Problems on General Relativity: 4

December 13, 2020

Problems. Given a metric tensor g and a co-tangent frame e_0, e_1, e_2, e_3, calculate the torsion free, metric connection and its curvature:

1. $$g = -dt^2 + a^2(t)(dx^2 + dy^2 + dz^2)$$
$$e_i = dt, dx, dy, dz$$
where you may use the formula derived at Exercises 8.

2. $$g = g_{ij} e^i e^j, \qquad g_{ij} = \text{const}$$
knowing that
$$de^i = c^i{}_{jk} e^j \wedge e^k.$$

3. $$g = -dt^2 + (e^1)^2 + (e^2)^2 + (e^3)^2, \qquad e_0 = dt$$
knowing that
$$de^1 = e^2 \wedge e^3, \;\; de^2 = e^3 \wedge e^1, \;\; de^3 = e^1 \wedge e^2.$$