

Problems on General Relativity - a special set

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Problem 1. The Kerr space-time of zero mass becomes the Minkowski space-time at any value of the rotation parameter a , but looking at the appropriate metric tensor, namely

$$g = -cdt^2 + \frac{r^2 + a^2 \cos^2 \theta}{r^2 + a^2} dr^2 + (r^2 + a^2 \cos^2 \theta) d\theta^2 + (r^2 + a^2) \sin^2 \theta d\phi^2 \quad (1)$$

it may not be obvious. Find the coordinate transformation that turns the metric tensor g into the Minkowski one written in terms of the spherical coordinates.

Problem 2 Prove that every map

$$f : M \rightarrow M \quad (2)$$

of Minkowski spacetime (M, g) into itself, such that

$$g(f(m') - f(m), f(m') - f(m)) = g(m' - m, m' - m) \quad (3)$$

for every pair of points $m, m' \in M$, is affine (that is it is a combination of a translation and a linear map with respect to a fixed $m_0 \in M$).