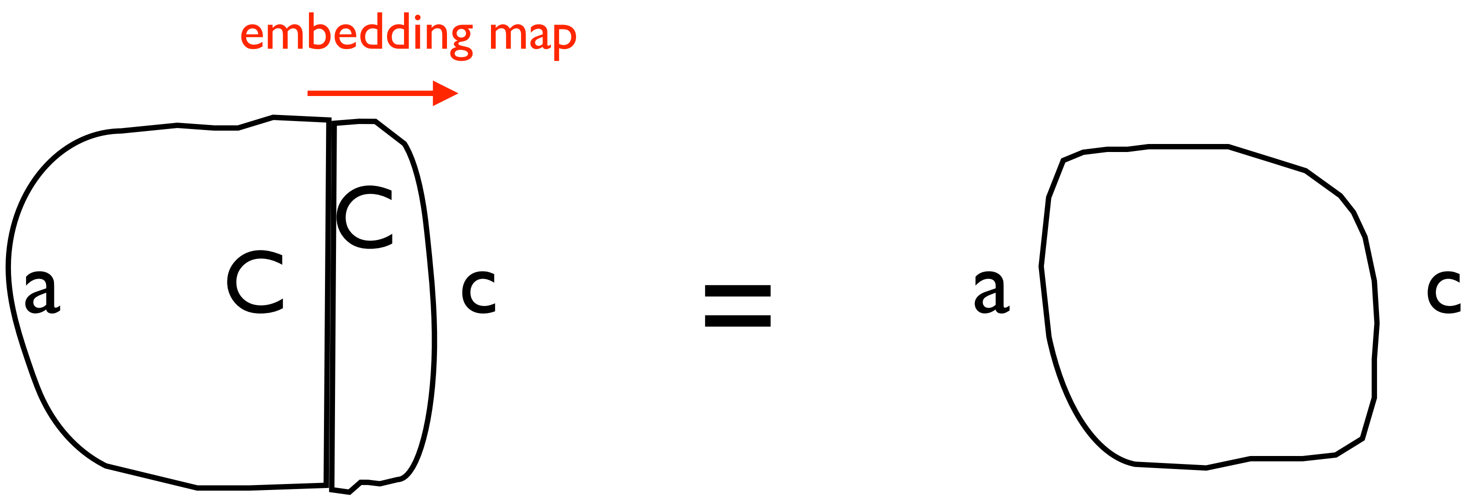
Identifies the most relevant (coarse grained) boundary observables.

A stronger condition

[BD, Hoehn 2011-13, Hoehn 2013]

(particularly relevant for quantum gravity)

[BD, Steinhaus NJP 2014]



$$\sum_C \mathcal{A}(a \cup C)\ \iota(C \triangleright c) \ = \ \mathcal{A}_{\text{finer}}(a \cup c)$$

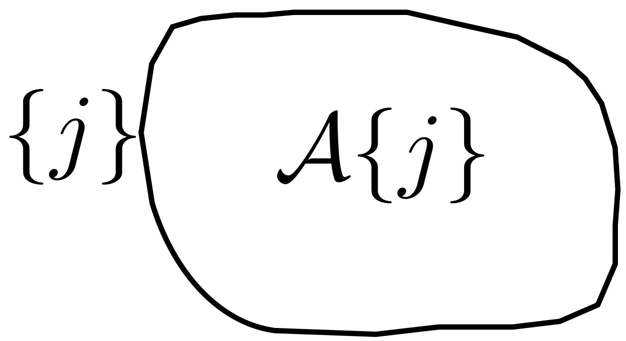
In this case embedding map coincides with (vacuum) amplitude.

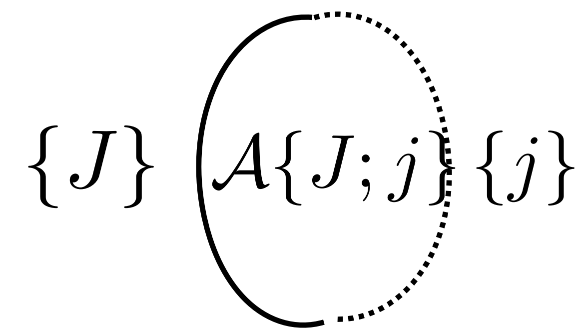
Thus embedding map is clearly decided by the dynamics of the system.

Example: BF theory

[see also Marc Geiller's talk]

Take spin networks as boundary states.





Amplitude given by "spin network evalutaion".

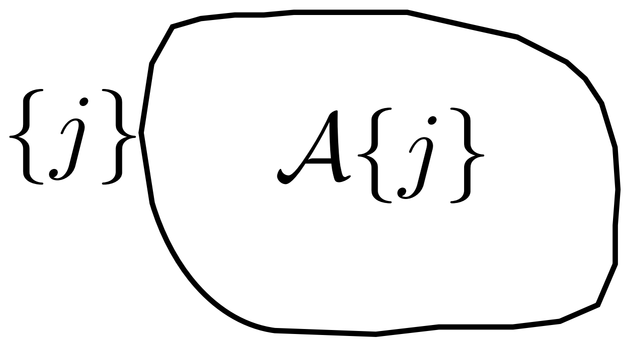
Embedding map given by amplitude itself

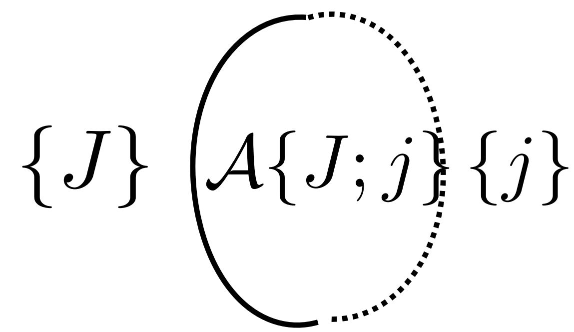
Consistency conditions follow from triangulation invariance of partition function.

Example: BF theory

[see also Marc Geiller's talk]

Take spin networks as boundary states.





Amplitude given by "spin network evalutaion".

Embedding map given by amplitude itself

Consistency conditions follow from triangulation invariance of partition function.

But spin network states are quite bad in "compressing" BF amplitudes!

Another huge disadvantage: gauge invariant spin networks not preserved under coarse graining.

[see Etera Levine's talk]

Fusion and Curvature basis:

[see Clement Delcamp's and Aldo Riello's talk]

(a new gauge invariant alternative to SNW)

[Delcamp, BD, Riello JHEP 2016, JHEP 2017, BD 2017, BD, Delcamp to appear]

(2+1)D:

Fusion basis labeled

by (Drinfeld double) representations

$$\mathcal{A}\{\rho\} = \delta_{\{\rho\},\mathrm{triv}}$$

Embedding map given

by amplitude itself:

$$\rho_{\mathrm{finer}} \to \mathrm{triv}$$

(3+1)D:

Curvature basis labeled

by class angles

(for quantum group: spins)

$$\mathcal{A}\{j\} = \delta_{\{j\},\mathrm{triv}}$$

Crane-Yetter amplitude much simplified

[BD 2017]

Fusion and Curvature basis:

[see also Clement Delcamp's and Aldo Riello's talk]

[related: Florian Girelli's talk]

(a new gauge invariant alternative to SNW)

[Delcamp, BD, Riello JHEP 2016, JHEP 2017, BD 2017, BD, Delcamp to appear]

Expect this basis to be much more practical also for spin foam coarse graining:

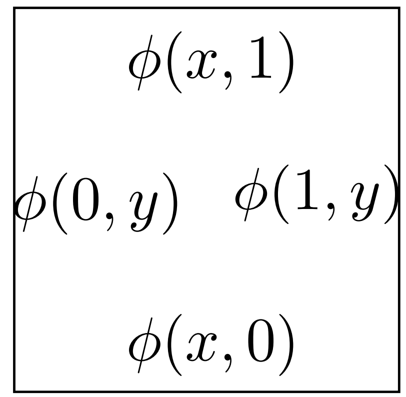
- Fusion basis has a built in coarse graining scheme.
- Much less memory required than for SNWs, in particular for configurations near flatness.
- Labels allow immediate insight into geometric observables: curvature and torsion

Dynamics decides on suitable boundary states

and on how to coarse grain or refine such states.

Example: Free scalar field

(massless on 2D Euclidean space)



Boundary value problem on square

can be solved in continuum.

Solution constructed as superposition of

-piecewise linear part (zero mode)

-Fourier modes $k>0$, for each side separate!

[Minkowski space: Asante, BD to appear: embedding can be based on piecewise linear decomposition only.]

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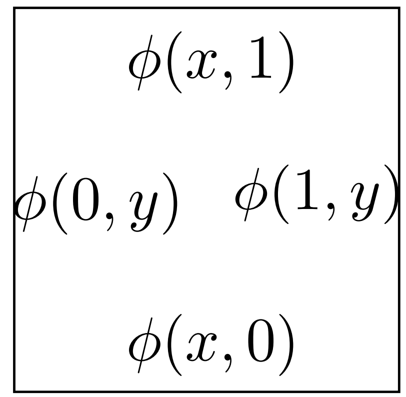


Boundary value problem on square

can be solved in continuum.

Solution constructed as superposition of

-piecewise linear part (zero mode)

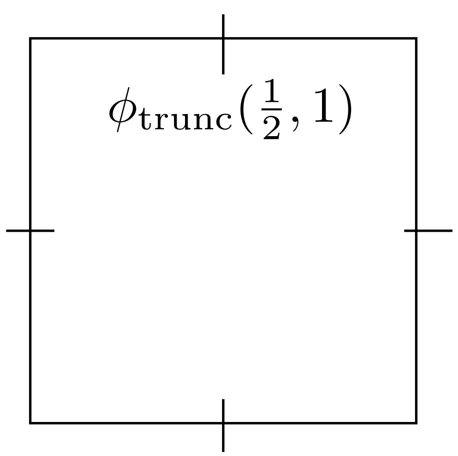
-Fourier modes k>0, for each side separate!

Fourier mode decomposition

of boundary values for each side.

Project all higher modes to zero.

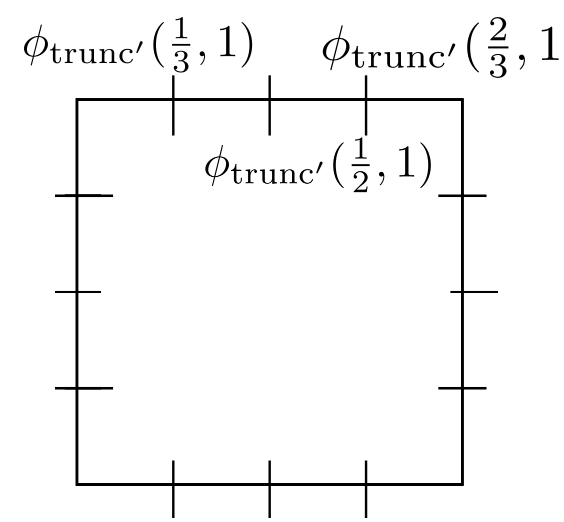
Go back to real space.



Add higher modes

with zero amplitude

(in vacuum state).



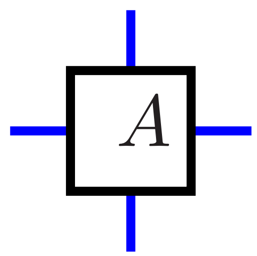
Embedding maps are non-local. (Suprise: sides are decoupled for non-zero modes.)

Fourier modes do indeed provide a useful scale parameter for free theories.

[Minkowski space: Asante, BD to appear: embedding can be based on piecewise linear decomposition only.]

Tensor network renormalization methods

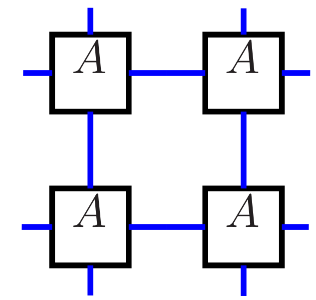
Identifying dynamically preferred boundary states and preferred coarse grainings.



Amplitude of a disk region

with edges representing

boundary data.



Amplitude for a region

with finer boundary

data via gluing.

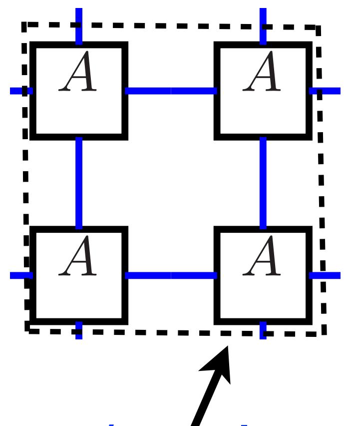
How to compare these?

Need to 'coarse grain' boundary data.

Tensor network renormalization methods

Identifying dynamically preferred boundary states.

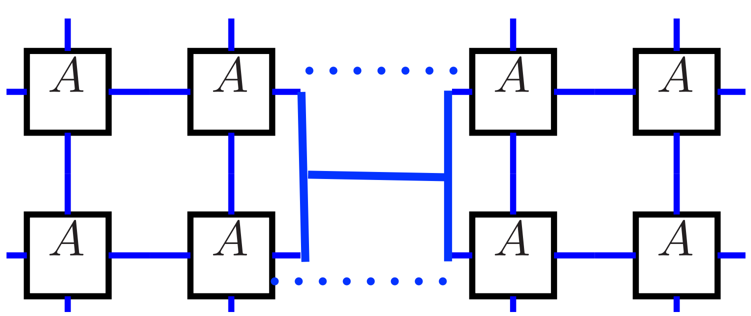
Coarse grain



bare/initial amplitude
depending on four variables

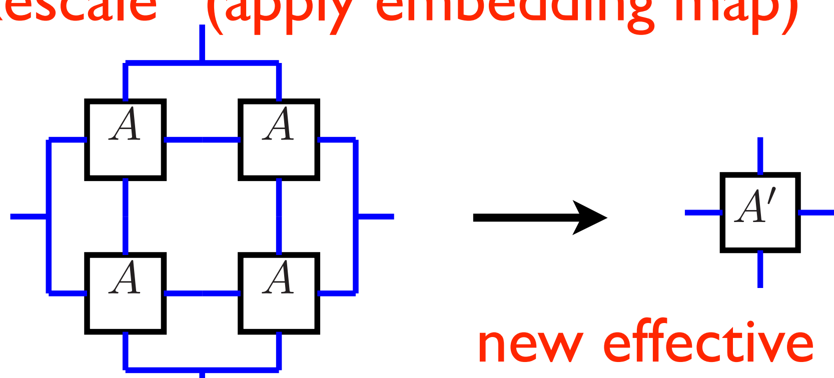
Contract initial amplitudes (sum over bulk variables). Obtain "effective amplitude" with more boundary variables.

Truncate /determine embedding map



Find an approximation (embedding map) that would minimize the error as compared to full summation (dotted lines). For instance using singular value decomposition, keeping only the largest ones.
Leads to field redefinition, and ordering of fields into more and less relevant.

"Rescale" (apply embedding map)

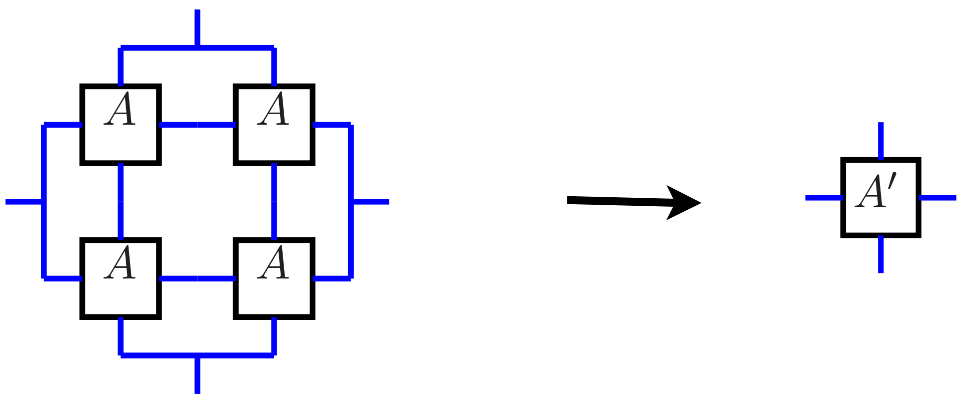


new effective amplitude

Use embedding maps to define coarse grained amplitude with the same (as initial) number of boundary variables.

Tensor network renormalization methods

Identifying dynamically preferred boundary states.



Renormalization flow in a huge space of models:
(almost) arbitrary tensors

Number of (coupling) parameters: $\sim \chi^4$

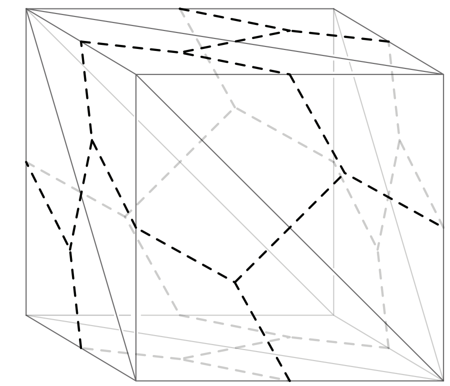
Advantage: Do not make assumptions about form of amplitudes.
(But allows a test of such assumptions.)

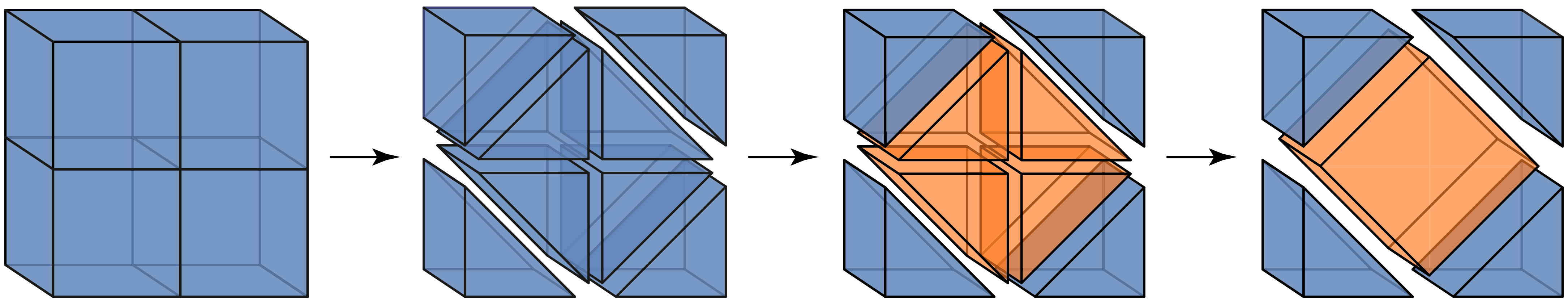
Decorated tensor networks

[BD, Mizera, Steinhaus, NJP (Best of 2016)]

Allow for more flexibility in type of boundary data, e.g. SNW for lattice gauge theories.

Flow: in space of arbitrary amplitude functions of fixed boundary data structure.





Can deal with non-Abelian lattice gauge models and spin foams. 3D: [Delcamps, BD, 2016]

Coarse graining with fusion basis and curvature basis.

Expect these bases to be much more effective (computer resource saving).

Immediate access to interesting observables: curvature and torsion.

Summary

Aim:

A feasible approximation scheme in which the dynamics can be accessed reliably.

Key:

Truncation determined by dynamics.

Type of boundary states determined by dynamics.

Let to / Motivated / Related to the development of lots of techniques:

New vacua for LQG: providing better starting points for coarse graining process. Marc Geiller's talk

Self-dual, doubly-finite version of LQG.

Fusion basis and Curvature basis. Clement Delcamp, Aldo Riello's talk

Coarse graining (Regge) geometries. Seth Asante's talk

Tensor network renormalization for lattice gauge theories and spin foams. Sebastian Steinhaus' talk

Holographic properties of (3D) LQG. Etera Livine's talk, Christophe Goeller's talk

Expect much more!